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WALSH'S BUSINESS ARITHMETIC

JOHN HENRY WALSH

GIFT OF
Miss E. S. Adams

WALSH'S
BUSINESS ARITHMETIC

BY

JOHN H. WALSH

ASSOCIATE SUPERINTENDENT OF SCHOOLS, THE CITY OF NEW YORK

THE GREGG PUBLISHING COMPANY

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PREFACE

In this work the author has kept in mind the needs of boys and girls that have taken up a commercial course. The latter generally requires the completion of the arithmetical portion by the end of the first year of high school, and care has been taken to keep the problems within the capacity of pupils at this stage.

Section I, Recording Business Transactions, presents briefly the clerical tasks likely to confront boys and girls upon their entrance into the business world; the calculations they are expected to make, the simple accounts they may be required to keep, the commoner "forms" they will use, etc.

In Section II, Business Calculations, are given computations in percentage, trade discount, interest, and bank discount, made in the ways employed by business men.

The arrangement that assigns Numbers and Processes to Section III does not mean that practice in these topics should be deferred until pupils have completed the two previous sections. It is expected that the teacher will take from this portion of the book oral exercises for short daily drills, and abstract written ones weekly, for a longer period, to give facility and accuracy.

In this section the pupil is shown the business way of reading numbers, some short cuts used in the several

processes, methods of combining two operations, and the manner in which results should be tested.

In the remaining four sections are presented the arithmetical treatment of conditions arising in the various departments of business, taken in the order of their importance and of their availability for the instruction of the young student. Every boy and every girl, regardless of his or her subsequent career, will be benefited by the work done in Problems of the Consumer, Chapter One of Section IV. In Chapter Two of this section, Problems of the Producer, those of the farmer have been chosen as typical. Later on, the baker is made the typical manufacturer. Broad treatment of conditions readily understandable has been the aim.

It is not expected that all of the work here presented will be completed in a year. The teacher should make intelligent selections from the material offered.

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WALSH'S BUSINESS ARITHMETIC

SECTION I

RECORDING BUSINESS TRANSACTIONS

CHAPTER ONE

LEARNING BY DOING

While Edward Kerr was still attending school he obtained employment during his spare time in Hiram Hunt's general store.

Being very methodical, Edward kept an account of his receipts and expenditures in a pocket memorandum book, in the following form:

A PAGE OF A BOY'S CASH BOOK

1919						
Sep.*	1	On hand	15	74		
"	"	Penknife				75
"	"	War Savings Stamps			10	—
"	2	Moving Pictures				10
"	3	Tie				30
"	6	Wages	3	50		
"	"	Hair cut				25
"	7	Church				25
"	8	*Balance*			7	59
			19	24	19	24
Sep.	8	On hand	7	59		
"	"	School supplies			4	45
"	"	Car fare				10
"	"	Lunch				20
"	"	Athletic dues				25

* The abbreviations for the names of the months are those used by business houses.

The first entry on this page shows the cash on hand, the amount being written in the first double money column. The next four items are expenditures, the amount of each being placed in the second money column. Then follow a cash receipt and two expenditures. In writing each *debit* item Edward began close to the date column, and began each *credit* item one-half inch to the right.

BALANCING THE ACCOUNT

He closed the account at the end of the week by writing the word "Balance" in red ink as the last item. He then drew a line across the money columns, and below it wrote 19.24, the total of the debits, in the debit (Dr.) money column, and the same amount in the credit (Cr.) column. He inserted the balance, 7.59, in red ink in the Cr. column. This balance he obtained as follows:

Adding downward, he thought — 10 (5 + 5), 15 (adding 5) and 9 (writing 9) are 24;

9 (carrying 2), 10 (adding 1), 13 (adding 3), 15 (adding 2), 17 (adding 2), and 5 (writing 5), are 22;

12 (carrying 2), and 7 (writing 7), are 19.

He tested the correctness of the balance by covering the second total, 19.24, with a strip of paper, on which he wrote the total obtained by adding the second column upward.

He then counted his cash on hand, and finding that it amounted to 7.59, he felt sure that he had entered all of the cash transactions for the week.

He next drew a double line below the totals and also across the date column, and on the line below he reopened the account by the entry "Sep. 8, On hand, 7.59."

If he desired to ascertain his available cash, he would add 7.59 to the balance shown by his savings-bank book, his War Savings Stamps, etc.

WRITTEN EXERCISE

Copy the foregoing account, and supply additional items to cover the transactions of the remainder of the second week. Close the account on the morning of September 15, by inserting the balance, and reopen it the same day.

Use a sheet of journal paper, or one ruled in the form shown in the foregoing account. On the top line write the word "Cash," but omit the abbreviations "Dr." and "Cr." Do not use the dollar sign ($).

PAGE OF A SINGLE-ENTRY LEDGER

Mr. Hunt's bookkeeping was limited to the accounts he kept with the few customers to whom he extended credit. To each of these he assigned a page in a ledger. He kept this book by single entry; that is, he entered each transaction but once.

The following shows the account of John McKeon, which was kept on page 15:

		JOHN McKEON		*Dr.*	*Cr.*
1920					
May	1	25 lb. Sugar .08	2.—		
		5 " Tea .35	1.75		
		20 yd. Muslin .12	2.40		
		1 bbl. Flour	9.50	15 65	
	6	8 doz. Eggs .35	2.80		
		Cash	10.—		12 80
	9	10 lb. Coffee .29		2 90	
	27	Cash in full			5 75
				18 55	18 55

The foregoing shows that Mr. McKeon on May 1, 1920, received goods to the amount of $15.65, for which he

did not pay at the time; and that on May 9 he similarly received goods to the amount of $2.90. It shows that on May 6 he paid a total of $12.80 in cash and produce, and that on May 27 he settled in full by a cash payment of $5.75.

Only the footing of a day's purchases was carried to the money columns, the separate extensions being written in the space to the left.

The first double money column (Dr.) shows the *debits* of John McKeon to the store, the second shows the *credits* due by the store to him.

When Mr. McKeon called on May 27 to pay the balance he owed, this was determined by deducting his credits of $12.80 from $18.55, the sum of his debits. This difference, $5.75, was then paid, and entered as a credit.

In the foregoing account, the items 2.—, 1.75, 2.40, 9.50, etc., are called *extensions*. The total, 15.65, is called a *footing*.

WRITTEN EXERCISE

Make out an account similar to the foregoing. Use prices prevailing in the vicinity of the school. Make the total of the Dr. money column agree with that of the Cr. column by the insertion of the necessary amount. Draw a line across the page to show that the account is closed.

NOTE: Omit unnecessary words, figures, etc. Do not write *Dr.* and *Cr.* above their respective columns. Omit the dollar sign ($).

ANTIQUATED METHODS

Mr. Hunt made no entries of his transactions with the merchants from whom he purchased goods. When he bought on credit, he placed the bill in a tray until it became due, and when it was receipted, he filed it away with other receipted bills.

He kept no cash account. At the close of business each day, he counted the money in the drawer, deducted from the total the amount placed in the drawer in the morning to be used in making change, and added to the balance the total of the cash payments taken from the drawer. If the final result agreed with the total cash sales for the day, as shown on the strip in the cash register, he was satisfied.

Not having had experience with anything different, Mr. Hunt felt only vaguely the advantages of a system that would enable him to ascertain his business condition at any time, the expense of selling goods, and the like.

CHARGING THE WRONG ACCOUNT

It happened once in a while that in the hurry of business the account of one customer was debited with goods that had been bought by a different one. When the former complained of the overcharge shown in the monthly statement rendered him, Mr. Hunt was disturbed, not so much at the possible loss of the sum involved as by the feeling that a customer might consider him dishonest rather than unbusinesslike. His annoyance was further increased by being unable at times to determine who was the purchaser.

OMITTED ENTRIES

One day a customer who wished to settle his account called attention to the omission of a charge for goods bought on credit early in the month. His inability to recall the details of the purchase or its total showed Mr. Hunt once more the need for a better system.

A complaint within a day or two from another customer that his monthly statement showed no credit for a cash payment on account made the previous week, confirmed Mr. Hunt's resolution to take up at once the matter of employing business methods.

A BETTER WAY

He called upon Edward's teacher of commercial subjects (Mr. Brown), and asked him if he could suggest a method of keeping accounts that Edward could handle in the short time that he was able daily to devote to the store. Mr. Hunt stated that neither he nor his two regular clerks had had any experience in real bookkeeping, and that his credit transactions were too few in number to warrant the employment of an experienced bookkeeper. He said, too, that he had begun to realize the need of the possession of more information about business conditions than he could get from a ledger that showed merely the store's dealings with a few customers.

Mr. Brown explained the advisability of including among the ledger accounts one showing Mr. Hunt's relation to the business, a cash account, a merchandise account, one giving the expense of conducting the business, and one with each individual or firm from whom goods were purchased on credit.

He showed also that the proper keeping of some of the foregoing accounts would require the employment of the double-entry method, the advantages of which he briefly stated.

He recommended the use of a journal daybook, in which transactions are entered in the order of their occurrence. In the case of a complaint that goods had been charged to a person other than the purchaser, a reference to the daybook would supply the name of the latter.

The advantages of journalizing the entries, as a preliminary to their transfer to the ledger, were explained.

WRITTEN EXERCISE

Make out the bill rendered to John McKeon by Hiram Hunt on May 1, 1920 for the six articles purchased by the former on that day.

A RECORDING REGISTER

Mr. Brown stated that the keeping of the books could easily be done by Edward in the time he was then giving the store. The important thing was to put him in possession of all the data required to make the entries. These details, Mr. Brown said, could best be supplied by a register that would record every transaction, giving the sum involved and its character, distinguishing each as either a sale for cash, a sale on credit, a cash payment made at the store for goods previously purchased, or a purchase of goods by the store on credit.

He showed Mr. Hunt that the employment of a register in which every transaction was "rung up" would prevent an omission to charge a customer's account with goods sold or a neglect to give him credit for a payment made.

THE INVENTORY

Mr. Brown stated that the ledger account which should be kept with the proprietor, Mr. Hunt, would require that an inventory should be made to determine the condition of the business on the day the new system was commenced, by ascertaining the amount of cash on hand, the value of the stock and fixtures, and the condition of unsettled accounts.

As the new books were to be opened on June 1, an inventory was made as of the close of business on May 31. The cash in the safe was found to be $61.14, and that in the bank $2284.75, a total of $2345.89. The value of the mer-

chandise was taken to be $1836.54, and that of the fixtures $1372.50. There was a balance of $147.85 due by J. H. Richards and one of $137.84 payable to R. A. Black.

The total assets (resources) were thus found to be $5702.78, and the liabilities (debts) $137.84, showing a balance of $5564.94, which constituted the net resources.

The following is the first entry in the journal daybook, the statement of the assets, of the liabilities, and of the balance, covering the daybook portion. In making this part Edward did not use the money columns, confining all the details to the space to the left.

Jun. 1, 1920 (p. 1)

	Statement of the assets and liabilities at the beginning of business to-day			
	Assets			
√	Mdse. as per inventory	1836.54		
√	Fixtures, etc. (see list)	1372.50		
√	Cash on hand (and in bank)	2345.89		
√	Due from J. H. Richards	147.85		
		5702.78		
	Liabilities			
√	Due R. A. Black	137.84		
√	Net Resources	5564.94		
		5702.78		
2	Mdse.		1836 54	
5	Fixtures		1372 50	
6	Cash		2345 89	
10	J. H. Richards		147 85	
11	R. A. Black			137 84
1	Hiram Hunt			5564 94

JOURNALIZING

The form and the details of the first entry were suggested by Mr. Brown, who recommended that the fixtures should be made a separate item in the inventory, and that the ledger should contain a Fixture Account.

He showed Edward that in the journal entry the *assets* should appear as debits and the *liabilities* as credits, the

amount of the *net resources* being credited to an account that should be opened with Mr. Hunt, the proprietor.

In writing the journal portion Edward was careful to insert the amount of each debit in the first money column, and that of each credit in the second money column. As he journalized each daybook item, he placed a check mark (√) before the latter. When he completed the journal entry he tested the correctness of his figures by finding the sum of the debits and of the credits, and comparing the two results.

The next task was the making of the ledger entries ("posting") called for by the inventory. These are shown in the following:

FIRST LEDGER ENTRIES

HIRAM HUNT (p. 1)

							1920 Jun.	1	Investment	1	5564	94

(p. 2) *MERCHANDISE*

1920 Jun.	1	Inventory	1	1836	54								

FIXTURES (p. 5)

1920 Jun.	1	Value (See list)	1	1372	50								

(p. 6) *CASH*

1920 Jun.	1	On hand	1	2345	89								

(p. 10) *J. H. RICHARDS*

1920 Jun.	1	Balance	1	147	85								

R. A. BLACK (p. 11)

							1920 Jun.	1	Balance	1	137	84

STARTING THE NEW LEDGER

Edward made the account with the proprietor the first one in the new ledger, writing "*HIRAM HUNT*" at the top of page 1. Since the latter's name appeared in the inventory among the credit items, he first wrote 5564.94 in the credit money column. In the first journal column, on a line with this item, he wrote the ledger page. He then wrote "Investment" to specify the character of the item, after which he inserted the date. He completed the posting of this item of the inventory by writing 1, the journal page, in the ledger column to the left of the money one. This he did to facilitate a reference to the journal daybook entry should it become necessary.

He opened the ledger accounts called for by the other items of the journal entry, in the order in which they appeared in the latter, writing *MERCHANDISE* at the top of page 2. Noting that the journal specified a debit entry, he made it on the debit side of the page, the left half, proceeding in the manner followed in entering the Hunt item.

Because of the likelihood that the number of merchandise transactions would be large, he left pages 3 and 4 blank for later uses, and opened the *FIXTURES* account on page 5. Here he entered the required debit, and then opened the *CASH* account on page 6. After making the necessary credit entry, he left three blank pages and opened accounts with J. H. Richards and R. A. Black, respectively, on pages 10 and 11, and made the proper entries.

This work he completed before school hours on June 1.

A DAY'S TRANSACTIONS

In accordance with Mr. Hunt's orders, each of the fifty-six transactions of June 1 was "rung up" on the register. This recorded the number of the transaction, its amount, the date, a letter to denote the person by whom it was handled, and its classification under one of five types, indicated as follows:

* An ordinary sale for cash
Pd. A cash payment made by the store
Rec. Cash received to be credited to a debtor
Ch. A sale made on credit
Bt. A credit purchase by the store

Transactions registered by Mr. Hunt were denoted by A, those by the two clerks by B and C, respectively.

Of the day's transactions, forty-eight were cash sales; the other eight were the following, numbered in the order of their occurrence:

1	Payment of June rent (check)	$125.—
3	Sale to H. A. Gaynor on a/c	9.08
6	Payment of freight bill (check)	19.44
9	Check received from J. H. Richards	147.85
11	Purchase of sugar on account	515.20
18	Payment to R. A. Black (check)	137.84
21	Bank deposit	207.85
24	Payment of expressage (cash)	1.50

THE RECORD STRIP

Each of these was recorded in its appropriate column on the strip. The following portion shows these eight, and includes a few of the cash sales:

*	PD.	REC.	CH.	BT.
2– 5.75–B	1–125.00–A	9–147.85–A	3–9.08–B	11–515.20–A
4– 3.94–B	6– 19.44–A			
5–10.87–C	18–137.84–A			
7– .95–B	21–207.85–A			
etc. etc.	24– 1.50–C			

THE CARDS

Besides making the foregoing records on the strip, the register printed a card in connection with each transaction. On the following, which was issued in connection with the rent payment, the register placed this heading:

Pd. A 125.00 0001 Jun-1-20

Mr. Hunt, who made the record, noted on the card, for the information of the bookkeeper, that the disbursement was a payment of rent made by check, and placed the card in a drawer.

The card for transaction 2, a cash sale, was given to the customer. It contained this heading:

* B 5.75 0002 Jun-1-20

In connection with the third transaction, a credit sale, the register printed the heading at the top of a bill and its carbon duplicate in this form:

Ch. B 9.08 0003 Jun-1-20			
Hiram Hunt Sold to *H. A. Gaynor* *223 Warren St.*			
24 lb. Butter	.32	7	68
5 " Coffee	.28	1	40
Amount of this purchase		9	08

Clerk B filled in the details as shown herewith, gave Mr. Gaynor the bill, and placed the duplicate in a drawer of the register.

Transactions 4 and 5, cash sales, were registered in the same way as No. 2, and the cards given to the purchasers.

Mr. Richards, transaction 9, not having sent with his check the bill it was intended to settle, Mr. Hunt registered the heading on a blank receipt and its duplicate, which he filled out in the form shown herewith:

Rec. A 147.85 0009 Jun-1-20
Received of J. H. Richards One Hundred Forty-seven 85/100 Dollars in full of account to date. Hiram Hunt

He mailed the original to Mr. Richards, and placed in the drawer the carbon duplicate, noting on the latter "By check."

If Mr. Richards had inclosed the bill, this would have been receipted and returned, and the transaction recorded on a card.

The bank deposit, transaction 21, while not being one calling for an entry in the daybook, was "rung up" to account for the withdrawal of cash from the drawer and the safe. It appeared thus:

Pd A 207.85 0021 Jun-1-20

On the card, which Mr. Hunt placed in the drawer, he noted

"Deposit: Cash, $60; Check, $147.85"

THE BOOKKEEPER

When Edward reached the store, after school, he first counted the cash in the register, which he found to be $37.12. He then examined the register slip to ascertain the number of transactions recorded, and their character. He learned that there were up to that time 44 in all, 36 of which were sales for cash, amounting, according to the register, to $37.48. To this he added $21.14, which had been placed in the drawer in the morning for change, which made a total of $58.62. From this he deducted $1.50, cash paid for expressage, leaving $57.12 to be accounted for.

The difference between this sum and the $37.12 in the drawer showed that $20 had been taken, presumably for the deposit. An examination of the safe, in which $40 had been left in the morning, showed that this had been used to make up the $60 deposited in cash.

Finding the money correct, Edward proceeded to make the entries in the journal daybook. As a preliminary, he collected the necessary data: cards, bills, bank book, check book, etc.

Taking up the first transaction recorded on the strip, the payment of $125, he learned from the card that it covered the June rent, and that it had been made by check. From the check-book stub he found that a check for that amount had been drawn, which had been correctly deducted from the previous balance.

He then made the following entry of the first transaction, using a ditto mark (") to show that the date was the same as that of the previous entry (June 1):

		"			
✓	Paid rent for June	125.—		125—	
	Expense				125—
	Cash				

The first line constitutes the daybook portion. This he "journalized" in the form shown in the next two lines. He first entered 125.—, the amount, in each of the two money columns, once as a debit and once as a credit. The transaction, involving an expenditure of cash, requires that the Cash account in the ledger be credited with this sum. The journal entry, therefore, placed "Cash" in the credit place. Inasmuch as expenditures for such items as freight, expressage, postage, taxes, clerk hire, rent, etc., were to be entered in the ledger under the general title of "Expense," Edward debited the Expense account with $125.

He began the debit entry item close to the first vertical line and the credit entry about an inch to the right.

Finding from the record strip that transaction 2 was a cash sale, Edward went to the next. From the carbon duplicate placed in the drawer, he obtained the details, for the daybook portion, which are given in the first three lines of the following entry:

		"			
✓	Sold H. A. Gaynor on account				
	24 lb. Butter .32	7.68			
	5 " Coffee .28	1.40	9.08	9 08	
	H. A. Gaynor				9 08
	Mdse.				

In journalizing it, he wrote "9.08" twice, as in the preceding entry. On the first line (the debit one) he wrote "H. A. Gaynor," whose ledger account was to be debited with $9.08 for goods bought. On the credit line, he wrote "Mdse.," the ledger account of which was to be credited.

Transactions 4 and 5, cash sales, were passed over, and after Edward had verified the extension of the freight bill, examined the check-book stub, etc., he made the following entry of transaction 6:

	"					
	Paid freight	19.44				
	Expense			19	44	
	Cash					19	44

The following are the entries for the remaining transactions other than the sales for cash:

	"					
✓	Received check from J. H. Richards in full of account	147.85				
	Cash			147	85	
	J. H. Richards					147	85
	"					
✓	Bought from Franklin Refinery on a/c 25 bbl. Sugar, as per invoice	515.20				
	Mdse.			515	20	
	Franklin Refinery					515	20
	"					
✓	Paid R. A. Black in full of account to date	137.84				
	R. A. Black			137	84	
	Cash					137	84
	"					
✓	Paid Cash for expressage	1.50				
	Expense			1	50	
	Cash					1	50

When business was over for the day, the register showed that the number of cash sales was 48, totaling $63.15. The following was the final entry for Jun. 1:

	"					
✓	Received from 48 cash sales, as per register	63.15				
	Cash			63	15	
	Mdse.					63	15

POSTING THE DAY'S TRANSACTIONS

While waiting for remaining sales to enable him to make the final daybook entry, the cash sales for June 1, Edward began to make the ledger entries. Turning to the journal portion of the first transaction he found it to be:

12	Expense	125 —	
6	Cash		125 —

which required that the Expense account in the ledger be debited with $125, and that the Cash account be credited with the same sum. Since the ledger did not as yet contain the former account he opened one on page 12, and entered the required debit, inserting in the journal "12," the ledger page of the Expense account. He completed the posting of this transaction by crediting the Cash account with $125, and inserting in the journal "6," the ledger page of the Cash account.

He then took up the other journal entries in regular order, opening accounts with H. A. Gaynor and Franklin Refinery on pages 13 and 14, respectively.

T[illegible] accounts appeared as follows after the posting of the las[illegible]tion of the day. A line was drawn under the accour[illegible] H. Richards and one under that of R. A. Black to show that each had been settled.

THE LEDGER PAGES

HIRAM HUNT (p. 1)

						1920 Jun.	1	Investment	1	5564	94

(p. 2) *MERCHANDISE*

1920 Jun.	1	Inventory	1	1836	54	1920 Jun.	1	H. A. Gaynor	1	9	08
"	1	Frank. Ref.	1	515	20	"	1	Cash Sales	1	63	15

FIXTURES (p. 5)

1920 Jun.	1	Value	1	1372	50								

(p. 6) *CASH*

1920 Jun.	1	On hand	1	2345	89	1920 Jun.	1	Rent	1	125	—
"	1	J. H. Richards	1	147	85	"	1	Freight	1	19	44
"	1	Sales for day	1	63	15	"	1	R. A. Black	1	137	84
						"	1	Express	1	1	50

(p. 10) *J. H. RICHARDS*

1920 Jun.	1	Balance	1	147	85	1920 Jun.	1	Cash	1	147	85

R. A. BLACK (p. 11)

1920 Jun.	1	Cash	1	137	84	Jun.	1	Balance	1	137	84

(p. 12) *EXPENSE*

1920 Jun.	1	Rent	1	125							
"	1	Freight	1	19	44						
"	1	Express	1	1	50						

H. A. GAYNOR (p. 13)

1920 Jun.	1	Mdse.	1	9	08						

(p. 14) *FRANKLIN REFINERY*

						1920 Jun.	1	Mdse.	4	515	20

ORAL EXERCISES

1. State why each item (*a*) on the debit side of the merchandise account is entered therein. (*b*) On the credit side.

2. What does (*a*) the debit side of the cash account show? (*b*) The credit side?

3. How would you journalize (*a*) the purchase of a platform scales from the Fairbanks Company on credit? (*b*) The purchase of 2000 postage stamps for cash? (*c*) The payment by check of a bill for electric light? (*d*) The purchase of 10 tons of coal on credit?

4. (*a*) What is shown by the difference between the totals of the two sides of the cash account? (*b*) How can the correctness of this difference be determined? (*c*) Which side must always have smaller total? Why?

THE BANK ACCOUNT

Mr. Hunt kept his account with the Newaygo County Bank on the stubs of the check book, as shown on another page.

In making a deposit he filled out a slip like the accompanying form, which he sent to the bank with the money and the bank book. When the messenger returned, Mr. Hunt examined the book to see that the proper entry had been made; then, on the stub he added \$207.85, the amount of the deposit, to the previous balance, \$2159.75, making a total of \$2367.60 to his credit in the bank.

Deposited in
The Newaygo County Bank,
White Cloud, Mich.
by Hiram Hunt
Address: 4 Court Square.

Jun. 1, 1920

Bills	40	—
Coin	20	—
Check	147	85
"		
"		
	207	85

List each check separately

PAGE OF CHECK BOOK

VI-1-1920

No. 458

To *John Whalen*

For *Rent in advance*

Balance brought forw'd	2284.75
Deposit	
Total	
Amount this check	125.—
Balance carried forw'd	2159.75

No. 459

To

For

Balance brought forw'd	2159.75
Deposit	
Total	
Amount this check	
Balance carried forw'd	

No. 460

To

For

Balance brought forw'd	
Deposit	
Total	
Amount this check	
Balance carried forw'd	

No. 458 White Cloud, Mich., Jun. 1, 1920

NEWAYGO COUNTY BANK

Pay to the order of John Whalen $ 125 00/100

One Hundred Twenty-five 00/100 Dollars

Hiram Hunt

No. 459 White Cloud, Mich., _____ 192__

NEWAYGO COUNTY BANK

Pay to the order of _____ $ ____ 00/100

_____ 00/100 Dollars

_____

No. 460 White Cloud, Mich., _____ 192__

NEWAYGO COUNTY BANK

Pay to the order of _____ $ ____ 00/100

_____ 00/100 Dollars

_____

When Mr. Hunt made out check No. 458 in payment of the June rent, he detached it, leaving in the book the portion on the left, called the *stub*. On the latter he wrote the name of the person in whose favor the check was drawn (the payee), the item covered by the payment, its amount, and then deducted this amount from the previous balance, 2284.75. He brought down the remainder, 2159.75, to the stub for check No. 459.

WRITTEN EXERCISES

1. On a sheet of paper of the proper size, make a copy of the next two checks with their accompanying stubs. Fill out one check to cover a freight payment, and the other to settle Mr. Hunt's account with R. A. Black. Use the proper number for each, and insert the amounts specified in the entries. Fill out each stub properly, inserting in the last one the deposit of $207.85, made before the check is drawn.

2. (*a*) How much cash should Mr. Hunt have in the store at the close of business on Jun. 1? (*b*) What should be his bank balance? (*c*) Compare the sum of (*a*) and (*b*) with the difference between the Dr. and the Cr. side of the cash account in the ledger.

BALANCING AN ACCOUNT

(p. 16) *WM. WINKLE*

1920						1920					
Jun.	2	To Mdse.	2	27	65	Jun.	20	By Cash	20	50	—
"	8	" "	9	8	43	"	27	" "	32	25	—
"	10	" "	12	19	64	"	30	" Bal.		28	10
"	15	" "	18	37	53						
"	23	" "	27	9	85						
				103	10					103	10
Jul.	1	To Bal.		28	10						

3. Find (*a*) the total of the Jun. 1 journal debits, including those of the inventory. (*b*) The total of the credits. (*c*) The total of the debits in the ledger entries. (*d*) The total of the credits.

On the first day of July Edward balanced the June accounts. The method is shown in the foregoing ledger account with Wm. Winkle. Noting that the Dr. side was the greater, he wrote under the former its total, 103.10, and placed the same total as the footing of the Cr. side, writing it on a line with the other total.

He then made the entry "By Balance," writing this in red ink, also the date, "Jun. 30," and the amount of the balance, "28.10." This he ascertained by deducting the sum of $50 and $25 from $103.10. He then drew a line under both totals, closing the account, which he reopened by the entry of Jul. 1, "To Bal., 28.10," which is the amount due from Wm. Winkle.

He then mailed to Mr. Winkle the following monthly statement, omitting the details of the purchases, since Mr. Winkle had received a bill with each.

MONTHLY STATEMENT

WHITE CLOUD, MICH., Jul. 1, 1920

Mr. WM. WINKLE

In account with HIRAM HUNT

General Merchandise

4 Court Square

1920					
Jun.	2	To Mdse. as per bill rendered	27 65		
	8	" " " " " "	8 43		
	10	" " " " " "	19 64		
	15	" " " " " "	37 53		
	23	" " " " " "	9 85	103	10
		Cr.			
Jun.	20	By Cash	50 —		
	27	" "	25 —	75	—
		Due		28	10

ANOTHER LEDGER PAGE

(p. 10) *J. H. RICHARDS*

1920						1920					
Jun.	8	To Mdse.	9	13 48		Jun.	18	By Mdse.	21	48	66
"	13	" "	15	6 86		"	28	" "	33	87	95
"	16	" "	19	27 95							
"	20	" "	23	42 63							
"	27	" "	32	18 04							
"	30	" Bal.		27 65							
				136 61						136	61
						Jul.	1	By Bal.		27	65

In balancing this account, Edward observed that the credit total was in excess of that of the debits. This required a balance entry in the debit column, for which he left a line. The final debit total, therefore, appeared two lines below the last debit entry; on this line, on the credit side, he entered 136.61, and wrote it on the debit side also. Adding the debits and subtracting them from ~~108.96 in~~ one operation, he entered the balance, 27.65, in red ink, and also the date and the word "Bal." He closed it by drawing the necessary lines, and reopened it by entering a credit balance of 27.65.

136.61

When a bill was received from Mr. Richards, on Jul. 2, Edward compared it with the account, and notified Mr. Hunt to send a check in settlement.

WRITTEN PROBLEMS

1. Make a copy of the foregoing account, balance, etc., as it appears in the ledger of J. H. Richards.

Write the heading "Hiram Hunt." Credit this account with the items that appear as debits in Mr. Hunt's ledger, and vice versa. Insert journal pages other than those found in Mr. Hunt's ledger.

2. Make out the monthly statement sent to Mr. Hunt by J. H. Richards.

3. Write the check sent by Mr. Hunt in settlement of the account.

4. (*a*) Find the total weekly pay of 73 graduates of a boys' technical school who receive weekly compensation as follows during the first year of employment:

2	receive	$6	16	receive	$9
21	"	7	8	"	10
20	"	8	6	"	11

(*b*) What is the average weekly pay?

5. Find the average weekly pay of graduates in the second year of employment who received weekly compensation as follows:

8	received	$7	11	received	$11
19	"	8	11	"	12
10	"	9	9	"	13
9	"	10	7	"	14

6. Find the average weekly pay of graduates in the third year of employment who receive weekly compensation as follows:

4	received	$10	16	received	$15
6	"	11	6	"	16
4	"	12	10	"	17
9	"	13	2	"	18
9	"	14	2	"	20

CHAPTER TWO

A GIRL IN BUSINESS

Miss White began as sales girl in a department store. Her first customer bought, for cash, the goods shown on the accompanying *sales slip*. In writing this, Miss White made a carbon duplicate. In the space at the bottom of the slip marked "Cash," Miss White wrote 2.31, the total of the transaction, and in the one marked "Rec'd," she wrote 5.—, the denomination of the bill handed her by the customer. She then sent both slips, the $5 bill, and the goods to the wrapper. The latter compared the duplicate slip with the original, and the latter with the articles. Finding everything correct, she placed her check mark on the original and sent the two slips, with the $5, to the cashier. The latter returned the duplicate to Miss White with the change, and sent the original to the auditing department. The wrapper made the goods into a neat parcel

SALES SLIP

No. 1 VII-1-20

W. S. Julius & Co.

Sold to

Cash

3 Hat Pins	.05		15
1 pr. Shields			33
3 Brushes	.24		72
3 Blue Boxes	.35	1	05
1 Pins			06
		2	31

CASH	REC'D	CH'G.
2.31	5.—	C.O.D.

and sent it to Miss White, who gave it to the purchaser, together with the duplicate slip and the change. To be certain of the correctness of the latter, she counted it out to the purchaser, saying: "2.31 and 4, 2.35; and 5, 2.40; and 10, 2.50; and 50, 3 dollars; and 2, 5 dollars"; handing over, as each item was specified, the 4 cents, the nickel, the dime, the half-dollar, and the $2 bill, supplied by the cashier.

The next sale was a credit one. This is shown by the entry of the total in the space marked "Chg." (charge) at the bottom of the slip.

To make sure that the name and residence of the purchaser were correctly written, they were read to the latter by Miss White, from the slip. This, with the duplicate and the goods, was sent to the wrapper. The original slip after being checked went to the charge clerk, and the duplicate with the parcel to the delivery department.

SALES SLIP

No. 2 VII-1-20
W. S. Julius & Co.
Sold to
Mrs. J. Carroll Payne
8502 Hamilton Boulevard

2 Paste	.20	40
2 Ammonia	.23	46
1 pr. Scissors		25
1 " "		45
6 Soap	.05	30
		1 86

CASH REC'D CH'G. 1.86

C.O.D.

A GIRL'S DAILY SALES

On a daily sales card, Miss White entered the amount of each sale, classifying it as cash, charge, or C.O.D. At the close of business she entered the total of each type, the grand total, and the number of sales made. This card went to the department of audits.

DAILY SALES

Date VII–1–20 Sold by M. White

CASH		CH'G.		C.O.D.	
No.	Am't.	No.	Am't.	No.	Am't.
1	2 31	1	1 86	1	1 35
2	79	2	1 27	2	1 65
3	15	3		3	1 87
4	3 48	4		4	
5	2 67	5		5	
6	1 10	6		6	
7	50	7		7	
8	25	8		8	
9	3 11	9		9	
10		10		10	
11		11		11	
12		12		12	
Tot.	(*a*)		(*b*)		(*c*)
No. of Sales *14*			Grand Tot.		(*d*)

In order to determine how she was succeeding in her work, Miss White kept a memorandum of the number of her daily sales and their total. She was gratified to perceive that her promptness, courtesy, knowledge of the stock, etc., enabled her to make a steady increase in the number of transactions she was able to handle in a day.

WRITTEN PROBLEMS

1. From Miss White's Daily Sales slip for **Jul. 1,** find the total of the sales (*a*) made for cash, (*b*) charged, (*c*) C.O.D., and (*d*) the grand total.

2. From the accompanying list of the sales made during the week ended Jul. 6, by the five girls at the

notion counter, find the weekly sales of each, (*a*) to (*e*); the total sales for each day, (*f*) to (*k*); and the total sales of notions for the week, (*l*).

Day	Miss W.	Miss V.	Miss U.	Miss T.	Miss S.	Tot.
Monday	22.36	38.75	32.63	30.48	36.12	(*f*)
Tuesday	11.17	19.05	17.38	15.72	18.69	(*g*)
Wednesday	43.82	31.56	27.73	29.84	28.11	(*h*)
Thursday	36.09	24.12	22.34	25.56	23.89	(*i*)
Friday	32.77	19.45	21.67	20.90	18.23	(*j*)
Saturday	38.04	32.78	34.88	30.76	36.65	(*k*)
Totals	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)	(*l*)

Find (*l*) by adding the line totals (*f*) to (*k*). Check (*l*) by adding the column totals (*a*) to (*e*).

3. Miss White's weekly pay is \$6 plus a commission of 5% on sales in excess of \$100. What did she receive for a week during which her sales amounted to \$184.25?

METHOD

\$6 + 5% of \$84.25 (\$184.25 − \$100.00) = \$6 + \$4.21
= \$10.21, Ans.

4. Find the compensation received by each of the other girls at the notion counter for the week from Jul. 1 to Jul. 6, at \$6 plus 5% on sales in excess of \$100.

THE BILL CLERK

Miss White's quickness at figures obtained for her a promotion to a place in the dress goods department. Her ability to fill out correctly the extensions in a sales slip, without making "side" calculations, enabled her to save the time of her customers, as well as to make in a day a larger number of sales than were made by some of her fellow clerks of greater experience.

Her next promotion was to a place in the office of the cashier, to check up the extensions on the sales slips. From this she went to the billing department.

SIGHT EXERCISES

1. Give to the nearest cent the extension of each of the following items in a bill:

a 8	yd. Muslin	@ 6¢	*b* 8	yd. Tape	@ ½¢
c 8	" Cambric	" 6½¢	*d* ½	" Cambric	" 6½¢
e 8½	" "	" 6½¢	*f* 3½	" Mohair	" 30¢
g 4	" Sateen	" 14½¢	*h* 10	" Cashmere	" 85¢
i 5	" Cashmere	" 96¢	*j* 2½	" "	" 96¢
k 4½	" Twill	" 8½¢	*l* 16	" Sateen	" $1¼
m 24	" Sateen	" 25¢	*n* 25	" "	" 24¢
o 12½	" "	" 24¢	*p* 1¼	" Alpaca	" 32¢
q 2	" Padding	" 84¢	*r* ¼	" Padding	" 84¢
s 2¼	" "	" 82¢	*t* 4	" Dress Goods	" 42¢
u ½	" Dress Goods	" 42¢	*v* 3¾	" " "	" 42¢
w 2¼	" Cambric	" 19¢	*x* 64	" Muslin	" 6¼¢
y 36	" Silk	" 99¢	*z* 99	" Cashmere	" 96¢

2. (*a*) From 10 times 44, take ¼ of 44; (*b*) multiply 44 by 9¾.

3. Give products:

a $44 \times 19\frac{3}{4}$	*b* 44×25	*c* $44 \times 24\frac{3}{4}$
d $44 \times 24\frac{1}{2}$	*e* $44 \times 49\frac{3}{4}$	*f* $44 \times 99\frac{3}{4}$
g $32 \times 19\frac{7}{8}$	*h* $32 \times 24\frac{7}{8}$	*i* $32 \times 99\frac{7}{8}$

MONTHLY BILL OF A DEPARTMENT STORE

The following is the heading of the bill rendered monthly by W. S. Julius & Co. to his "Charge" customers:

DATE, Jul. 31, 1920 FOLIO 35814

NAME, Mrs. J. Carroll Payne

ADDRESS 8502 Hamilton Boulevard, Tucson.

AMOUNT OF PAYMENT, $——

.....

Tucson, Ariz., Jul. 31, 1920

W. S. Julius & Co.

SOLD TO Mrs. J. Carroll Payne

8502 Hamilton Boulevard.

TERMS: Settlements required the first part of each month.

When the bill is paid, the cashier enters the sum received on the coupon at the top, which he detaches and sends to the department of customers' accounts. He then receipts the bill, which he returns to the customer.

The body of the bill contains three money columns, two for the debits and one for the credits. The second debit column gives the total of the purchases of the day. The credit entries are made in red ink.

WRITTEN EXERCISES

Make out a monthly bill covering the following purchases. Fill in the missing items (*a*) to (*s*). Make yourself a purchaser and a local firm the seller.

Date			Amount		Daily Total		Credit	
Jul.								
1	2 Paste	.20		40				
	2 Ammonia	.23		46				
	1 pr. Scissors			25				
	1 " "			45				
	6 Soap	.05		30	1	86		
2	1 yd. Cretonne			38				
	4 Towels	.19		(*a*)				
	2¼ yd. Padding	.82		(*b*)		(*c*)		
6	½ doz. Plates	2.70		(*d*)				
	½ " "	1.50		(*e*)		(*f*)		
8	1 yd. Cretonne							38
10	1 Wrapper		3	95				
11	1 Skirt			98		(*g*)		
	3¾ yd. Dress Goods	.42		(*h*)				
	6 Napkins	.05		(*i*)		(*j*)		
13	1 pr. Hose				1	35		
15	3 doz. Napkins	.35			1	05		
18	3¾ yd. Dress Goods	.42					1	58
	1 Wrapper						3	95
20	4 yd. White Goods	.20		(*k*)				
	3 pr. Hose for		1	—				
	2¼ yd. Cambric	.19		(*l*)				
	6 Handkerchiefs	.35		(*m*)		(*n*)		
27	1 Smock		1	29				
	3⅞ yd. Embroidery	.08		(*o*)				
	1 Dress		2	75		(*p*)		
			Total			(*q*)		(*r*)
			Less			(*r*)		
			Due			(*s*)		

Paid Aug. 3, 1920
W. S. Julius & Co.
by M. E. K.

Do not use the price column when a single article is bought; write the price only in the "amount" column. When but one purchase is made on any day, write the amount only, in the "daily total" column.

THE CAREFUL CUSTOMER

Upon the arrival of each purchase, Mrs. Payne examines the sales slip to be sure that she has received all of the articles charged to her account. She then files away the slips until the arrival of the monthly bill, which she "checks" by means of the slips.

She also examines the credit column to ascertain that the proper reductions have been made for the articles returned, as shown by her "Customer's Receipts."

When she finds that goods received are unsatisfactory or unnecessary, she notifies the store to send for them. The driver gives her a receipt in the accompanying form, which is a carbon copy of the "Call Check" filled out by the driver and brought by him to the store with the articles returned.

CUSTOMER'S RECEIPT

DATE VII-17-20 F. 32748
RECEIVED OF Mrs. J. Carroll Payne
ADDRESS 8502 Hamilton Blvd.

Dress Goods Wrapper		

RETAIN THIS RECEIPT OF GOODS RETURNED

DEP'T.—— WHY RETURNED——
ORDER NO.—— CALL NO.——
Paid or Charge Check Entry——
W. S. JULIUS & Co.
per T. E. B.

THE OFFICE ASSISTANT

Miss White's desire to become an efficient employee caused her to devote much of her spare time to a review of the commercial branches. Now that she was daily brought face to face with the advantages of

training, she brought a new interest to her studies, and the latter held for her a new meaning.

When, one day, she was offered the position of office assistant to the purchasing agent of The Harrison Company, she accepted it, feeling competent to perform the duties, and glad of the opportunity for more varied work than she would be likely to get in a larger business.

THE HARRISON CO.
Office of the Purchasing Agent

Happy Valley, Ariz.
Nov. 16, 1921

No. 5837
Be sure to place this order number on your bill.

Messrs. Barrett and Jones
1364 Water Street
Cincinnati, Ohio.

BILL REC'D. Nov. 24, 1921
GOODS REC'D. Nov. 27, 1921

KINDLY SHIP BY Freight

6 2-in. Brass Valves, F247
2 3-in. IBB " F600
2 Victor Gate "

W. A. Humbridge,
Purchasing Agent.

Mail bills in duplicate when goods are shipped
Mail statements the last of every month

THE ORDER BOOK

One of her duties is to make stenographic notes of goods to be purchased, and to fill out the necessary order slips. She writes each on a perforated sheet of the order book, a carbon copy being made on a page remaining in the book. When the bill is received, she stamps the date on the order. This she also does when the goods arrive.

CARD INDEXES

She enters each order on two cards, one headed with the name of the article and the other with the name of the firm. Each set she files alphabetically in the proper file.

ARTICLE		VALVES		
Date	Firm	Location	Quantity	Order No.
1921				
Jul. 16	Barrett and Jones	Cincinnati, O.	12	4386
Aug. 4	Delancey Mfg. Co.	Denver, Colo.	6	5234
Nov. 16	Barrett and Jones	Cincinnati, O.	10	5837

The foregoing card shows all the orders given for valves. The entry of the order number enables a person to turn at once to the proper place in the order book, if details are desired.

To the following card Miss White turns to obtain the address of Barrett and Jones, from which firm she has been directed to order valves.

NAME Barrett and Jones
ADDRESS 1364 Water St., Cincinnati, O.
BUSINESS Plumbers' Supplies
SALESMAN Aldcroft
REMARKS See letter of VI–16–21; prices
ORDERS 4386, 5837

On this card she enters the number of the order last sent, then files it away.

INCOMING BILLS

When the two invoices (original and duplicate) reach the office of the purchasing agent, Miss White stamps on the order sheet and on both invoices the date they are received. On the original invoice she also stamps a form to be initialed by the proper persons to certify (*a*) that the specified number of articles has been received; (*b*) that each is of the proper quality; (*c*) that the prices are those agreed upon; and (*d*) that the extensions, etc., are correctly made.

When goods arrive, she stamps the date on the order, and on both invoices, and sends the "original" for certification to the persons passing, respectively, upon quality, quantity, and price. When the invoice is returned with the required initials, she adds hers as to the correctness of the extensions, etc., and passes it along to the company's treasurer, retaining the duplicate in her files.

THE INVOICE

CINCINNATI, O., Nov. 21, 1921

BARRETT AND JONES

Plumbers' Supplies

SOLD TO The Harrison Company
Happy Valley, Ariz.

Via Freight DATE OF ORDER XI–16–1921 YOUR No. 5837

6	2″ Brass Valves, F247	7.50	45 —		
		Plus 5%	2 25	47	25
2	3″ IBB Valves, F600	15.00	30 —		
		Less 47%		15	90
2	Victor Gate Valves	22.50	45 —		
		Less 35%		29	25
				92	40
	Bill Received, Nov. 24, 1921				
	Goods Received, Nov. 27, 1921				

Quantities Correct... *J. B.*
Qualities Correct.. *H. A.*
Prices Correct. *W. A. S.*
Extensions Correct. *M. W.*

WRITTEN PROBLEMS

1. Copy and complete the following invoice. Try to make all extensions without the use of "side" calculations.

BAILY AND MIDDLESEX

Hotel Sundries

Folio 649
Terms: Net Cash

Butte, Mont., May 26, 1920

SOLD TO Hotel Burgundy
Lorton Valley, Mont.

Your Order 8502

H B	16	100 doz.	Tea Cups #4982	1.05	105	—	
		100 "	" Saucers	.78			
	17	43 "	" Dishes 6″ #4993	1.80			
		19 "	Dishes 10″	4.25			
		5 "	" 12″	7.09			
		16 "	Celery Trays	4.40			
	18	80 "	Tea Cups #4994	1.05			
		80 "	Saucers	.78			
		30 "	Double Egg Cups	1.58			
	19	141 "	Fruits 4″ #4995	.72			
		78 "	" 5″	.90			
	20	38 "	Bakers 3″	1.58			
		70 "	Plates 7″ #4996	1.58			
		40 "	Bakers 6″	2.25			(a)
			5 Crates	2.50			12 50
							(b)

Write extensions in the first double column, and the footing of the articles at (*a*) in the second. Write at (*b*) the total amount due.

2. Make out a bill for the following articles for a hotel:

78	doz.	Towels	@ $4.50
10	"	Unbl. Sheets 54 × 90	7.92
12	"	Pillow Cases 44 × 36	1.74
50	"	Red End Towels	1.34
25	"	Sheets 72 × 108	11.—
256	yd.	Cheesecloth, P. red	.06¼
204	"	Pantry Toweling	.19¾
200	"	Dish " P. blue	.19¾
400	"	Glass " 200 H	.10¼
408	"	Side " F. white	.24¾
40½	"	Sheeting, 1¼ Utica	.42½
49	"	Fine Glass Toweling	.21

THE TIME CLOCK

Miss White ascertains the weekly service of each of the 12 employees by his or her time card.

As each arrives in the morning, he takes his card from the "OUT" rack, inserts it in the recording instrument in the time clock, and pulls a lever. By doing

this, he stamps in the first column the time of his arrival. He then places the card in the "IN" rack. When he leaves at noon, he takes his card from the "IN" rack, has the time stamped in the second column, and places it in the "OUT" rack. When he returns, he replaces the card in the "IN" rack, after having had the time recorded in the third column. When he leaves for the day, he places the card in the "OUT"

rack, after the time has been recorded in the fourth column, or in the sixth column in the case of "overtime."

THE TIME CARD

When the time card is completed by the insertion of the number of hours of daily service, the weekly total, the amount due, and the employee's receipt, it presents the following appearance:

Number 4
Name D. Marquard

Beginning Dec. 1, 1921	A.M.	Noon		P.M.	Overtime		Hours
	In	Out	In	Out	In	Out	
M.	7.55	12.05	12.55	5.00			8
Tu.	7.57	12.01	1.03	5.02			7[3]
W.	7.56	12.08	12.56	5.00			8
Th.	7.59	12.02	12.57	5.01			8
F.	8.00	12.02	12.58			6.02	9
S.	8.03					1.02	4[3]
						Total	45[2]

TIME 45½ HOURS. RATE $12
Due for week $12.41

I hereby acknowledge receipt in full,
(Signed) DORA MARQUARD

These employees are paid a weekly rate based upon 44 hours of service, 4 on Saturday and 8 on each of the other 5 working days. When opportunity offers, Miss White enters in the last column the number of hours of service rendered each day.

Miss Marquard, not having been late or absent on Monday, Wednesday, or Thursday, Miss White enters the number of hours for each of these days as 8. She

enters 7¾ for Tuesday, owing to the arrival of Miss Marquard after 1. She makes a similar deduction on Saturday after allowing the overtime of an hour. Friday's entry shows 9 hours, which includes the overtime.

On Monday, Miss White completes the time cards, entering Saturday's time on each, the total for the week, and the amount due. She also completes the time sheet, shown below, having made as many of the daily entries as possible in her spare time the preceding week.

As a check on the accuracy of the total service entered on each time card, and the amount due, she calculates these once more from the time sheet, and compares the results with those shown on the card.

TIME SHEET — WAREHOUSE

Dec. 1, 1921 to Dec. 6, 1921

No.	Name	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.	Hours	Rate	Pay
1	Cutshaw, G.	7^3	8	8	7^2	8^3	4	44	15.—	15.—
2	Daubert, J.	8	7^3	8	8^2	8	4	44^1	12.—	12.07
3	Johnson, J.	7^2	8	8	8	9	4	44^2	12.—	12.14
4	Marquard, R.	8	7^3	8	8	9	4^3	45^2	12.—	12.41
5	Meyers, J.	7	8	8	8	7^3	5	43^3	12.—	11.93
6	Miller, O.	8	8	7^2	8	8	4	43^3	11.—	10.94
7	Mowery, H.	9	8	8	7^3	—	—	32^3	10.50	7.82
8	Olsen, I.	8^2	8	8	8	8^2	4	45	10.—	10.23
9	Pfeffer, E.	6	—	8	8	6	4^3	32^3	9.—	6.70
10	Rucker, N.	8	8	8	—	8	4	36	8.50	6.95
11	Stengle, C.	8^2	8	8	8	9	5	46^2	8.—	8.45
12	Wheat, Z.	8	8	7^2	8	8^2	4	44^1	6.—	6.03
	Total	94^1	87^2	95^1	87^3	90^2	47^2	50^3		120.67

The small figures above the others and to the right denote *quarters;* 94^1 meaning 94¼, 87^2 meaning 87½, and 90^3 meaning 90¾.

CASH FOR THE PAY ENVELOPES

After verifying the correctness of the total, Miss White prepares a check for the amount. This she sends with the time sheet to the purchasing agent. When the latter has affixed his signature to the check, he sends it to the treasurer.

THE CHECK

Payroll of Warehouse Employees for week, ended Dec. 6, 1921.
W. A. PLUMBRIDGE, *Purchasing Agent.*

Happy Valley, Arizona, Dec. 8, 1921 No. 8502

THE BATH COUNTY NATIONAL BANK

Pay to the Order of Maurice J. Moore 120^{67}/_{100}$

One Hundred Twenty $^{67}/_{100}$ ..... Dollars

Donald Campbell
Treasurer

When the check is returned to Miss White with the necessary signatures she sends Maurice Moore to the bank to obtain the cash for the pay envelopes, giving him the accompanying "change slip."

After Maurice Moore's arrival at the bank, he indorses the check and presents it with the "change slip" to the paying teller. The latter gives him the specified bills and the smaller change, which he counts. Finding the amount correct, Maurice returns with the money to Miss White. She distrib-

CHANGE SLIP

BATH COUNTY NATIONAL BANK

Kindly send by Maurice Moore the following:

7	$10's	$70.—
6	5's	30.—
5	2's	10.—
5	1's	5.—
5	halves	2.50
6	quarters	1.50
12	dimes	1.20
5	nickels	.25
22	pennies	.22
	Total	$120.67

utes it in the pay envelopes, writing on the back of each the name of the employee and the amount contained. Each employee, on receiving his envelope and counting its contents, signs the receipt on the time card. When all the employees have been paid, the cards are sent to the auditor.

To determine the denomination of the bills and coins needed for the different envelopes, Miss White makes a memorandum in the following form:

CHANGE MEMORANDUM

No.	Pay	$10	$5	$2	$1	50¢	25¢	10¢	5¢	1¢
1	15.—	1	1	–	–	–	–	–	–	–
2	12.07	1	–	1	–	–	–	–	1	2
3	12.14	1	–	1	–	–	–	1	–	4
4	12.41	1	–	1	–	–	1	1	1	1
5	11.93	1	–	–	1	1	1	1	1	3
6	10.94	1	–	–	–	1	1	1	1	4
7	7.82	–	1	1	–	1	1	–	1	2
8	10.23	1	–	–	–	–	–	2	–	3
9	6.70	–	1	–	1	1	–	2	–	–
10	6.95	–	1	–	1	1	1	2	–	–
11	8.45	–	1	1	1	–	1	2	–	–
12	6.03	–	1	–	1	–	–	–	–	3
Tot.	120.67	7	6	5	5	5	6	12	5	22

WRITTEN EXERCISES

Writing in a column the amounts due the different employees, she inserts on a line with each the denominations required for the envelope. The footings at the bottom give the total number of each denomination. This she verifies when she makes out the change slip.

WAGE TABLES

To insure the correctness of the pay rolls, Miss White obtains the results by two different methods: one by performing the calculations in the common way, and the other by the use of the wage tables.

PORTION OF WEEKLY WAGE TABLE

Hrs.	Rate per 44-hour week						Hrs.
44	$15	$12	$11	$10	$9	$8	44
¼	.0852	.0682	.0625	.0568	.0511	.0455	¼
½	.1704	.1364	.125	.1136	.1023	.0909	½
¾	.2557	.2045	.1875	.1705	.1534	.1364	¾
1	.3409	.2727	.25	.2273	.2046	.1818	1
2	.6818	.5455	.50	.4545	.4091	.3636	2
3	1.0227	.8182	.75	.6818	.6136	.5455	3
4	1.3636	1.0909	1.—	.9091	.8182	.7273	4
5	1.7045	1.3636	1.25	1.1364	1.0228	.9091	5
6	2.0455	1.6364	1.50	1.3636	1.2273	1.0909	6
7	2.3864	1.9091	1.75	1.5909	1.4318	1.2727	7
8	2.7273	2.1818	2.—	1.8182	1.6364	1.4545	8
9	3.0682	2.4545	2.25	2.0455	1.8410	1.6364	9
10	3.4091	2.7272	2.50	2.2727	2.0454	1.8182	10
20	6.8182	5.4545	5.—	4.5454	4.0909	3.6364	20
30	10.2273	8.1818	7.50	6.8182	6.1364	5.4545	30
40	13.6364	10.9091	10.—	9.0909	8.1818	7.2727	40
50	17.0454	13.6364	12.50	11.3636	10.2272	9.0909	50

WRITTEN EXERCISES

1. From the foregoing table, find the pay at the rate of $15 per 44-hour week for (*a*) 36 hours, (*b*) 27 hours, (*c*) 43½ hours, and (*d*) 39¼ hours.

METHOD

(*a*) 30	hr.	$10.2273	(*b*) 20	hr.	
add 6	"	2.0455	add 7	"	
36	hr.		27	hr.	
(*c*) 44	hr.	$15. —	(*d*) 40	hr.	
less ½	"		less ¾	"	
43½	hr.		39¼	hr.	

To obtain the answer to (*a*), take from the $15 column of the table the amount payable for 30 hours, and to this add the amount for 6 hours.

2. Find the wages payable on a weekly basis of 44 hours, for (*a*) 37 hours at $15, for (*b*) 42 hours at $12, for (*c*) 37½ hours at $10, for (*d*) 49¾ hours at $9, for (*e*) 54 hours at $8.

3. At the rate of $11 per week of 44 hours, find the amount due for (*a*) 43½ hours, for (*b*) 28¾ hours, for (*c*) 39¼ hours, for (*d*) 54¾ hours.

METHOD

At $11 for 44 hours, the hourly rate is $¼. Multiply $¼ by (*a*) 43.5, (*b*) 28.75, etc.; that is, the quotient of these by 4 gives the wages in dollars.

4. At the specified rates for 44 hours, find the wages due for services rendered as follows:

a 48 hours at	$12	per week	*d* 36 hours at	$16	per week
b 40 " "	$13	" "	*e* 55 " "	$21	" "
c 52 " "	$14	" "	*f* 33 " "	$24	" "

5. From the following time card calculate the pay due Miss Jones for the week, deducting ¼ hour for an absence of each 15 minutes, or less.

NUMBER 14
NAME *MARY E. JONES*

Day	A.M.	NOON		P.M.	Overtime		Hours
	In	Out	In	Out	In	Out	
M.	7.56	12.01	12.55	5.01			
Tu.	8.03	12.05	1.02	5.04			
W.	7.59	12.03	12.59	5.06	5.30	7.30	
Th.	8.31	12.02	1.03			6.01	
F.	7.57	12.04	12.56	5.03	5.30	6.30	
S.	7.59	12.03	12.58			2.31	

6. From the following pay slip determine the hourly rate for each day:

NAME — Marguerite Carter
OPERATION — Seaming Coats

Day	Jan.	Hr.	Earnings
Mon.	4	8¼	$2.31
Tues.	5	7½	2.25
Wed.	6	8¾	2.80
Thurs.	7	8¾	2.45
Fri.	8	8¼	2.64
Sat.	9	4¼	1.36

7. Make a graph[1] showing the fluctuations during the year in the monthly earnings of a girl employed in a "seasonal" occupation:

Jan.	$48	May	$27	Sep.	$44
Feb.	$50	Jun.	$36	Oct.	$40
Mar.	$49	Jul.	$42	Nov.	$28
Apr.	$35	Aug.	$47	Dec.	$30

[1] For a description of graphs, see Section III, p. 123.

CHAPTER THREE

SOME BUSINESS FORMS

INVOICES AND BILLS

WRITTEN EXERCISES

1. Copy and complete the following invoice:

Akron, Ohio, Apr. 7, 1920

CLARK, STOWE, & CO.
BUILDERS' SUPPLIES

SOLD TO Mr. Albert Janson.

		2450 Red Brick	30.—	73	50		
		85 bags White Sand	.30				
		85 Sand Bags	.06				
		140 bdl. Laths	4.85	67	90		

In a bill or an invoice begin with a small letter the word denoting the quantity; lb., bu., etc. Begin with a capital the name of the article. Do not use "of" or "@."

Rule your paper as shown above. Write each *extension* in the first double money column. Write the *footing* in the second double money column on the line below the last footing.

2. When 2450 bricks cost $73.50, (*a*) what is the price of one brick? (*b*) How many can be bought for $30?

3. At $4.85 per thousand, (*a*) how many laths can be bought for $67.90? (*b*) If this quantity is contained in 140 bundles, how many laths are there to a bundle?

4. When 52 bags of Portland cement cost $30.55, (*a*) what is the cost per bag? (*b*) What is the cost of a barrel of four bags?

5. One item on an invoice is 60 bags of K. W. cement, the rate per unit being $12.50, and the extension $37.50. (*a*) How many pounds are there in the purchase if a bag contains 100 pounds? (*b*) How many units are charged for? (*c*) How many pounds are there in the unit? (*d*) What is the unit?

Invoice clerks do not write "per pound," "per gross," "per ton," "per thousand," etc., the assumption being that the buyer knows the quantity represented by the given rate.

MADISON, WIS., Apr. 30, 1920

MR. ALBERT JANSON

Bought of J. P. Duffy Company

Lime, Lath, Brick, Cement, etc.

Terms, Net 30 days.

Apr.	8	1 cu. yd. Sand		3	—	
		5 bags Port. Cement	2.35	2	94	
	8	4 bbl. Marble Dust	1.75			
		104 bags Port. Cement	2.30			
	12	72 " " "	2.30			
	13	3 bbl. N. A. Plaster	1.95	5	85	
	17	12 bags K. W. Cement	12.50	7	50	
		24 " Port.	2.40			
		1 bbl. Atlas "		5	—	
	21	4500 Bricks	30.00			
		150 bdl. Laths	4.80			
		4 bags Atlas Cem.		5	—	(*a*)
		Cr.				
	27	175 M. T. bags Portland	.08			
	28	72 " " " K. W.	.06¾			
		3 " " " R. W.	.06			(*b*)
			Balance			(*c*)

In the last invoice the given price of brick is by the M; of laths, by the M, each bundle containing 100 laths; of Portland cement, a barrel of four bags; of K. W. cement, a ton of 2000 pounds, each bag containing 100 pounds.

6. Complete the foregoing invoice, which shows credits for empty (M. T.) bags returned. It differs from a *statement* by itemizing the purchases made during the month.

Write the total debits at (*a*), the total credits at (*b*), and the balance at (*c*).

This concern renders invoices (bills) once a month to regular customers. When the goods are delivered, the driver obtains a receipt in the accompanying form, from Mr. Janson's representative, to whom he gives a carbon duplicate.

In case there is any question as to articles, quantities, etc., the receipts are referred to.

MEMORANDUM

RECEIVED OF
J. P. Duffy Company
Apr. 3, 1920.

1 cu. yd. Sand
5 bags Port. Cem

Signed..A. Janson.....
per M. M. W.

The foregoing invoices are frequently called bills, the former name being more particularly applied to the next form, which designates by number the case in which each item is packed. This enables the purchaser to locate a particular article.

The goods in the next invoice are contained in two cases; one marked A. S. 53, and the other A. S. 54. Each case contains 6 pieces.

7. Complete the following invoice. Insert at (*a*) the total number of yards in the second three pieces; at (*b*) the total in the next six pieces; at (*c*) the exten-

sion for the first two lots; at (*d*) for the next three; etc. Place the footing at (*g*).

SAN FRANCISCO, Aug. 27, 1919

BUSSEY & TAYLOR

Wholesale Woolens

SOLD TO Albert Shields,
Phoenix, Arizona.

A. S.		2 c/s Woolen Mantlings		53/54			
	53	48¼ yd.					
		46¼ "	94½ yd.	1.64	(*c*)		
		47¼ "					
		47¾ "					
		45¼ "	(*a*)	1.26	(*d*)		
		61 "		2.25	(*e*)		
	54	47½ yd.					
		62¾ "					
		66¼ "					
		62½ "					
		61 "					
		48¾ "	(*b*)	1.26	(*f*)		(*g*)

When a monthly invoice contains a great number of items, several are placed on a line to economize space, as in the following.

KNOXVILLE, TENN., Oct. 1, 1920.

Mr. Henry Schlaefer

Bought of Richard H. Wattles

GRAIN, HAY, STRAW, MILL FEED

Interest charged on overdue accounts

Sep.	1	100# Perfection 1.95; 1 bag, .05	2	—
		100# Wh. Bran, 1.80; 100# Middlings, 2.— 1.09 97½	3	80
		2 bu. Corn, 2.18; 2 bu. Cr. Corn, 1.95 .05 .80	4	13
		2 bags, .10; 2 bags G. A. Salt, 1.60	1	70

NOTE: The character # before a number means "Number"; after a number, it indicates "pounds."

8. Complete the foregoing invoice by adding the following: Sep. 4, 2 bags of cracked corn, 1 bag; Sep. 8, 100 pounds Wheat Bran, 600 pounds of corn bran, 2 bushels of corn, 7 bags; Sep. 9, 100 pounds of Perfection feed, 1 bag; Sep. 10, 2 bushels of cracked corn, 1 bag; Sep. 11, 200 pounds of wheat bran, 100 pounds of middlings, 400 pounds of beet pulp, 400 pounds of corn bran, 4 bushels of corn, 2 bushels of middlings, 7 bags; Sep. 16, 100 pounds of Perfection feed, 2 bushels of cracked corn, 2 bags; Sep. 17, 200 pounds of wheat bran, 200 pounds of cotton seed meal, 200 pounds of beet pulp, 200 pounds of gluten, 100 pounds of middlings, 4 bushels of corn, 2 bags; Sep. 22, 500 pounds of gluten, 200 pounds of wheat bran, 200 pounds of beet pulp, 200 pounds of corn bran, 2 bushels of corn, 2 bushels of cracked corn, 1 bag; G. A. salt, 4 bags; Sep. 28, 200 pounds of cotton seed meal, 200 pounds of wheat bran, 200 pounds of gluten, 400 pounds of corn bran, 100 pounds of middlings, 2 bushels of corn, 2 bushels of cracked corn, 6 bags.

Use the following prices:

Cracked corn,	$ .97½ per bu.	Middlings,	$2.00 per 100 lb.
Corn,	$1.09 " "	Beet pulp,	$1.35 " " "
Wheat bran	$1.80 " 100 lb.	Cottonseed meal,	$1.85 per 100 lb.
Corn "	$1.30 per 100 lb.		
Perfection feed	$1.95 " 100 "	Gluten,	$1.85 per 100 lb.
Bags, 5 cents each		G. A. salt,	$ .80 per bag

While every invoice may be called a *bill*, bills containing items for services rendered are not *invoices*.

In bills of this kind the heading "Bought of" or "Sold to" is changed to the form given in the following

BILL FOR SERVICES AND MATERIALS

TOPEKA, KAN., Jun. 11, 1920

Mr. Robert P. Webb

85th Street and Ridge Boulevard

To *JOHN TODD*, Dr.

Plumbing Contractor

1 Range $268		48	—
2 " Couplings, ¾"	.37½		
1 Nipple, ¾"			10
10# Galv. Fittings	.15		
3 Unions	.37		
19 ft. 2 in. Pipe	.12		
1 Boiler Coupling			60
2 Couplings, water back	.62½		
10 Nipples	.13		
2 Els, ¾"	.15		
1 Tee, 1"			15
2 Elbows, 45 degrees	.15		
3 lengths Pipe	.10		
1 Damper, 6"			30
2 Black Elbows, 6"	.20		
1 Brass Ring		9	—
1 Galv. Cross			20
2 " Ells	.15		
4 " Elbows	.10		
2 Black Ells	.15		
1 Plate Rack		3	—
Time, 2¼ days	4.—		

Received Payment
June 15, 1920
JOHN TODD
per W. H. M.

9. Copy and complete the foregoing bill.

10. Make out a check on the School Bank in settlement of the foregoing bill.

RECEIPTS

A person to whom an express package is delivered, a telegram, a special delivery letter, etc., acknowledges the delivery by writing his name in the proper place in the receipt book carried by the messenger, driver, carrier, etc.

The receipt of Mr. Brown's money by a bank, on deposit, is shown by the teller's entry in the passbook. When Mr. Webb settled Mr. Todd's bill, the latter indicated the fact by "receipting" the bill. If Mr. Webb did not have the bill with him, Mr. Todd's clerk would give him the following

RECEIPT IN FULL

TOPEKA, KAN., Jun. 15, 1920.

RECEIVED OF MR. .....Robert P. Webb.....

.....100 DOLLARS

in full of account to date.

JOHN TODD

$..... per.....

11. Copy and complete the foregoing receipt, inserting the dollars in words on the third line, and in figures on the last line, expressing the cents as a fraction of a dollar in each place. Use your own initials as the clerk who receives the money for Mr. Todd.

12. Write John Whalen's receipt for $125, sent him by Hiram Hunt for the rent of his store for Jun., 1920. Insert on the third line "For rent of premises No. 4 Court Square for June, 1920."

CHECKS AS RECEIPTS

To save the time and expense of mailing receipted bills to thousands of customers making monthly settlements by check, many business concerns print at one end of the bill a coupon to be detached therefrom and inclosed with the check sent in payment, unless the customer prefers to send the bill, and to have it returned to him receipted.

If no further receipt is desired, detach this coupon and mail with your check.
The canceled check is your receipt.

Date, Jul, 31, 1920 — W. S. Julius & Co.
Mrs. J. Carroll Payne, — Folio 35814
8502 Hamilton Boulevard. — Amt. $114.42

Some bills contain a form similar to the one on the right in which the customer makes the necessary entries. If a second bill for the same purchases should be presented, the record on the original bill will furnish the number of the check. The presentation of the canceled check with its indorsement showing that it was collected by the merchant will be satisfactory evidence that the bill was paid, and that he made an error in rendering the second bill.

CUSTOMER'S RECORD
Paid by Check No. 476.....
Bank Arizona School.....
Date, Aug. 3, 1920.....

It is unnecessary to state that receipts, receipted bills, and canceled checks should be carefully preserved.

13. Copy and complete the following check by which Mrs. Payne pays the bill of A. D. Winkle for $84.75. She notes in the left the purpose of the check and

signs the check with her own name. She also notes upon her check-book stub the purpose of the check, and mails it, with the coupon. She fills out the "Customer's Record" on the bill and files away the latter.

In settlement of bill dated July 31, 1920	Tucson, Ariz., Aug. 3, 1920 No. 476 ARIZONA SCHOOL BANK Pay to the order of.....$..... .....Dollars. .....Elizabeth Payne.....

ORDERS FOR GOODS

When a merchant orders goods by mail he retains a carbon copy of the order slip in the order book (see p. 32). When he gives a verbal order to a seller's agent, the latter makes out a slip. In the accompanying one Mr. Yates, a salesman for the Fairfax Furniture Company, sells Jervis Johnson & Co. the specified articles at the prices

ORDER SLIP

Fairfax Furniture Co.
Brockton, N. Y.
Dressers and Chiffoniers
VI–9–1920

Salesman *Yates*. Order No. 231
Ship to Jervis Johnson & Co.
Address — 255 Columbia Ave., Passaic, N. J.
Ship via Penn. R. R. Terms — 60; 5/30
F. o. b. Brockton.

Quantity	No.	Finish	Price
3	2784	Mah. Dress	11 75
3	2658	" Chiff.	12 50
3	3062	Oak Dress.	12 25
3	3817	" Chiff.	12 75

J. Johnson & Co.

given. The slip shows the terms of credit and that the goods are to be delivered to the railroad company.

Mr. Yates obtains the signature of J. Johnson & Co. to the original, which he sends to the Fairfax Furniture Co. He gives a carbon duplicate to J. Johnson & Co. for their files.

The following is the invoice:

No. 647

Fairfax Furniture Company
Manufacturers of Dressers & Chiffoniers

Brockton, N. Y., June 11, 1920

Order No. 231. Dated VI–9–1920. Salesman Yates.
Sold to Jervis Johnson & Co.
255 Columbia Ave., Passaic, N. J.

Shipped by Penn. R. R. Car P. R. R. 85026
F. o. b. — R. R. Terms: 60 da. net; 5%, 30 da.

		3 Mah. Dress.	11.75			
		3 " Chiff.	12.50			
		3 Oak Dress.	12.25			
		3 " Chiff.	12.75			

NOTE; F.o.b. — R. R. means that the goods are delivered to the railroad company without charge for cartage — "Free on Board."

WRITTEN EXERCISES

1. Copy and complete the foregoing invoice.

2. Make out a check on the Passaic National Bank for the amount of the bill, dating the check about 40 days after Jun. 11.

3. Make out a check to the order of the Pennsylvania Railroad Company for the freight on 2700 pounds at 45 cents per 100 pounds.

BILL OF LADING

When the goods are delivered to the railroad, the freight agent gives the shippers a Bill of Lading, which is a form of receipt, acknowledging that the goods have been delivered to the railroad. The Fairfax Company mails this bill of lading to Jervis Johnson & Co., who present it to the freight agent at Passaic as evidence that they are the owners.

The following is an abbreviated form of a

BILL OF LADING

Pennsylvania Railroad Company #22999
Date Jun. 11, 1920

Received at Brockton, N. Y., from Fairfax Furniture Co. the property described below, in apparent good order except as noted, contents and condition of contents of package unknown.

The rate on freight from Brockton, N. Y., to Passaic, N. J. is in cents per 100 pounds

1st class	2d class	3d class	4th class	5th class	6th class	Special
			45			

Consigned to Jervis Johnson & Co., 255 Columbia Av.
Destination, Passaic, State of New Jersey
Route N. Y. C., Penn. Car Initial P. R. R. Car No. 85026

Number of Package	Description	Weight
6	Crates Dressers	750 #
6	" Chiffoniers	600 #

ANDREW JAVINS, Agent
per M. E. K.

Upon presentation of the bill of lading and the payment of the freight the twelve crates are delivered to the consignees.

FREIGHT BILL

Consignees: Jervis Johnson & Co. No. 1669
255 Columbia Av. Date VI–16–1920
To Pennsylvania Railroad Company, Dr.
Passaic Station, from Brockton, N. Y.
Waybill No. 92 Date VI–11–1919
Via N. Y. C., Penn. Car P. R. R. 85206

FREIGHT BILL

Shipper: Fairfax Furniture Co.
Original Point of Shipment: Brockton, N. Y.

Description of Articles	Weight	Rate	Charges	Total
6 crt. Dressers	1500			
6 " Chiffoniers	1200			
	2700	45		

Claims for loss or damage must be promptly made in writing to Freight Agent accompanied by this bill.
Make check payable to Pennsylvania Railroad Company

Received Payment for the Company
Jun. 16, 1920
John Carrol
Freight Agent

WRITTEN EXERCISES

1. Find the amount of the foregoing freight bill.

2. Make out a bill for the following articles bought of the Wolverine Manufacturing Company of Detroit, Mich.: 4 mahogany library tables, at $9.65; 4 at $7.75, 4 at $8.75; 8 golden oak parlor tables at $1.50; and 8 mahogany parlor tables at $1.55.

Insert catalogue numbers to denote the styles. Use Mah. for mahogany, G. O. for golden oak, Lib. for library, Par. for parlor, Tab. for table.

3. Make out a freight bill for the delivery of the foregoing goods in the home town. Obtain from the local freight agent the rate on furniture from Detroit, also a copy of a bill of lading and a blank freight bill.

Assume that the articles are shipped in 28 boxes; the legs in 8 boxes weighing, with the contents, 15 pounds each; each library table top in a box, weighing with its contents 70 pounds; the 16 parlor table tops in 8 boxes, each weighing with its contents 85 pounds.

4. Make out an order slip for the forgoing articles, using the form shown on p. 52. Insert the catalogue numbers, but not the prices.

5. Make out a check on the School Bank for the amount of the bill.

Be careful in your selection of dates to allow a proper interval to elapse between the date of the order, that of the bill, and that of the freight bill.

For other business forms see Statements, Notes, Drafts, Bills of Exchange, Trade Acceptances, etc.

SECTION II

BUSINESS CALCULATIONS

CHAPTER ONE

PERCENTAGE

PREPARATORY EXERCISES

1. A sales girl received, as part of her pay \$6 on sales of \$120. (*a*) What fraction of the amount of her sales did she receive in this way? How much should she receive, at the same rate, on sales (*b*) of \$150? (*c*) Of \$100?

2. A man received \$9 on sales of \$150. (*a*) What fraction of the amount of his sales did he receive? How much should he receive on sales (*b*) of \$200? (*c*) Of \$100?

3. (*a*) How many problems out of 20 should a boy solve who solves 95 out of 100? (*b*) How many out of 25 should a girl solve who solves 96 hundredths of her problems?

A rate of \$6 on \$100 is stated in business as 6 *per cent*, which means 6 hundredths, or .06. It is written 6%.

Any decimal may be written as a per cent by expressing it in hundredths, omitting the decimal point, and placing after it the per cent sign. Thus 5 tenths, which is equal to 50 hundredths, is written 50%; 125 thousandths, which is equal to 12½ hundredths,

is written 12½%; 36 thousandths, which is equal to 3.6 hundredths, is written 3.6%.

FINDING THE PERCENTAGE

WRITTEN EXERCISES

1. (*a*) How much does a man receive who is given 6% of $347.50? (*b*) How much weight is lost during the winter by 256 tons of hay, if the loss in weight is 3%? (*c*) What is the cost of insuring a store for $7500 when the rate is ½ of 1%? (*d*) How many pounds of butter fat are there in 231 pounds of milk, when it contains 3.6% of butter fat?

	METHOD		
Base	(*a*) $347.50	(*b*) 256 T.	(*c*) $7500
Rate	× .06	×.03	× .00½
Percentage	$20.85ϕϕ Ans.	7.68 T.	$37.50

To find the percentage, multiply the *base* by the *rate* expressed as a decimal.

2. A salesman receives a commission of 3% on the amount of his sales. How much does he receive on sales of $1575?

3. A man bought a house for $3500 and sold it at a profit of 35%. (*a*) What was his profit? (*b*) How much did he receive for the house?

4. In a school of 425 pupils 96% of them are present. (*a*) How many are present? (*b*) What per cent are absent? (*c*) How many are absent?

5. In this school 48 % of the pupils are boys. (*a*) How many boys are in the school? (*b*) how many girls?

6. How many hits does a player make in 480 attempts, when 35 % of his attempts are successful?

7. How much tax does an owner pay when he pays ¾ % of $7800 which is the valuation of his farm for purposes of taxation?

8. A contractor agrees to do a piece of work in 140 days. How many days should he require to do 65 % of the work?

9. A dealer bought a suit of clothes for $15 and sold it at an advance of 66⅔ %. (*a*) How much was the advance? (*b*) The selling price? It takes 30 % of the selling price to do business. (*c*) What did it cost him to sell the suit? (*d*) What was his net profit?

10. A man whose income is $1500 a year, spends 24 % of it for rent. How much is his rent (*a*) for a year? (*b*) For a month?

SIGHT EXERCISES

1. Change to common fractions, lowest terms:

a .4	*b* .14	*c* .124	*d* .3125
e .8	*f* .32	*g* .328	*h* .5625

2. Express as per cents:

a ½	*b* ⅓	*c* ¼	*d* ⅕	*e* ⅙	*f* ⅛
g 1/9	*h* 1/10	*i* ⅔	*j* ¾	*k* ⅖	*l* ⅗
m ⅘	*n* 1/25	*o* 1/50	*p* 3/25	*q* ⅜	*r* ⅝
s ⅞	*t* ⅚	*u* 3/10	*v* 9/50	*w* 9/25	*x* 7/50

3. Express as common fractions, lowest terms:

a 25 %	*b* 50 %	*c* 33⅓ %	*d* 87½ %
e 66⅔ %	*f* 6¼ %	*g* 75 %	*h* 80 %
i 37½ %	*j* 83⅓ %	*k* 62½ %	*l* 8⅓ %

4. Find (*a*) 25 % of 36, (*b*) 6⅔ % of 75

METHOD

(*a*) 25 % of 36 = ¼ of 36 = 9, Ans.

(*b*) 6⅔ % of 75 = 1/15 of 75 = 5, Ans.

Change per cents to fractions.

5. Give answers:

a 25 % of 96	*b* 33⅓ % of 69	*c* 12½ % of 248
d 6¼ % of 176	*e* 75 % of 72	*f* 66⅔ % of 99
g 37½ % of 480	*h* 8⅓ % of 252	*i* 50 % of 83
j 87½ % of 88	*k* 62½ % of 840	*l* 6⅔ % of 165.

6. Find (*a*) 4% of 375; (*b*) 6% of 450.

ONE WAY

(*a*) 4 % of 375 = .04 × 375 = 4 × 3.75 = 4 × 3¾ = 15, Ans.

(*b*) 6 % of 450 = .06 × 450 = 6 × 4.5 = 6 × 4½ = 27, Ans.

Instead of taking the rate in hundredths, divide the base by 100, changing the quotient to a mixed number.

7. Give answers:

a 4 % of 975	*b* 12 % of 633⅓	*c* 8 % of 937½
d 6 % of 850	*e* 16 % of 412½	*f* 6 % of 566⅔
g 8 % of 725	*h* 24 % of 216⅔	*i* 9 % of 833⅓

8. Find (*a*) 69% of 33⅓; (*b*) 88% of 37½.

METHOD

(*a*) 69 % of 33⅓ = 33⅓ % of 69 = ⅓ of 69 = 23, Ans.
(*b*) 88 % of 37½ = 37½ % of 88 = ⅜ of 88 = 33, Ans.

9. Give answers:

a 99 % of 33⅓	*b* 88 % of 25	*c* 72 % of 16⅔
d 48 % of 12½	*e* 84 % of 75	*f* 66 % of 66⅔
g 32 % of 37½	*h* 92 % of 50	*i* 56 % of 62½

WRITTEN EXERCISES

1. A merchant's sales were $14,880 last month. How much will be this month's increase at the rate (*a*) Of 25%? (*b*) Of 33⅓%? (*c*) Of 6¼%?

METHOD

(*a*)	(*b*)	(*c*)
$14,880	$14,880	$14,880
× 25	.33⅓	.06¼
$3,720 Ans.	$4,960 Ans.	$930 Ans.

Write the given per cents as shown above, but obtain the result by dividing $14,880, the base, by 4 in (*a*), by 3 in (*b*), by 16 in (*c*): that is, multiply the base by ¼, ⅓, and 1/16, the fractional equivalents of the respective rates.

2. Write answers from the book:

a 25 % of 24,672	*b* 33⅓ % of 34,569	*c* 6¼ % of 17,632
d 50 % of 17,976	*e* 11⅛ % of 96,543	*f* 8⅓ % of 12,396

METHOD BY ALIQUOT PARTS

While at school a pupil should accustom himself to the employment of methods used in the business world, specimens of which are given in the following examples:

3. Find (*a*) 37½% of 872, (*b*) 62½% of 984, (*c*) 27½% of 548, (*d*) 36⅔% of 936.

METHOD

(*a*) 37½% of 872
25% = 218
+ 12½% = 109
37½% = 327 Ans.

(*b*) 62½% of 984
50% = 492
+ 12½% = 123
62½% = 615 Ans.

(*c*) 27½% of 548
25% = 137
+ 2½% = 13.7
27½% = 150.7 Ans.

(*d*) 36⅔% of 936
33⅓% = 312
+ 3⅓% = 31.2
36⅔% = 343.2 Ans.

In (*a*) find 25% of 872 by taking ¼ of it; find 12½% of 872 by taking ½ of the one-fourth. Test (*a*) by multiplying 109 by 3; (*b*) by multiplying 123 by 5; (*c*) by multiplying 13.7 by 11; (*d*) by multiplying 31.2 by 11. Why?

4. Find answers:

a 37½% of 392	*b* 62½% of 664	*c* 27½% of 680
d 36⅔% of 780	*e* 31¼% of 384	*f* 56¼% of 768
g 52½% of 760	*h* 36⅔% of 690	*i* 18¾% of 524

5. Find (*a*) 17½%, (*b*) 68¾%, (*c*) 81¼%, (*d*) 43¾%, respectively, of 1760.

In (*a*), take 12½%, 5% and 2½%
In (*b*), take 50%, 12½% and 6¼%

6. Find answers:

a 17½ % of 564 *b* 68¾ % of 932 *c* 81¼ % of 676
d 43¾ % of 896 *e* 18¾ % of 736 *f* 37½ % of 684

7. Find (*a*) 18⅓ % of 972, (*b*) 67½ % of 784, (*c*) 38½ % of 496.

In (*a*) take 10 %, 5 %, and 3⅓ % (⅓ of 10 %)
In (*b*) take 50 %, 12½ %, and 5 % (1/10 of 50 %)
In (*c*) take 25 %, 12½ % and 1 %

8. Find answers:

a 13½ % of 864 *b* 17⅞ % of 396 *c* 122½ % of 444
d 34⅓ % of 675 *e* 18⅓ % of 555 *f* 67½ % of 712

9. There are five schools in a district. The largest has a register of 1296 pupils. The registers of the others are (*a*) 75 %, (*b*) 66⅔ %, (*c*) 87½ %, and (*d*) 83⅓ %, respectively, of the foregoing. Find the register of each.

Find ¾, ⅔, ⅞, and ⅚, respectively, of 1296 by deducting from the latter ¼, ⅓, ⅛, and ⅙, respectively, of itself.

10. Find answers:

a 75 % of 976 *b* 66⅔ % of 687 *c* 87½ % of 672
d 83⅓ % of 876 *e* 75 % of 872 *f* 66⅔ % of 876

FINDING THE RATE

PREPARATORY EXERCISES

1. When a ball player makes 100 hits in 300 attempts, (*a*) what fraction of his attempts are successful? (*b*) What per cent?

2. When a girl has worked 19 examples out of 20 (*a*) what fraction of her work has she completed? (*b*) What decimal? (*c*) What per cent?

3. A man pays 27 dollars taxes on property valued at 36 hundred dollars. (*a*) What fraction of a dollar

does he pay on each $100 of valuation? (*b*) What is the rate per cent?

4. A dealer sold at a profit of $160 an article that cost him $400. (*a*) What fraction of the cost was the profit? (*b*) What decimal? (*c*) What per cent?

5. 50 is what fraction (*a*) of 150? (*b*) Of 300? (*c*) Of 250? (*d*) Of 75? (*e*) Of 60?

6. 150 is what (improper) fraction (*a*) of 100? (*b*) Of 90? (*c*) Of 60? (*d*) Of 125?

SIGHT EXERCISES

1. What is the rate of profit (*a*) on suits costing $15 each and sold at a profit of $10 each? (*b*) On land costing $60 per acre, and sold at a profit of $75 per acre?

METHOD

(*a*) $^{10}/_{15} = \frac{2}{3} = 66\frac{2}{3}\%$, Ans. (*b*) $^{75}/_{60} = \frac{5}{4} = 125\%$, Ans. Divide the profit (percentage) by the cost (base). Express the fractional (or decimal) result as a per cent.

2. Give rates per cent:

a What per cent of 36 is 18?
b 36 is what per cent of 18?
c 23 is what per cent of 69?
d What per cent of 23 is 69?
e What per cent of 32 is 24?
f 32 is what per cent of 24?
g 99 is what per cent of 66?
h What per cent of 99 is 66?
i What per cent of 80 is 30?
j 80 is what per cent of 30?

WRITTEN EXERCISES

1. (*a*) What is a dealer's profit when he gains 15½% of his investment of $16,000? (*b*) What is his rate of profit when he gains $2480 on an investment of $16,000?

METHOD

(*a*) \$160ØØ
15½
\$2480 Ans.

(*b*) 16ØØØ)2.480
.15½
15½% Ans.

In (*a*) cancel the decimal point in one factor and two ciphers in the other. Use 160 as the multiplier (see p. 307).

In (*b*) divide \$2480, the *percentage*, by \$16,000, the *base*. Reject the dollar signs, cancel the three ciphers in the divisor and set off three decimal places in the dividend (see p. 309). Change 15½ hundredths (.15½), the quotient, to 15½ per cent (15½%).

In (*a*) are given the base, \$16,000, and the rate, 15½%, from which the percentage is to be found. In (*b*) are given the base, \$16,000, and the percentage, \$2480, from which the rate is to be found.

(*a*) may be expressed thus: 15½% of \$16,000 = ?

(*b*) may be expressed thus: ? % of 16,000 = 2480

To find how many times 16,000 equals 2480, find the number of times the former is contained in the latter.

To find the RATE, *divide the* PERCENTAGE *by the* BASE. *Express the quotient as hundredths, and replace the decimal point by the per cent sign.*

2. A salesman received \$94 commission on sales of \$3760. What was the rate?

3. A man received $26.40 yearly interest on a loan of $4800. What was the rate?

4. In a year, the population of a village increased from 720 to 768. What was (*a*) the increase for the year? (*b*) The rate per cent of increase?

5. What was (*a*) the decrease when the population fell off in a year from 768 to 720? (*b*) The rate per cent of decrease?

6. (*a*) What % of 496 is 217? (*b*) 527 is what % of 465?

METHOD

(*a*)
$$\begin{array}{r} .4375 \\ 496\overline{)217.0} \\ 1860 \\ 3720 \\ 248 \end{array}$$
43¾%, Ans.

(*b*)
$$\begin{array}{r} 1.13\frac{1}{8} \\ 465\overline{)527.} \\ 620 \\ 1550 \\ 155 \end{array}$$
113⅛%, Ans.

Test by finding (*a*) 43¾% of 496, (*b*) 113⅛% of 465

NOTE. For this form of division, see p. 307.

The foregoing tests show merely that the division has been correctly made; they do not determine that the proper number has been taken as the base (the divisor).

The latter is more clearly indicated to some pupils when they state the problem in the form of an equation as,

(*a*) ? % of 496 = 217; ? % × 496 = 217
(*b*) 527 = ? % of 465; 527 = ? × 465

This shows them that in (*a*) 217 is the percentage and that in (*b*) the percentage is 527, the base in each being the number following the word "of," which may be replaced by the sign of multiplication (×).

7. Find answers:

a 486 is what per cent of 3888?
b What per cent of 284 is 710?
c $7.77 is what per cent of $129.50?
d What per cent of 384 A. is 320 A.?
e 12½ is what per cent of 37½?
f What per cent of 12½ is 83⅓?
g $137.70 is what per cent of $17,000?

8. What per cent of hits is made by a ball player who makes (*a*) 134 hits out of 443 attempts? (*b*) 146 hits out of 487 attempts?

METHOD

	.3024			.2998	
(*a*)	443)134.0		(*b*)	487)146.0	
	1 100			48 60	
	214			4 770	
	30.2%	Ans.		30.0%	Ans.

Carry out the division to the fourth decimal place. Drop the latter if less than 5; increase the third place by 1 if the fourth figure is 5 or more.

A rate expressed as a mixed decimal generally indicates an approximation. Thus 30.2% does not necessarily mean exactly 30⅕%; 30.0% may mean that a more exact rate is somewhere between 29.95% and 30.05%.

To find the number of hits represented by 30.2% of 443, change 133.786, the product of $.302 \times 443$, to the nearest integer, 134. To find the number represented by 30% of 487, change 146.1, the product of $.3 \times 487$, to the nearest integer, 146.

Rates given in statistical tables are sometimes carried to the nearest hundredth, one or two decimal ciphers being annexed, for the sake of uniformity, even when the rate is an integer.

Baseball records are generally printed as three-place decimals; such as, .380, .295, .400, etc. These are generally spoken of as 380, 295, 400, etc., omitting the denomination "thousandths." A thoughtless person may say "380 per cent," and a careless newspaper may use the expression "Per Cents" at the head of the table of records, when the latter contain the prefixed decimal point.

9. Find the records of each of the following players, giving the rate as a decimal to the nearest thousandth.

a Cobb, 195 hits out of 507 times at bat
b Sisler, 187 hits out of 530 times at bat
c Felsch, 160 hits out of 511 times at bat
d Speaker, 159 hits out of 457 times at bat
e Veach, 149 hits out of 491 times at bat
f Chapman, 149 hits out of 488 times at bat
g Lewis, 143 hits out of 466 times at bat
h Roth, 130 hits out of 429 times at bat
i Whose is the better record, Roth's or Veach's?

10. Find results as per cents correct to one decimal place:

In these examples carry the division out to only three decimal places. Give each answer as a per cent and a tenth. The *nearest* tenth is not called for.

a What per cent of 1325 is 476?
b \$380.50 is what per cent of \$250.75?
c What per cent of 187 A. is 83 A.?

d 692 bu. is what per cent of 463 bu.?
e What per cent of $191.75 is $47.50?
f 165 lb. is what per cent of 3329 lb.?
g What per cent of 365 da. is 56 da.?
h $33.92 is what per cent of $283.11?
i What per cent of 431 mi. is 653 mi.?
j 823 gal. is what per cent of 237 gal.?

FINDING THE BASE

PREPARATORY EXERCISES

1. What should be the amount of a girl's sales to entitle her to a commission of $6 when the rate is 5%?

2. How much must a man invest to obtain an annual income of $1200 when the investment pays 4% a year?

3. Give the cost of an article when $30 is (*a*) ⅔ of the cost; (*b*) ¾ of it; (*c*) ⅚ of it; (*d*) ⅓ of it; (*e*) ½ of it; (*f*) ⅞ of it.

4. What is the cost of goods when profits of $120 are (a) 25% of the cost? (*b*) 33⅓%? (*c*) 20%? (*d*) 16⅔%? (*e*) 12½%? (*f*) 6¼%?

SIGHT EXERCISES

1. Give the base:

a 25 is 12½% of what?
b 34 is 20% of what?
c 32 is 33⅓% of what?
d 66 is 30% of what?
e 24 is 37½% of what?
f 72 is 40% of what?
g 36 is 66⅔% of what?
h 48 is 60% of what?
i 30 is 83⅓% of what?
j 56 is 70% of what?

2. Give answers:

a 24 is 3% of what?
b 24 is 120% of what?
c 32 is 4% of what?
d 30 is 125% of what?
e 30 is 5% of what?
f 50 is 200% of what?
g 36 is 6% of what?
h 99 is 110% of what?
i 21 is 7% of what?
j 60 is 300% of what?

3. Give answers:

a Base, \$300; rate, 6%. Percentage?
b Rate, 25%; percentage, \$30. Base?
c Percentage, \$60; base, \$120. Rate?
d Base, \$250; percentage, \$50. Rate?
e Rate, 33⅓%; percentage, \$12. Base?
f Percentage, \$24; rate, 20%. Base?

WRITTEN EXERCISES

1. A dealer's profit of \$2480 is 15½% of his investment. What is his investment?

METHOD

.15½ × Investment = \$2480
Investment = \$2480 ÷ .155
\$16,000 Ans.
/155)\$2480/000.
930
0

Since .15½ times the investment is \$2480, the investment is found by dividing \$2480 by .155.

To find the BASE, *divide the* PERCENTAGE *by the* RATE *expressed as a decimal.*

2. How much insurance can a property owner obtain for \$37.50 when the rate is ¾%?

.00¾ of ? = \$37.50 ? = \$37.50 ÷ .00¾

3. How much must be an agent's sales to give him a commission of \$106.20 when the rate is 2¼% of the sales?

4. How much must be loaned at 5½% per year to realize \$27.50 interest annually?

5. This year's register is 15% greater than that of last year. What was last year's register if there is an increase this year of 24 pupils?

6. There were promoted at the end of the term 240 pupils, which was 96% of the register. What was the register?

NOTE: If the resulting per cent is not an integer, express it as a mixed number when the fractional part contains small numbers; otherwise, express it as a mixed decimal to nearest tenths, even when the latter is a cipher (0).

7. Find the per cent of butter fat in milk when 2480 pounds of the latter yield 136½ pounds of butter fat.

8. A manufacturer expended in a year $43,625 for materials, $46,750 for labor, and $12,375 for other manufacturing expenses. (*a*) Find the total cost of the product. What per cent of the total cost was paid (*b*) for materials? (*c*) For labor? (*d*) For other manufacturing expenses?

9. When the dressed weight of a steer is 56% of its live weight, what should be the live weight to give the butcher 868 pounds of meat?

10. What per cent of a long ton (2240 pounds) is a short ton (2000 pounds)?

11. A merchant has $27,500 with which to pay debts amounting to $35,000. What per cent of his indebtedness can he pay?

12. A school having 725 pupils on register transferred 87 of them to a neighboring school. What per cent of the pupils were transferred?

13. (*a*) Last year a planter raised an average of 360 pounds of cotton to the acre. This year's yield is

54 pounds greater. What is the rate of increase? (*b*) What is the rate of decrease when a yield of 414 pounds to the acre is a decrease of 54 pounds?

14. What does a merchant receive for a parlor set bought for $184.50 and sold (*a*) at an increase of 40%? (*b*) At a loss of 6%?

METHOD

(*a*) $184.50	(*b*) $184.50
+ 40% 73.80¢	− 6% 11.07¢¢
Ans. $268.30	Ans. $173.43

(*a*) Find 4 tenths of $184.50 by multiplying the latter by 4, writing the first figure of the product as thousandths (3 decimal places).

(*b*) Find 6 hundredths by multiplying $184.50 by 6, writing the first figure of the product as ten-thousandths (4 decimal places).

15. Find the selling prices of the following:

a Cost, $47.60; gain, 30% of cost.
b " $23.40; loss, 15% " "
c " $37.50; gain, 60% " "
d " $92.64; loss, 25% " "
e " $83.20; gain, 30% " "
f " $76.15; loss, 20% " "
g " $41.50; gain, 18% " "
h " $56.25; loss, 12% " "
i " $83.90; gain, 40% " "
j " $12.50; loss, 24% " "

16. (*a*) What per cent of the cost is gained by a man when he sells for $420 a horse that cost him $360?

(*b*) What per cent of the cost does a man lose by selling for \$360 a horse that cost him \$420?

17. What per cent of the cost is made on an article bought for \$183.75 and sold for \$221.75?

18. Find the per cent of the cost that is gained or lost on each of the following:

a Cost, \$356; selling price, \$475
b Selling price, \$129.50; cost, \$87.75
c Cost, 36¢; selling price, 46¢
d Selling price, \$123; cost, \$150
e Cost, 87½¢; selling price, 83¢
f Selling price, \$250; cost, \$260
g Cost, \$25.60; selling price, \$30
h Selling price, \$84; cost, \$75
i Cost, \$1.50; selling price, \$1.63
j Selling price, 33⅓¢; cost, 37½¢

19. What per cent of the cost is made on goods sold for \$260, in which (*a*) the gain was \$40? (*b*) The loss was \$30?

First find the cost, which is in (*a*) \$40 *less* than the selling price, in (*b*) \$30 *more* than the selling price.

20. What does a merchant lose on an article sold for \$118.90, which was 18% less than cost?

$$\text{Cost} = \frac{\$118.90}{.82}; \quad \text{loss} = 18\% \text{ of cost} = \frac{\$118.90 \times .18}{.82}$$

Do not find the cost. After writing it as shown above, write .18 after it as a multiplier.

21. Find the selling price of goods costing \$125 and sold at an advance of 36%.

22. What per cent above the cost was made on sales amounting to \$896.10, on which the profit was \$26.10?

23. How much was lost on sales of $142.80 when goods were sold 15% below cost?

24. What per cent of the cost was realized on goods costing $245 and sold at $300?

25. When 20% of the cost was lost on goods sold for $324, what was the cost?

RATE OF PROFIT

A bank official who had bought a house for $3600 and sold it for $4500 would possibly consider that, in making $900 by the transaction, he had realized 25% on his investment of $3600. A merchant, however, would look upon it as a gain of 20%, meaning that of the $4500 received 20% was profit.

A dry goods dealer who fixes his selling price of silk at an advance of 25% above the invoice cost of $1.60 per yard, thinks of his profit as 20% of the selling price of $2. When, therefore, he makes an estimate of his gross profits on sales of $4000, he takes 20% of the latter as the profit, even though he may have been taught in school that he should first find the cost of the goods, $3200, by dividing $4000 by 1.25, and then deduct this quotient from $4000 to ascertain the profit.

SIGHT EXERCISES

1. When Mr. Jones sells for $27 a table that cost him $18, what fraction (*a*) of the selling price is the cost? (*b*) Of the cost is the selling price? (*c*) Of the cost is the profit? (*d*) Of the selling price is the profit?

NOTE: The fraction may be an improper one.

2. Give the fraction the selling price is of the cost in each of the following:

	Cost	*S. P.*		*Cost*	*S. P.*		*Cost*	*S. P.*
a	$24	$36	*b*	21¢	28¢	*c*	$1.60	$2.00
d	$15	$18	*e*	18¢	21¢	*f*	$1.60	$1.80

3. Give the per cent the selling price is of the cost in each of the following:

	Cost	*S. P.*		*Cost*	*S. P.*		*Cost*	*S. P.*
a	$32	$48	*b*	18¢	24¢	*c*	$2.80	$3.50
d	$60	$72	*e*	54¢	63¢	*f*	$1.20	$1.35

4. For each of the following transactions, state what fraction the profit is (I) of the cost, (II) of the selling price:

	Cost	*S. P.*		*Cost*	*S. P.*		*Cost*	*S. P.*
a	$50	$75	*b*	36¢	48¢	*c*	$1.20	$1.50
d	$20	$24	*e*	60¢	70¢	*f*	$2.40	2.70

5. For each of the following, state what per cent the profit is (I) of the cost, (II) of the selling price:

	Cost	*S. P.*		*Cost*	*S. P.*		*Cost*	*S. P.*
a	$40	$60	*b*	60¢	80¢	*c*	$3.60	$4.50
d	$30	$36	*e*	36¢	42¢	*f*	$4.20	$4.80

6. Give the per cent of profit on the selling price that corresponds to each of the following per cents of the cost:

a 150%	*b* 133⅓%	*c* 125%	*d* 120%
e 116⅔%	*f* 114⅔%	*g* 112½%	*h* 111⅑%

NET PROFIT

In the foregoing examples the *gross* profit has been considered. This is the difference between the invoice price of an article and the sum received for it. In determining the *net* profits of a business, all expenses incurred in buying and in selling must be taken into account.

WRITTEN EXERCISE

The books of H. L. Mathews & Co. show the following receipts and expenditures:

Gross receipts from sales		$159,000
Expenditures for merchandise	$106,000	
Salaries, commissions, etc.	31,800	
Rent, taxes, etc.	6,360	
Other expenses	2,120	(*a*)
Net profit		(*b*)

Find (*a*) total expenditures, and (*b*) the net profit. (*c*) What per cent of the total receipts is the net profit? What per cent of the total receipts is expended (*d*) for merchandise? (*e*) For salaries, commissions, etc.? (*f*) For rent, taxes, etc.? (*g*) For other expenses?

CHAPTER TWO

COMMERCIAL DISCOUNTS

CASH DISCOUNT

The following is an invoice for a half chest of gunpowder tea and one of imperial tea. The accompanying marks designate the grade of each.

The gross weight of each is given, also the tare and the net weight.

The terms of the sale state that payment is due in 4 months, and that a discount of 3% of the face of the bill will be allowed if payment is made in 10 days.

San Francisco, Cal., January 21, 1920

Messrs. John Ahern & Co.,
San Bernardino, Cal.

Bought of S. Collard & Co.,
104 Front St.

Terms 4 mo.; cash 10 da., less 3%

	1 h/c Gunp. 93–22–71	T 26	.38
	1 h/c Imp. 96–22–74	L.W.C.	.35

WRITTEN EXERCISES

1. Copy and complete the foregoing invoice.

2. (*a*) How much discount will be allowed if the invoice is paid in January, 1920? (*b*) What sum will settle the invoice on this date?

The terms are sometimes expressed in a shortened form; 60–2/30, meaning that a credit of 60 days is granted, with discount of 2% for payment within 30 days.

3. Find the sum that will settle each of the following bills (invoices) on the date specified:

a	*b*
Bought Nov. 16, 1920	Bought Mar. 23, 1921
2 library tables at $9.65	7 cases milk $3.50
4 parlor tables at $2.50	3 " " 4.20
Terms 60–2/30	Terms 30–1½/10
Paid Dec. 4, 1920	Paid Apr. 4, 1921

The terms of the following invoice, 60 – 2/10 – 1/30 indicate a credit of 60 days, a discount of 2% for payment within 10 days, or 1% for payment within 30 days.

4. W. S. Goodnough buys of John Ziegler & Co., on January 7, 1920, 1½ doz. milk kettles @ $18 per dozen, and 2½ doz. dippers @ $2.10 per dozen. What sum will settle the bill (*a*) on February 14, 1920? (*b*) On January 12, 1920? (*c*) On March 12, 1920?

5. A grocer bought on March 1, 1921, 1500 pounds of coffee at 18.75¢ per pound. Find (*a*) the net amount of the bill; that is, the sum due at the expiration of the credit period. (*b*) The sum required to pay the bill on March 10 with 1% discount. (*c*) The sum payable on March 13, if the seller allows a discount for 48 days at the rate of 6% per year.

TRADE DISCOUNTS

"List Prices"

Many manufacturers issue catalogues describing their products. The prices given in these catalogues (*list prices*) are much higher than those actually charged to dealers, being subject to a *trade discount,*

which is not specified in the catalogue, but is contained in a *discount sheet* supplied only to customers. When rates are changed, a new discount sheet is sent out.

The following bill (invoice) for sewer pipes provides for specified trade discounts. A cash discount of 2% of the net amount is offered for payment within 15 days.

The *net* amount of a bill is generally taken as the sum required to settle the bill at the end of the credit period, viz., $188.24 in the one given below; that is, the sum remaining after the deduction of the trade discounts.

Denver, Colo., April 26, 1920

Messrs. Tully & Larkin
Manitou, Colo.

Order #53516

Bought of

AMERICAN SEWER PIPE COMPANY

Terms: 30 da.; 15 da. less 2%

Buyer's Order No.	Pieces	Size	Kind	List Price	Gross Amt.	Net	Total
1149	400	6″	Pipe #2	80	320—		
	15	15″	" "	2 70	(*a*)		
	15	24″	" "	6 50	(*b*)		
					458—		
		Dis	ct. 72%		(*c*)	128 24	
	250	8″	Pipe #3	80	(*d*)		
		Dis	ct. 70%		140—	60—	
							188 24

WRITTEN EXERCISES

1. Copy the foregoing bill, filling out the missing extensions; (*a*), (*b*) and (*d*), also (*c*) the missing discount.

2. What sum will pay this bill on May 25?

3. Find the net amount of a bill for 300 pieces of 8″ pipe at $1.10, and 50 pieces of 12″ pipe at $2, less 74%.

SIGHT EXERCISES

1. What is (*a*) the discount on a purchase of pipe listed at $250 when the rate is 72%? (*b*) The net price? (*c*) What per cent of the list price is the net price?

2. When the discount is 70% (*a*) what per cent of the list price is the net price? (*b*) What is the net price of an article listed at $333?

3. When the discount is 90%, what is the net price of an article listed at $475?

4. Give answers:

a	List price,	$150;	discount rate,	15%	Discount?
b	" "	203;	" "	30%	Net price?
c	" "	320;	" "	20%	Discount?
d	" "	110;	" "	40%	Net price?
e	" "	284;	" "	50%	Discount?
f	" "	560;	" "	25%	Net price?
g	" "	675;	" "	10%	Discount?
h	" "	222;	" "	60%	Net price?
i	" "	102;	" "	18%	Discount?
j	" "	313;	" "	90%	Net price?

COMPOUND DISCOUNTS

Some discount sheets offer two, three, or more, successive discounts on a given article: 25 and 5%, for example; 33⅓, 15, and 10%; 35, 10, 5, and 2½%; etc.

In expressing these, the per cent sign (%) is written only after the last rate of a series. On bills, these compound discounts are frequently written thus: 25/5, 33⅓/15/10, 35/10/5/2, without the per cent sign.

The general method of determining the net price of an article subject to a compound discount is shown in the following example. The first discount is taken

on the list price, the next is taken on the remainder left after the deduction of the first discount, the next is taken on the remainder left after the deduction of the second discount, etc.

WRITTEN EXERCISES

1. Find the net price of an article "listed" at \$102, and subject to discounts (*a*) of 25 and 5%; (*b*) of 33⅓, 15, and 10%; (*c*) of 35, 10, 5, and 2½%.

METHOD

(*a*)

List price	\$102.
Less 25%	25.50
Remainder	\$76.50
Less 5%	3.825
Net price	\$72.68

(*b*)

List price	\$102.
Less 33⅓%	34.
1st Remainder	\$68.00
Less 15%	10.20
2d Remainder	\$57.80
Less 5%	2.89
Net price	\$54.91

(*c*)

List price	\$102.
Less 35%	35.70
1st Remainder	\$66.30
Less 10%	6.63
2d Remainder	\$59.67
Less 5%	2.983
3d Remainder	\$56.687
Less 2½%	1.417
Net price	\$55.27

Test each result by taking the separate discounts in a different order; in (*a*) 5 and 25%; in (*b*) 5, 15, and 33⅓%; and in (*c*) 2½, 5, 10, and 35%.

2. Find the net price of each:

List price	*Discount Rate*	*List price*	*Discount rate*
a $104	33⅓ and 15 %	*b* $200	35, 10, and 5 %
c 220	25 and 10 %	*d* 300	33⅓, 15, and 10 %
e 310	45 and 5 %	*f* 100	45, 10, and 2½ %
g 201	15 and 10 %	*h* 150	15, 5, and 3 %
i 142	35 and 5 %	*j* 400	25, 10, and 10 %

3. What is the net price of an article listed at $275 and subject to a discount of 60 and 20 %?

METHOD

List price $275. (*a*)
40 % of (*a*) 110. (*b*)
80 % of (*b*) $ 88. Net price

Use as successive multipliers the *complements* of the per cents constituting the discount rate.

Check the work by reversing the order of the multipliers.

To obtain the *complement* of a per cent, deduct it from 100 %. Thus: 25 % is the complement of 75 %, 95 % is the complement of 5 %, etc.

4. Write from the book the net price of each of the following:

List price	*Discount*	*List price*	*Discount*	*List price*	*Discount*
a $123.40	20 %	*b* $312.20	40 %	*c* $369.36	66⅔ %
d 211.15	30 %	*e* 156.84	50 %	*f* 248.24	87½ %
g 486.40	75 %	*h* 215.25	60 %	*i* 486.12	83⅓ %

5. Using the complements of the given rates, find the net price of each of the following:

a $420; 60 and 20 % *b* $465; 50 and 30 % *c* $450; 66⅔ and 20 %
d 352; 50 and 30 % *e* 352; 75 and 30 % *f* 864; 87½ and 30 %
g 275; 40 and 40 % *h* 576; 60 and 30 % *i* 648; 83⅓ and 20 %

6. Copy and complete the following bill for iron pipes. Take the quantity given in feet and inches at the list price per foot.

Use the "gross" column only when more than one item is subject to the same discount.

INTERNATIONAL TUBE COMPANY

Birmingham, Ala.

March 29, 1921

SOLD TO Thomas N. DeLaney, Wilmington, N. C.

Agency Order 5188
Customer's Order 3716
Car S. A. L. 75190
F.o.b. Wilmington

Terms 60-2/10. Route S. A. L.

Bdls.	Size	Description	Feet	In.	List Price	Gross	Total	Dis.	Net
15	$\frac{3}{4}$″	Wro't Pipe	5322	6	.10		532 25	60–20	(*e*)
15	$\frac{3}{4}$″	" "	2093	3	.20	418 65			
25	1″	" "	2 82	4	.30	804 70	1223 35	70–20	(*f*)
15	$\frac{1}{2}$″	" "	3652	9	.16		(*a*)	75–20	(*g*)
25	$1\frac{1}{4}$″	" "	1562	–	.40	(*b*)			
25	$1\frac{1}{2}$″	" "	1534	8	.50	(*c*)	(*d*)	70–30	(*h*)
									$ (*i*)

NOTE: *F. o. b. Wilmington* means that the goods are delivered at the R. R. station at Wilmington without charge for cartage at Birmingham or freight charges to Wilmington. The buyer is expected to remove them promptly from the freight car upon notification of the arrival of the latter. The invoice gives the designation of the car S. A. L. (Seaboard Air Line) and its number.

SIGHT EXERCISES

1. Give the per cent of the list price equal to:

a 40 % of 80 % of it.
b 33⅓ % of 80 % of it.
c 50 % of 70 % of it.
d 12½ % of 70 % of it.
e 25 % of 70 % of it.
f 16⅔ % of 80 % of it.

2. What per cent of the list price is the net price when the discount rate is:

a 20 and 10 %?	*b* 30 and 20 %?	*c* 75 and 20 %?
d 40 and 20 %?	*e* 50 and 10 %?	*f* 66⅔ and 10 %?
g 30 and 10 %?	*h* 40 and 30 %?	*i* 87½ and 20 %?
j 60 and 20 %?	*k* 70 and 10 %?	*l* 83⅓ and 40 %?

3. Give the net price of each of the following:

List price	*Discount*	*List price*	*Discount*
a $444	75 and 20 %	*b* $312	66⅔ and 10 %
c $695	87½ and 20 %	*d* $547	83⅓ and 40 %

4. What single discount equals a double discount of 40 and 20 %?

When the discount is 40 and 20 %, the net price is 60 % of 80 % of the list price; that is, it is 48 % of the list price. The discount is, therefore, 52 % of the list price (100 %–48 %).

A shorter method to obtain the latter is to deduct from the sum of the successive discounts their product.

$$(40\% + 20\%) - (40\% \text{ of } 20\%) = 60\% - 8\% = 52\%$$

5. Give the single discount equal to each of the following:

a 60 and 10 %	*b* 50 and 40 %	*c* 90 and 10 %	*d* 33⅓ and 10 %
e 80 and 20 %	*f* 60 and 30 %	*g* 70 and 20 %	*h* 66⅔ and 10 %
i 70 and 10 %	*j* 80 and 10 %	*k* 60 and 40 %	*l* 83⅓ and 10 %

6. Which is the better discount for the buyer?

a 60 and 10 % or 50 and 20 %	*b* 40 and 20 % or 30 and 30 %
c 80 and 20 % or 70 and 30 %	*d* 50 and 40 % or 60 and 30 %
e 50 and 30 % or 40 and 40 %	*f* 30 and 20 % or 40 and 10 %

WRITTEN EXERCISES

1. Two manufacturers list a certain grade of piano at $975. One offers a discount of 60 and 20 %; the other offers 50 and 30 %. (*a*) What per cent of the list price does the purchaser save by taking the better

offer? (*b*) How much money does he save on each piano purchased at the lower rate?

2. What single discount is equal to a discount of 45, 10, and 5%?

METHOD

A discount of 45 and 10% = 45% + 10% − (10% of 45%) = 55% − 4.5% = 50.5%; a discount of 50.5 and 5% = (50.5% + 5%) − (5% of 50.5%) = ?

First combine two of the successive discounts into an equivalent single discount; then combine the latter and the third successive discount into an equivalent single discount.

3. Find the single discount equivalent to each of the following:

a 40, 10, and 10%
b 50, 20, 10, and 5%
c 30, 20, and 10%
d 60, 30, 10, and 5%

4. What per cent of the list price is the net price when the discount rate is 50, 30, and 20%?

METHOD

Using the complements, take 50% of 70% of 80%, which can be simplified by taking 50% of 80% of 70%.

5. What per cent of the list price is the net price when the discount rates are, respectively?

a 75, 20, and 10%
b 87½, 20, and 10%
c 66⅔, 10, and 5%
d 83⅓, 40, and 5%
e 45, 10, and 5%
f 60, 30, and 10

METHOD

	(e)
	55 %
less 1/10	5.5
	49.5 %
less 1/20	2.475
	47.025 %

Many accountants prefer the deduction of 1/10 to the multiplication, by 90 %. All prefer the deduction of 1/20 to the multiplication by 95 %.

Begin with 55 %, the complement of 45 %.

6. What per cent of the list price is the net price when the discount rates are, respectively?

a 55, 15, and 5% *b* 45, 15, 10, and 5%

To be enabled to make calculations more rapidly, bill clerks use tables showing the per cent of the list price to be taken in determining the net price of articles subject to a compound discount. The table also gives the equivalent single discount.

The following shows a portion of one of the pages:

COMPOUND DISCOUNT TABLE

Supplementary Discounts			30		40		50		60	
			Dis.	Net	Dis.	Net	Dis.	Net	Dis.	Net
			.30	.70	.40	.60	.50	.50	.60	.40
5			.335	.665	.43	.57	.525	.475	.62	.38
10			.37	.63	.46	.54	.55	.45	.64	.36
10	5		.4015	.5985	.487	.513	.5725	.4275	.658	.342
15			.405	.595	.49	.51	.575	.425		
15	5		.43475	.56525	.5155	.4845	.59625	.40375		
15	10		.4645	.535	.541	.459	.6175	.3825		
15	10	5	.491275	.508725	.56395	.43605	.636625	.363375		
20			.44	.56	.52	.48				
20	5		.468	.532	.544	.456				
20	10		.496	.504	.568	.432				
20	10		.5212	.4788	.5896	.4104				

7. Calculate (*a*) the missing net rates; (*b*) the missing equivalent single discounts.

Check results by finding the total of (*a*) and (*b*) in each case.

8. Find the net price of goods subject to a discount of 30, 15, 10, and 5 % and listed (*a*) at \$4812; (*b*) at \$481.20.

9. Find the net price of each of the following:

List price	*Discount*	*List price*	*Discount*
a \$568	30, 15, 10, and 5 %	*b* \$4812	40, 15, and 5 %
c \$328	50, 15, 10, and 5 %	*d* \$6408	50, 15, and 5 %

10. Find the net price of goods subject to a discount of 30, 15, 10, and 5 % and listed (*a*) at \$3876; (*b*) at \$387.60.

11. Find the net price of each of the following:

List price	*Discount*	*List price*	*Discount*
a \$678	30, 15, 10, and 5 %	*b* \$4929	40, 15, and 5 %
c \$547	50, 15, 10, and 5 %	*d* \$6457	50, 15, and 5 %

QUANTITY DISCOUNTS

Manufacturers of certain staples frequently offer discounts dependent upon the quantity purchased.

The following bill shows a deduction of 20 cents per 100 pounds from the rate charged to smaller buyers.

To withhold from other customers information as to the *jobber's discount*, it is inserted by means of a rubber stamp only upon the bills of such purchasers as are entitled to receive it.

San Francisco, Cal., Jan. 8, 1921

Messrs. Jno. Ziegler & Co.

Bought of

THE PACIFIC SUGAR REFINING COMPANY

Terms 10 days; cash less 1% 7 days

Bbl.	Cases	Bags		Net	Price				
40			Fine #5	12790	7.34	(a)			
10			" #2	3200	7.44	(b)			
				(d)		(c)			
			Less 5¢ and 10¢ per 100 lb.			(e)			
			" special 5¢ " "			(f)		(g)	

WRITTEN EXERCISES

1. Copy the foregoing bill, filling in the omitted items.

2. Give the sum that will settle it if payment is made on Jan. 12.

Sometimes the discount is conditioned upon payment within a specified time.

Indian Rock, Va., Aug. 2, 1919

Messrs. Popkins & Appich
Meadow Springs, Ohio

Bought of *SWIFT & STEVENSON*
Manufacturers of Building and Agricultural Lime

130 bbl. Lime 1.35 $

Terms:

5¢ per bbl. discount if paid in 10 days from date of bill, 60 days net.

When the allowance of a discount, or its amount, depends upon the time of payment, the seller does not enter the discount upon the bill. The buyer makes the proper deduction, if any, and sends his check for the remainder.

CHAPTER THREE

SIMPLE INTEREST

LENDING MONEY

Banks, life insurance companies, individuals, etc., are always ready to loan their spare funds to reputable borrowers that furnish satisfactory security to pay a fair rate for the use of the money and to return the latter at a specified time.

The sum loaned is called the PRINCIPAL.

The sum paid for the use of the principal, is called the INTEREST.

The per cent of the principal to be paid for its use for a year, is called the RATE.

BONDS

Among the borrowers are the United States, individual states, counties, cities, railroads, etc. As evidence of the loan the lenders receive *bonds*. These documents specify the sum loaned, the rate of interest, the time of successive interest payments (which are generally made half yearly or quarterly), and the date for the repayment of the principal.

MORTGAGES, DEEDS OF TRUST

The owner of a house or a farm can borrow money by giving, as security, a *mortgage* or a *deed of trust*.

Either document provides that if the borrower defaults in making payments of interest or principal, the lender may cause the property to be sold, and the amount due to be paid him from the proceeds.

BANK BOOK

The evidence of the loan to a savings bank by a depositor is given by the entry made in the latter's *pass book.*

PROMISSORY NOTES

A common evidence of indebtedness is a *promissory note,* which every lender should require in the absence of other security. This should show the sum loaned, the date of the loan, the time for its repayment, the rate of interest, etc.

PREPARATORY EXERCISES

1. A man borrows $1000 on a mortgage, agreeing to pay annually for the use of the money 6% of the sum borrowed. (*a*) How much does he pay each year? (*b*) If he makes these payments every six months, how much is each semi-annual payment?

2. At 6% per year, how much should a person pay (*a*) for 1 month's use of $1000? (*b*) For 2 months' use? (*c*) For 5 months' use?

3. A woman has $400 on deposit in a savings bank. If the bank allows her 4% per year, how much will her money earn in 6 months?

4. How much interest does a girl receive every 3 months on a $50 Liberty Bond that pays 4 per cent interest each year?

SIGHT EXERCISES

1. Give the interest on \$1200 for 1 year at:

a 6% *b* 5% *c* 7% *d* 4½% *e* 5½% *f* 4¼%
g 8% *h* 4% *i* 3% *j* 6½% *k* 3½% *l* 5¼%

2. Give the interest at 6% for 1 year on:

a \$100 *b* \$150 *c* \$200 *d* \$250 *e* \$1200 *f* \$1250
g \$125 *h* \$225 *i* \$325 *j* \$425 *k* \$1500 *l* \$2100

3. Give the interest for 1 year on:

a \$150 at 5% *b* \$225 at 4% *c* \$1230 at 3%
d \$310 at 6% *e* \$510 at 8% *f* \$2010 at 7%

4. Give the interest at 6% on:

a \$150 for 2 yr. *b* \$300 for ½ yr. *c* \$400 for 6 mo.
d \$200 for 3 yr. *e* \$400 for ¼ yr. *f* \$600 for 4 mo.

5. Give the interest on:

a \$200 at 5½% for 3 yr. *b* \$300 at 5% for 6 mo.
c \$120 at 4½% for 2 yr. *d* \$500 at 4% for 3 mo.
e \$400 at 3½% for 4 yr. *f* \$100 at 6% for 8 mo.

6. Assuming the year to consist of 360 days, give the interest at 6% on \$300 for:

a 120 da. *b* 180 da. *c* 90 da. *d* 60 da. *e* 40 da. *f* 20 da.

WRITTEN EXERCISES

1. (*a*) In 3 years at 5½%, how much interest does a man pay on a mortgage of \$1580? (*b*) How much interest is paid on a loan of \$875 in 4½ years at 6%?

METHOD

(*a*)	Principal	$1580	(*b*)	Principal	875
	Rate	× .05½		Rate	× .06
		790		Interest for 1 yr.	$52.50
		7900		Time in years	× 4½
	Interest for 1 yr.	$86.90			2625
	Time in years	× 3			21000
	Int. for 3 yr.	$260.70		Int. for 4½ yr.	$236.25

Reverse the order of the factors, in (*a*) multiplying $1580 by 3 and this product by 5½; or, combine the factors into a single one, multiplying $1580 by 16½. In (*b*) multiply $875 by 27.

To find the interest multiply the PRINCIPAL *by the* RATE (*expressed as hundredths*) *by the* TIME (*in years*).

2. Find the interest on:

a $756 at 6 % for 4 yr.
b $968 at 5 % for 1½ yr.
c 359 at 4 % for 1 yr. 6 mo.
d 642 at 5 % for 3 yr.
e 495 at 7 % for 2¼ yr.
f 825 at 6 % for 2 yr. 3 mo.
g 508 at 7 % for 2 yr.
h 287 at 6 % for 3⅓ yr.
i 163 at 9 % for 3 yr. 4 mo.
j 753 at 4 % for 1 yr. 4 mo.

3. Find the interest at 6 % on:

a $120 for 5 mo,
b $480 for 12 da.
c $840 for 5 mo. 12 da.
d 240 for 7 mo.
e 600 for 18 da.
f 960 for 7 mo. 18 da.
g 360 for 9 mo.
h 720 for 24 da.
i 180 for 9 mo. 24 da.

4. Find the interest on \$180 at 6% for (*a*) 24 da. (*b*) 8 mo. 24 da.

METHOD

$$a\ \frac{\$1\not8\not0 \times .0\not6 \times \not2\not4}{3\not6\not0\ \ \not2} \qquad b\ \frac{\$1\not8\not0 \times .0\not6 \times 264}{3\not6\not0\ \ \not2}$$

Indicate the time in years by writing it in (*a*) as $^{24}/_{360}$, and in (*b*) as $^{264}/_{360}$, changing 8 mo. 24 da. to 264 da.

5. Find the interest on \$720 at 6% for (*a*) 25 da. (*b*) 68 da. (*c*) 2 mo. 8 da. (*d*) 6 mo. 26 da.

6. On March 1, 1917, Charles Wilcox borrowed \$475 of Arthur Washburn, which he agreed to repay on demand with interest at 6%. As evidence of the indebtedness he gave Mr. Washburn the following note:

Harvey, Neb., March 1, 1917

On demand after date, I promise to pay to the order of

Arthur Washburn

Four Hundred Seventy-five $^{0}/_{100}$.....Dollars
Value received, with interest at 6%.

\$475$^{0}/_{100}$ *Charles Wilcox*

Every six months Mr. Wilcox paid ½ year's interest. (*a*) How much was each payment? (*b*) How much had he paid in interest up to and including the payment on March 1, 1920?

(*c*) If he settled the indebtedness on Aug. 18, 1920, how much would be the sum of his interest payments? (*d*) Find the interest on $475 at 6% for 5 mo. 17 da.

CANCELLATION METHOD

(*c*) Find by compound subtraction the time for which interest is paid.

1920– VIII–18	
1917– III– 1	
3– V–17	

3 yr. = 1080 da.
5 mo. = 150 "
17 da. = 17 "

$\frac{1247}{360}$ yr.

Change the compound number 3 yr. 5 mo. 17 da. to days (1247). Write 360 as a denominator, thus expressing it in years (in the form of an improper fraction.)

Indicate the product of the principal, by the rate (in hundredths), by the time in years; cancel.

$$\frac{\overset{95}{\cancel{\$475}} \times \overset{.01}{\cancel{.06}} \times 1247}{\underset{\cancel{60}}{\cancel{360}}} = \frac{\$1184.65}{12} = \$98.72 \text{ Ans.}$$

(*d*) The interest for 167 days is indicated thus $\frac{\$475 \times .06 \times 167}{360}$

Test

Test both results by deducting (*d*) from (*c*). The difference should be the interest for 3 years.

When the time is given in years, months, and days, take each year as 360 days and each month as 30 days.

7. Find the interest at 6%:

a	$378	for	3	mo.	18	da.	*b*	$840	for	2	mo.	9	da.
c	156	"	4	"	20	"	*d*	252	"	6	"	8	"
e	618	"	8	"	17	"	*f*	507	"	5	"	3	"
g	405	"	9	"	28	"	*h*	936	"	7	"	6	"
i	864	"	1	"	25	"	*j*	738	"	2	"	5	"
k	534	"	2	"	20	"	*l*	351	"	1	"	7	"

8. Find the interest on:

a	$426.60	for	1	yr.	3	mo.	18	da.	at	4%
b	318.75	"	2	"	7	"	15	"	"	6%
c	563.10	"	3	"	4	"	24	"	"	5%
d	911.25	"	2	"	8	"	13	"	"	3%
e	123.45	"	1	"	5	"	16	"	"	8%
f	708.36	"	2	"	9	"	17	"	"	7%
g	245.70	"	3	"	6	"	11	"	"	9%
h	636.30	"	2	"	2	"	23	"	"	8%
i	824.40	"	1	"	7	"	14	"	"	5%
j	135.66	"	2	"	1	"	12	"	"	6%

9. Find the interest on:

a	$378	at	6%	for	140	da.	*b*	$156	at	3%	for	57	da.
c	405	"	5%	"	105	"	*d*	804	"	6%	"	69	"
e	980	"	6%	"	126	"	*f*	252	"	5%	"	87	"
g	536	"	4%	"	144	"	*h*	438	"	4%	"	36	"
i	618	"	6%	"	210	"	*j*	597	"	8%	"	75	"
k	734	"	7%	"	168	"	*l*	351	"	7%	"	96	"

10. Find (*a*) the interest on $386.50 for 3 yr. 7 mo. 18 da. at 4½%; (*b*) The amount of $485.60 at 5% for 2 yr. 9 mo. 25 da.

METHOD BY ALIQUOT PARTS

(*a*) Principal	$386.50
Int. for 2 yr.	34.7850
" " 1 "	17.3925
" " 6 mo.	8.6962
" " 1 "	1.4494
" " 18 da.	.8696
3 yr. 7 mo. 18 da.	$63.19 Ans.

Find the interest for 2 yr. by multiplying the principal by .09. Take 1/2 of this as the interest for 1 yr. etc. Note that the interest for 18 da. is 1/10 of that for 6 mo.

(*b*) Principal	$485.60
Int. for 2 yr.	48.56
6 mo.	12.14
3 "	6.07
20 da.	1.3489
5 "	.3372
Amt. 2 yr. 9 mo. 25 da.	$554.06 Ans.

For 6 mo. take 1/4 of 2 yr. For 20 da. take 1/9 of 6 mo. Since the *amount* is required, do not draw a line under the principal.

TEST

Find the interest by the cancellation method.

a $$\frac{\$386.50 \times .045 \times 1308}{360}$$ *b* $$\frac{\$485.60 \times .05 \times 1015}{360}$$

To the interest found in (*b*) by the cancellation method, add the principal. This should give the amount as obtained by the aliquot parts method.

Amount = Principal + Interest

11. Find the amount of:

a	$713.39	at	6%	for	1	yr.	2	mo.	12	da.
b	246.77	"	5½%	"	3	"	3	"	17	"
c	525.40	"	8%	"	2	"	8	"	15	"
d	536.81	"	9%	"	3	"	3	"	24	"
e	809.47	"	7%	"	4	"	7	"	13	"
f	234.56	"	3½%	"	3	"	9	"	18	"
g	923.75	"	3%	"	2	"	6	"	17	"
h	452.09	"	5%	"	3	"	9	"	14	"
i	227.80	"	6%	"	1	"	3	"	23	"
j	315.50	"	4½%	"	2	"	4	"	19	"

k	146.56	at	5 %	for	2	yr.	10	mo.	18	da.
l	245.70	"	3 %	"	3	"	11	"	9	"
m	318.75	"	3½ %	"	1	"	6	"	25	"
n	426.60	"	7 %	"	2	"	8	"	8	"
o	563.10	"	9 %	"	3	"	3	"	20	"
p	636.30	"	8 %	"	2	"	7	"	6	"
q	708.36	"	5½ %	"	1	"	5	"	24	"
r	824.40	"	6 %	"	2	"	4	"	7	"
s	911.25	"	4½ %	"	3	"	9	"	21	"

12. A loan for $2500 was made July 13, 1916. Find the amount that will pay it on May 6, 1920, including interest at 6 %.

Time between Dates

1920	V	6
−1916	VII	13

When dates are more than a year apart, the time is expressed in years, months, and days.

In the following examples find the time by compound subtraction.

13. Find the interest on:

a	$930.75	at	6 %	from	Dec.	20,	1915	to	Jan.	10, 1919
b	815.62	"	5½ %	"	Mar.	15,	1916	"	Feb.	29, 1920
c	732.98	"	4 %	"	Jan.	24,	1917	"	May	15, 1918
d	641.07	"	7½ %	"	Aug.	18,	1916	"	Jun.	14, 1919
e	552.96	"	8 %	"	Jan.	22,	1915	"	Aug.	20, 1918
f	463.85	"	3¾ %	"	May	20,	1916	"	Mar.	25, 1920
g	374.74	"	9 %	"	Feb.	21,	1917	"	Jun.	18, 1921
h	285.63	"	3½ %	"	Jul.	18,	1918	"	Apr.	22, 1922
i	196.52	"	4 %	"	Sep.	16,	1917	"	Jul.	30, 1920
j	207.41	"	6½ %	"	Apr.	25,	1916	"	Sep.	24, 1918
k	398.32	"	6 %	"	Dec.	20,	1915	"	Mar.	16, 1917
l	489.23	"	3¾ %	"	Mar.	19,	1914	"	Jan.	18, 1918

14. (*a*) How many months and days are there from July 25, 1919 to March 16, 1920? (*b*) How many days?

METHOD

(*b*) $6 + 31 + 30 + 31 + 30 + 31 + 31 + 29 + 16 = ?$

To the 6 (31–25) remaining days in July, add the number of days in the other months to February, inclusive, and the number of days (16) expressed by the March date.

Observe that 1920 is a leap year, which gives 29 days for February.

15. Find the interest on \$1864 at 6% (*a*) for 7 mo. 21 da. (*b*) For 235 da.

Since 360 is generally taken as the number of days to the interest year, it would be more reasonable to use 7 mo. 21 da. as the interest period on a loan made July 25, 1919, and paid March 16, 1920; but usage generally permits the employment of the number of days when the time is less than a year.

Ascertain the practice prevalent in your neighborhood.

In the following examples, use the number of days between the given dates:

16. Find the interest on \$1084 from Jul. 25, 1919, to March 16, 1920 (*a*) at 5%; (*b*) at 6%; (*c*) at 7%; (*d*) at 8%.

METHOD

(*a*) $\frac{\$1084 \times .05 \times 235}{360}$ (*b*) $\frac{\$1084 \times .06 \times 235}{360}$

(*c*) $\frac{\$1084 \times .07 \times 235}{360}$ (*d*) $\frac{\$1084 \times .08 \times 235}{360}$

TEST

In making the test by the use of aliquot parts, do not find the interest for a year unnecessarily.

	(*a*) 5%	(*b*) 6%	(*c*) 7%	(*d*) 8%
Principal	$1084	$1084	$1084	$1084
1 yr.			$75.88	
180 da.	$27.10	$32.52	$37.94	$43.36
45 "				
9 "				
1 "				
235 da.	Ans.	Ans.	Ans.	Ans.

(*a*) The interest for ½ yr. at 5% is 2½% of the principal, or 1/40 of it. Divide $1084 by 40.

(*b*) Find the interest for 180 da. by taking 3% of $1084.

(*c*) Write the interest for a year, then take ½ of it.

(*d*) Take 4% of $1084 for the interest for 180 da.

17. Find the interest on:

a	$1192	at 3%	from	Mar.	10,	1919	to	Jan.	29, 1920
b	1283	" 8%	"	Dec.	30,	1918	"	Mar.	18, 1919
c	1374	" 4%	"	Apr.	20,	1920	"	Sep.	26, 1920
d	1465	" 6%	"	Sep.	12,	1918	"	Jul.	21, 1919
e	1556	" 5%	"	Jul.	16,	1919	"	Apr.	12, 1920
f	1647	" 7%	"	Feb.	23,	1920	"	Jun.	30, 1920
g	1738	" 6%	"	May	29,	1919	"	Mar.	31, 1920
h	1829	" 5%	"	Jun.	15,	1920	"	Aug.	28, 1920
i	1910	" 7%	"	Aug.	14,	1919	"	Jun.	30, 1920
j	2029	" 4%	"	Jan.	18,	1921	"	May	20, 1921
k	2138	" 8%	"	Mar.	25,	1919	"	Feb.	29, 1920
l	2247	" 7%	"	Dec.	18,	1920	"	Jan.	31, 1921
m	2356	" 9%	"	Apr.	11,	1919	"	Feb.	16, 1920
n	2408	" 3%	"	Sep.	26,	1920	"	May	15, 1921

PREPARATORY EXERCISES

1. In finding the value of the following expressions, which product should first be obtained? Why?

a $125 \times 5\frac{1}{2} \times 8$ *b* $300 \times 3\frac{3}{4} \times 3\frac{1}{3}$ *c* $250 \times 6\frac{1}{2} \times 4$

2. Which product should first be found in finding the value of the following? Why?

a $4 \times 137 \times 2\frac{1}{2}$ *b* $6 \times 389 \times \frac{1}{6}$ *c* $6 \times 594 \times 1\frac{1}{3}$

3. Give the value of each of the foregoing.

SIGHT EXERCISES

1. Give the interest on:

a $125 at $5\frac{1}{2}\%$ for 8 yr. *b* $300 at $3\frac{3}{4}\%$ for $3\frac{1}{3}$ yr.
c 250 " $6\frac{1}{2}\%$ " 4 " *d* 137 " $2\frac{1}{2}\%$ " 4 "
e 389 " 6% " $\frac{1}{6}$ " *f* 594 " 6% " $1\frac{1}{3}$ "

2. Give the interest on \$963 at 6% (*a*) for ⅙ yr. (*b*) For 2 mo. (*c*) For 60 da.

3. Give the interest on \$480 for (*a*) 60 da. (*b*) 30 da. (*c*) 15 da. (*d*) 2 mo. (*e*) 1 mo. (*f*) 4 mo.

4. What fraction of 60 days is:

a 15 da.? *b* 20 da.? *c* 45 da.? *d* 6 da.? *e* 2 da.? *f* 4 da.?
g 10 da.? *h* 12 da.? *i* 30 da.? *j* 5 da.? *k* 3 da.? *l* 1 da.?

5. Give the interest on \$540 at 6% for:

a 15 da. *b* 60 da. *c* 20 da. *d* 1 da. *e* 5 da. *f* 3 da.
g 12 da. *h* 10 da. *i* 30 da. *j* 6 da. *k* 2 da. *l* 4 da.

6. What decimal of 60 days is:

a 54 da.? *b* 18 da.? *c* 42 da.? *d* 36 da.? *e* 24 da.?

7. Give the interest on \$500 at 6% for:

a 72 da. *b* 48 da. *c* 66 da.

METHOD

a 72 da.	= 1.2 of 60 da.	\$5 × 1.2 = \$6, Ans.
b 48 "	= .8 " " "	\$5 × .8 = \$4, Ans.
c 66 "	= 1.1 " " "	\$5 × 1.1 = \$5.50, Ans.

8. Give the interest on \$400 at 6% for:

a 54 da. *b* 18 da. *c* 42 da. *d* 36 da. *e* 24 da. *f* 6 da.

9. Give the interest at 6% on:

a \$249 for 60 da.	*b* \$240 for 5 da.
c 333 " 20 "	*d* 420 " 3 "
e 510 " 12 "	*f* 670 " 6 "
g 864 " 30 "	*h* 660 " 1 "
i 726 " 10 "	*j* 450 " 4 "
k 488 " 15 "	*l* 690 " 2 "

10. Using decimals, give the interest at 6% on:

a	$215 for 24 da.	*b*	$110 for 54 da.
c	310 " 66 "	*d*	210 " 48 "
e	330 " 18 "	*f*	120 " 36 "
g	120 " 42 "	*h*	300 " 72 "

60-DAY METHOD — RATE 6%

In calculating interest at 6% for short periods, business men generally point off 1% of the principal as the interest for 60 days.

WRITTEN EXERCISES

1. Find the interest on $583.20 at 6% (*a*) for 90 da. (*b*) For 80 da. (*c*) For 75 da. (*d*) For 72 da.

METHOD

(*a*)	60 da.		$5.832	(*b*)	60 da.		$5.832
	+ 30 "	(½)	2.916		+ 20 "	(⅓)	1.944
	90 da.		?		80 da.		?
(*c*)	60 da.		$5.832	(*d*)	60 da.		$5.832
	+ 15 "	(¼)	1.458		+ 12 "	(⅕)	1.166
	75 da.		?		72 da.		?

Write the interest for 60 days by moving the decimal point in the principal two places to the left.

Find the interest for 90 days, 80 days, 75 days, and 72 days, respectively, by adding to the interest for 60 days ½ of itself, ⅓ of itself, ¼ of itself, and ⅕ of itself, respectively.

Test results, using cancellation method.

2. Find the interest on \$465.30 at 6% for 100 da.

METHOD

Int. for 60 da. \$4.653
" " 30 " 2.3265 ½ of 60 da.
" " 10 " .7755 ⅓ " 30 "
Int. for 100 da. \$7.76 Ans.
Test by taking 60 da., 20 da., and 20 da.

3. Find the interest on \$695.40 at 6% (*a*) for 95 da. (*b*) For 84 da. (*c*) For 78 da. (*d*) For 74 da. (*e*) For 67 da.

METHOD

60 da.	60 da.	60 da.	60 da.	60 da.
30 "	20 "	15 "	12 "	6 "
5 "	4 "	3 "	2 "	1 "
(*a*) 95 da.	(*b*) 84 da.	(*c*) 78 da.	(*d*) 74 da.	(*e*) 67 da.

Test (*b*) by taking 60 da., 12 da., and 12 da.

4. Find the interest at 6% on:

a \$372 for 81 da. *b* \$477 for 93 da. *c* \$165 for 63 da.
d 513 " 72 " *e* 615 " 84 " *f* 723 " 67 "

5. What is the interest on \$394.75 at 6% for 86 days?

METHOD

60 da.	$3.9475		6 days, while not an
20 "	1.3158	⅓ of 60 da.	aliquot part of 20
6 "	.3948	1/10 " 60 "	days, is an aliquot
	?		part of 60 days.

Be careful to take 1/10 of $3.9475.

6. Using three aliquot parts, find the interest at 6% on:

a $684 for 73 da. *b* $495 for 77 da. *c* $783 for 86 da.

7. Find the interest on $834.36 at 6% (*a*) for 45 da. (*b*) For 48 da. (*c*) For 50 da. (*d*) For 54 da. (*e*) For 55 da. (*f*) For 59 da.

METHOD

(*a*)	(*b*)	(*c*)
60 da.	60 da.	60 da.
− 15 "	− 12 "	− 10 "
45 da.	48 da.	50 da.
(*d*)	(*e*)	(*f*)
60 da.	60 da.	60 da.
− 6 "	− 5 "	− 1 "
54 da.	55 da.	59 da.

From the interest for 60 days, deduct the interest as indicated.

Test (*b*) by multiplying the interest for 60 days by .8; test (*d*) by multiplying it by .9.

8. Find the interest at 6% on:

a $954 for 58 da. *b* $693 for 57 da. *c* $765 for 56 da.

9. Find the interest on $1232.73 at 6% (*a*) for 16 da. (*b*) For 33 da. (*c*) For 29 da. (*d*) For 53 da. (*e*) For 47 da. (*f*) For 44 da.

METHOD

	60 da.		60 da		60 da.
	15 da.		30 da.		30 da.
+	1 "	+	3 "	−	1 "
(*a*)	16 da.	(*b*)	33 da.	(*c*)	29 da.
	60 da.		60 da.		60 da.
−	6 "	−	10 "	−	10 "
−	1 "	−	3 "	−	6 "
(*d*)	53 da.	(*e*)	47 da.	(*f*)	44 da.

In (*a*), (*b*), and (*c*), draw a line under the interest for 60 days, excluding that portion. In (*d*) and (*e*), from the interest for 60 days deduct, in one operation, the sum of the two having a prefixed minus sign.

10. Find the interest at 6% on:

a $297 for 17 da. *b* $459 for 26 da. *c* $576 for 52 da.
d 708 " 35 " *e* 693 " 44 " *f* 351 " 19 "

11. Find the interest on \$972 for 87 days (*a*) at 6%; (*b*) at 5%; (*c*) at 4½%; (*d*) at 8%.

Use 1% of the principal as the interest for 72 days (⅕ year) at 5%, for 80 days (⅔ year) at 4½%, and for 45 days (⅛ year) at 8%.

METHOD

(*a*) 6%		(*b*) 5%	
60 da.	\$9.72	72 da.	\$9.72
+ 15 "	2.43	+ 12 "	1.62
+ 12 "	1.944	+ 3 "	.405
87 da.	\$14.09 Ans.	87 da.	\$11.75 Ans.

(*c*) 4½%		(*d*) 8%	
80 da.	\$9.72	45 da.	\$9.72
+ 8 "	.972	90 "	19.44
— 1 "	.1215	— 3 "	.648
87 da.	\$10.57 Ans.	87 da.	\$18.79 Ans.

12. Find the interest on \$429 for 97 days (*a*) at 6%. (*b*) At 5%. (*c*) At 4½%. (*d*) At 4%. (*e*) At 9%.

13. Find the amount of \$972 for 75 days (*a*) at 6%; (*b*) at 4%; (*c*) at 8%.

METHOD

	(*a*)		(*b*)
Principal	\$972.–		\$972.–
Int. 60 da.	+ 9.72	90 da.	+ 9.72
" 15 "	+ 2.43	15 "	— 1.62
Amt. 75 da.	\$984.15 Ans.		\$980.10 Ans.

For (*c*), take 45 da., 15 da., and 15 da.

14. Find the amount of \$864 for 98 days (*a*) at 5%; (*b*) at 6%; (*c*) at 4%.

Some clerks begin with the 6% rate in all interest calculations, making the necessary change in this result to conform with the rate specified in a given transaction.

15. Find the amount of \$972 for 66 days (*a*) at 7%; (*b*) 4½%.

METHOD

(*a*)

Int. 60 da. at 6%	\$9.72
" 6 "	+ .972
Int. 66 da. at 6%	\$10.692
" " " " 1%	+ 1.782
Int. 66 da. at 7%	\$12.47
Principal	972.–
Amount	\$984.47 Ans.

(*b*)

Int. 66 da. at 6%	\$10.692
" " " " 1½%	— 2.673
" " " " ¼%	— .4455
Int. 66 da. at 4¼%	+\$7.57
Principal	972.+
Amount	\$979.57 Ans.

16. Find the interest using any method. Test the result by using another method.

Principal	Rate	Time	Principal	Rate	Time
a \$306	5%	66 da.	*b* \$603	4%	68 da.
c 702	8%	83 "	*d* 801	3%	54 "

When you calculate interest by commencing with a rate other than the given one, be careful to find the interest at the latter rate before adding it to the principal to obtain the amount.

17. Find the interest on \$594 for 100 days (*a*) at 6%. From this result determine the interest (*b*) at 5½%. (*c*) At 6½%. (*d*) At 7%. (*e*) At 7½%.

METHOD

(*b*)	(*c*)	(*d*)	(*e*)
At 6%	At 6%	At 6%	At 6%
−½%	+½%	+1%	+1½%
5½%	6½%	7%	7½%

18. Find the amount:

Principal	Rate	Time	Principal	Rate	Time
a \$954	5½%	49 da.	*b* \$891	7%	96 da.
c 648	6½%	73 "	*d* 765	7½%	81 "

19. A man incurred a debt of \$465.30, which he repaid in 3 yr. 8 mo. 24 da. with interest at 5%. What was his total payment?

SIGHT EXERCISES

1. Give the interest at 6% on \$360 for 81 days.

NOTE: Change mentally the foregoing as follows: Give the interest on \$81 for 360 days at 6%.

2. Give the interest on \$360 at:

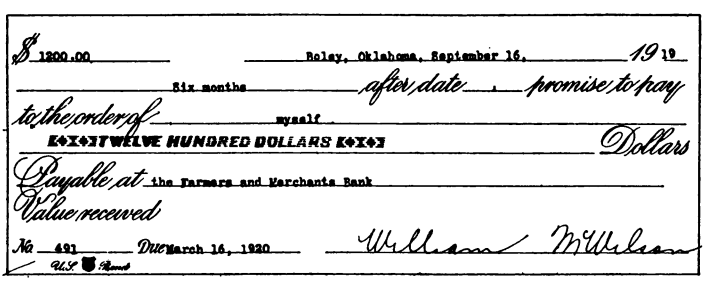
a 6% for 70 da.	*b* 4½% for 60 da.
c 5% " 46 "	*d* 5½% " 40 "
e 4% " 82 "	*f* 6½% " 80 "

CHAPTER FOUR

BANK DISCOUNT

William McWilson applies to his bank for a loan of $1200 to buy cattle he wishes to fatten. The bank authorities agree to make the loan, taking as security his note indorsed by a responsible person.

The following is the note:


$ 1200.00 Boley, Oklahoma, September 16, 1919

Six months after date I promise to pay

to the order of myself

XXTWELVE HUNDRED DOLLARS XX Dollars

Payable at the Farmers and Merchants Bank

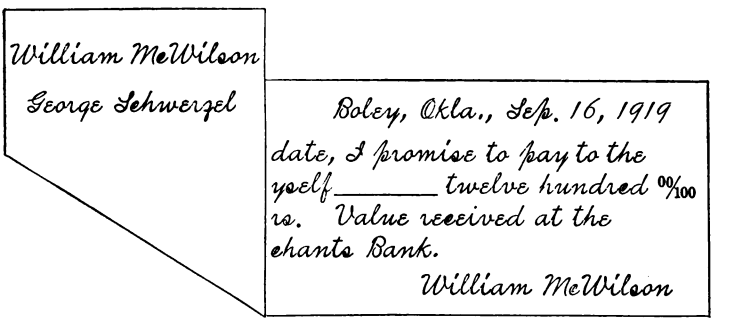
Value received

No. 491 Due March 16, 1920 William McWilson

U.S. Stamp

Mr. McWilson makes out the note to his own order thereby becoming maker and payee. He then becomes an indorser. Mr. McWilson then delivers the note to the bank by writing his name across the back. To furnish the required security, he obtains the indorsement of Mr. Schwerzel, which the latter gives by writing his name on the back of the note under that of Mr. McWilson.

INDORSEMENT



DISCOUNTING THE NOTE

The bank "discounts" the foregoing note by deducting in advance the interest from September 16, 1919, to March 16, 1920, the date the note is due.

From the date table the bank cashier finds the time to be 182 days; from the interest table he finds the interest (*bank discount*) to be $36.40. Deducting the latter from $1200 (*the face of the note*) he credits Mr. McWilson's account with the remainder, $1163.60 (*the proceeds*).

PROCEEDS = Face of Note − Bank Discount

When the note becomes due, March 16, 1920, the bank deducts $1200 from Mr. McWilson's account and places the note with his canceled checks, to be returned to him with the latter at the proper time.

To find the BANK DISCOUNT on a non-interest bearing note, find the interest on the face of the note from the day of discount to the day of maturity.

SIGHT EXERCISES

1. What is the interest on \$1000 at 6% for 60 days?

2. What is the interest on a 60-day loan of \$1000, at 6%?

3. If the interest is paid in advance by the borrower, how much remains for his use?

4. What is the bank discount on \$1000 for 60 days at 6%?

5. What are the proceeds of a note for \$1000 discounted at a bank for 60 days at 6%?

6. Give the bank discount (interest) at 6% on a note of \$1200 for:

a 60 da.	*b* 30 da.	*c* 6 da.	*d* 120 da.
e 10 "	*f* 15 "	*g* 1 "	*h* 180 "
i 24 "	*j* 12 "	*k* 5 "	*l* 240 "
m 66 "	*n* 54 "	*o* 9 "	*p* 300 "
q 18 "	*r* 36 "	*s* 2 "	*t* 150 "
u 42 "	*v* 48 "	*w* 7 "	*x* 210 "

7. Give the proceeds of a note for \$1200, discounted at a bank at 6% for:

a 30 da.	*b* 60 da.	*c* 3 da.	*d* 240 da.
e 90 "	*f* 24 "	*g* 6 "	*h* 120 "
i 15 "	*j* 36 "	*k* 9 "	*l* 180 "

WRITTEN EXERCISES

1. What is the bank discount of a note for \$475.20 discounted at a bank at 6% for (*a*) 48 da.? (*b*) For 72 da.?

PROCESS

Face of note	(*a*)	$475.20	(*b*)	$475.20
Int. for 60 da.		$4.752	60 da.	$4.752
" " 12 "		− .9504	+12 "	.9504
Ans.		$3.80	Ans.	$5.70

2. Find the bank discount at 6% on notes as follows:

Face of Note	Discount Period	Face of Note	Discount Period
a $443.89	108 da.	*b* $367.64	98 da.
c 855.78	119 "	*d* 665.73	66 "
e 627.39	164 "	*f* 122.58	32 "
g 435.87	100 "	*h* 146.52	58 "
i 625.34	105 "	*j* 346.53	65 "
k 837.25	102 "	*l* 635.89	97 "

3. Find the proceeds of a note for $378, discounted at a bank at 6% (*a*) for 63 days. (*b*) For 57 days.

METHOD

(*a*) $378. — Face of note

Int. for 60 da. − 3.78
" " 3 " − .189 } Deduct

Ans. $374.03

(*b*) $378. — Face of note

Int. for 30 da. 1.89
" " 27 " 1.701 } Deduct

Ans. $374.41

In (*b*) find the interest for 27 days by multiplying $1.89, the interest for 30 days, by .9.

4. Find the proceeds of the following notes discounted at 6%:

	Face of Note	Discount Period		Face of Note	Discount Period
a	$657.83	112 da.	*b*	$796.52	64 da.
c	868.35	108 "	*d*	473.76	37 "
e	285.72	130 "	*f*	985.31	59 "
g	432.12	107 "	*h*	523.46	38 "
i	524.63	101 "	*j*	418.52	67 "
k	774.34	123 "	*l*	293.75	94 "

DAYS OF GRACE — HOLIDAYS

When the day on which a note by its terms is made payable (*date of maturity*) is a Sunday or a holiday, the laws of many states defer its maturity to the following business day. In a state in which Saturday is a half-holiday, a note falling due on Saturday does not *mature* until Monday; or, when Monday is a holiday, until Tuesday.

Thus a note for 30 days, drawn June 3, is due July 3. When this falls on Saturday (as in 1920) the note runs for 33 days. In discounting the note, the bank takes note of these conditions and deducts interest for 33 days, the term of discount.

In some states an allowance of three days, called *days of grace*, is given after the note is due in accordance with its terms. These days are taken into consideration by the bank in determining the discount period in states in which days of grace prevail.

While the pupil should familiarize himself with the practice of the neighborhood, he may ignore in the following examples the matter of holidays, days of

grace, etc., and assume that the note is payable on the day specified by its terms for its maturity.

DATE OF THE MATURITY OF A NOTE

A note for 30, 60, 90, etc., days, is payable 30, 60, 90, etc., days after its date: that is, a note for 30 days dated February 17, 1919, is due March 19, 1919, and one dated February 17, 1920, is due March 18, 1920.

A note for 30 days has 30 days to run; a note for 1 month may be payable in 28 days, 29 days, 30 days, or 31 days.

A note for 1 month dated January 31, 1919, is payable February 28, 1919, 28 days; one dated January 31, 1920, is payable February 29, 1920, 29 days; a note for 1 month dated January 28 is payable February 28, 31 days; one dated June 25 is payable July 25, 30 days.

The expressions that a note is "drawn," "made," or "dated," on a specified day, have the same meaning.

SIGHT EXERCISES

1. Give the date when each of the following notes becomes due:

	Drawn	Time		Drawn	Time
a	Jan. 30, 1921	1 mo.	*b*	May 31, 1919	1 mo.
c	Oct. 13, 1920	30 da.	*d*	Feb. 23, 1921	30 da.
e	Jul. 31, 1919	2 mo.	*f*	Dec. 30, 1919	2 mo.
g	Feb. 18, 1920	60 da.	*h*	Feb. 16, 1921	60 da.
i	Nov. 30, 1919	3 mo.	*j*	Sep. 30, 1920	3 mo.
k	Jan. 31, 1920	90 da.	*l*	Aug. 15, 1919	90 da.
m	Mch. 16, 1919	4 da.	*n*	Feb. 29, 1920	4 mo.

2. Give the number of days between the date on which each of the following notes is drawn, and the date on which it falls due:

	Drawn	Time		Drawn	Time
a	Jan. 30, 1921	1 mo.	*b*	May 31, 1919	1 mo.
c	Jul. 31, 1919	2 mo.	*d*	Dec. 30, 1919	2 mo.
e	Nov. 30, 1920	3 mo.	*f*	Feb. 16, 1921	3 mo.
g	Mch. 16, 1919	4 mo.	*h*	Apr. 30, 1920	4 mo.

BANK DISCOUNT OF COMMERCIAL PAPER

J. R. Page & Co. buy machinery for $4000, giving in payment a note for 4 months, in the following form:

Ashland, Ore., October 31, 1919

Four months after date, we promise to pay
to the order of the Lidgewood Mfg. Co.

Four Thousand 00/100.....Dollars

Value Received, at the United States National Bank.

400^{00}/_{100}$ J. R. Page & Co.

The firm mails this note to the Denver Office of the Lidgewood Mfg. Co. On November 4 the latter has the note discounted at the Colorado National Bank.

SIGHT EXERCISES

1. (*a*) On what day is the foregoing note due? (*b*) How many days after October 31?

2. (*a*) How many days after it was drawn was it discounted? (*b*) How many days are there from the day of discount to the day it is due? (*c*) What is the interest on $4000 at 6% for this latter period?

3. On December 6, the Colorado National Bank "sells" this note to the Mississippi Trust Co., of St. Louis; that is, the latter discounts the note for the remainder of the term. (*a*) How many days was the note held by the Colorado National Bank? (*b*) How much interest did it earn in that time at 6%? (*c*) How many days will elapse from December 16, 1919 to the maturity of the note? (*d*) What is the bank discount on $4000 at 5% for this period?

NOTE: Bank Discount and Interest have the same meaning.

TERM OF DISCOUNT

In many discount examples the period (*term of discount*) for which the discounting bank deducts interest is not specified. When money is borrowed from a bank on a note, the latter is generally discounted on the day it is drawn, and the discount (interest) period corresponds with the time specified in the note. When this is stated in days, 30 days, 60 days, etc., the interest is taken for the specified number (omitting from consideration the question of holidays). When the time is expressed in months, 3 months, 6 months, etc., the number of days must be ascertained.

WRITTEN EXERCISES

1. Calculate (*a*) the bank discount and (*b*) the proceeds of a note for $864, for 4 months, drawn Dec. 31, 1920, and discounted at 5% on the same day.

METHOD

Find the date of maturity, 4 months after Dec. 31, 1920, viz., Apr. 30, 1920.

The time between those dates is 31 da. + 28 da. + 31 da. + 30 da.

Obtain (*a*) by finding the interest for the foregoing time.

Test.

2. Calculate the bank discount on the following notes, discounted at 6% on the day drawn.

	Face	Drawn	Time
a	$396.90	May 23, 1919	2 months
b	487.45	Oct. 31, 1920	4 "
c	543.60	Jan. 20, 1921	6 "
d	627.15	Jul. 16, 1920	3 "

DATE OF DISCOUNT

When a note is given in settlement of a business transaction, it may not be discounted for several days. In working examples of this kind, the careless pupil may make an error in the discount period. Among the data he finds that a note was made on March 31 and discounted May 13. Ignoring the item which states that the note was drawn for 90 days, or for 4 months, etc., he takes as the discount period the 43 days between the foregoing dates. This is the time for which interest is *not* deducted. The discount period begins May 13 and terminates on the day the note is due. The March 31 date is used only as the basis for the determination of the date of maturity.

SIGHT EXERCISES

1. A 60-day note is discounted 12 days after it is drawn. What is the term of discount?

2. Give the term of discount of each of the following:

	Time	Discounted
a	60 da.	15 days after date
b	30 "	9 " " "
c	90 "	32 " " "
d	80 "	56 " " "

3. Give the term of discount:

	Time	Dated	Discounted
a	60 da.	Jan. 13	Jan. 31
b	30 "	Feb. 10	Feb. 28
c	90 "	Mar. 25	Apr. 10
d	80 "	Apr. 16	Apr. 30

WRITTEN EXERCISES

1. Calculate the proceeds of the following notes:

	Face	Discounted	Rate
a	$783.90	48 days before maturity	6%
b	842.76	63 " " "	5%
c	927.54	75 " " "	8%
d	648.36	87 " " "	7%

2. Calculate the discount on the following notes:

	Face	Drawn for	Discounted	Rate
a	$567	90 da.	25 days after date	6½%
b	732	30 "	12 " " "	6%
c	984	60 "	15 " " "	5½%
d	652	80 "	42 " " "	7%

3. Calculate the bank discount on a note of $1236 drawn Feb. 18, 1920, for 90 days and discounted at 5%, Apr. 13, 1920.

METHOD

To find the term of discount in this case, it is not necessary to determine the date of maturity. From Feb. 18, 1920 to Apr. 13, 1920 the time is 11 da. + 31 da. + 13 da., or 55 days, which is the period the note has been withheld from discount. The remaining time, 35 days (90 days − 55 days), is the time during which the borrower has the use of the bank's money, and the period for which he must pay interest.

4. Calculate the bank discount on the following notes:

	Face	Time	Dated	Discounted
a	$1350	90 da.	Jan. 5, 1918	Mar. 4, 1918
b	2880	80 "	Dec. 6, 1919	Feb. 8, 1920
c	3240	60 "	Jul. 7, 1920	Jul. 9, 1920
d	4320	70 "	Sep. 6, 1921	Oct. 1, 1921

5. A note for $1560 drawn Oct. 31, 1920, for 4 months, was discounted Dec. 5, 1920, at 5½%. Find (*a*) the day the note is due; (*b*) the term of discount; (*c*) the discount.

METHOD

(*a*) The note is due 4 months after Oct. 31, 1920, which is Feb. 28, 1921.

(*b*) The term of discount is the number of days between the day of discount, Dec. 5, 1920, and the day the note is due, Feb. 28, 1921.

Observe that a note for 4 months drawn on Oct. 28, Oct. 29, Oct. 30, or Oct. 31, 1920, is due on Feb. 28, 1921.

6. Find (I) the date on which each of the following notes is due; (II) the discount period; (III) the bank discount; and (IV) the proceeds.

	Face	Time	Dated	Discounted	Rate
a	$1080	3 mo.	Feb. 16, 1918	Feb. 28, 1918	5%
b	3240	6 "	Jun. 28, 1919	Nov. 15, 1919	6%
c	2560	2 "	Sep. 15, 1920	Sep. 30, 1920	6%
d	4440	4 "	Jul. 30, 1921	Aug. 10, 1921	5%

DISCOUNT OF INTEREST-BEARING NOTES

As evidence of a loan to Percy Keating, Lester Johnson received the following note:

SHELDON, N. DAK., March 6, 1920

Ninety days after date I promise to pay to the order of Lester Johnson

Twelve Hundred 00/100..... Dollars

Value received, with interest at 5%.

1200\frac{00}{100}$ Percy Keating

Mr. Johnson loaned this money at a time he thought he would not need it for ninety days, and he was glad of the opportunity to lend it at a fair rate of interest.

The need for cash before the maturity of the note made it necessary for him to offer it to his bank as security for a loan. The bank agreed to discount the note at 6% on the amount of the note at maturity. On Apr. 5 Mr. Johnson transferred the note to the bank by indorsement, thereby agreeing to pay it on the failure of Mr. Keating to "take it up" at its maturity. On Apr. 5 the bank discounted the note at 6% and placed the proceeds to the credit of Mr. Johnson.

SIGHT EXERCISES

1. On what date is the foregoing note due?

2. (*a*) What is the interest on $1200 at 5% for ¼ year? (*b*) What is the amount of the note at its maturity?

3. (*a*) How many days are there between Apr. 5 and the maturity of the note? (*b*) What is the bank discount at 6% for this time on the amount of the note at maturity? (*c*) How much does the bank place to the credit of Mr. Johnson?

In discounting a note, the bank authorities calculate the bank discount on the sum the maker agrees to pay at its maturity. In the case of a note that does not call for the payment of interest, this corresponds with its "face." When the note provides for the payment of interest, the sum to be paid at its maturity must be determined. This is the "amount" of the specified sum, which includes the "face of the note" plus the interest.

The note Mr. Johnson offers for discount calls for

the payment of \$1200 + \$15 on Jun. 5, a total of \$1215. The bank discount is the interest at 6% on \$1215 from Apr. 6 to Jun. 5.

WRITTEN EXERCISES

1. Determine the proceeds of a note for \$1200, dated Jan. 9, 1922, payable in 60 days, with interest at 6%, and discounted at a bank on Feb. 1, 1922, at 6%.

METHOD

I. Find the amount of \$1200 at 6% for 60 days.
II. Find the date of the maturity of the note.
III. Find the time between Feb. 1 and the date of maturity.
IV. Find the interest for this period on the *amount* obtained in I.
V. From I (the *amount* of the note at maturity) deduct IV (the bank discount).

2. Find the proceeds of the following notes, which bear interest at 6% and are discounted at 6%:

	Face	Time	Dated	Discounted
a	\$1200	90 da.	Jul. 3, 1921	Aug. 16, 1921
b	2400	60 da.	Jan. 8, 1921	Feb. 25, 1921
c	3600	30 da.	Jun. 1, 1920	Jun. 15, 1920

SECTION III

NUMBERS AND PROCESSES

CHAPTER ONE

READING AND WRITING NUMBERS

ORAL EXERCISES

1. Read the following numbers: 18, 47, 94, 33, 56, 69.

2. Read 365. Say "Three hundred sixty-five." Do not use "and" in reading an *integer* (a whole number).

3. Read the following numbers: 390, 418, 962, 804, 700.

4. Read 4063. Say "Four thousand, sixty three."

5. Read the following numbers: 9000, 8007, 6052, 7329, 4010.

6. Read the following numbers: 49,000, 38,007, 26,052, 17,329, 349,000, 238,007, 126,052, 817,329.

7. Read, 1,234,000. Say "1 million, 234 thousand."

8. Read the following numbers:

1,249,000	3,238,007	5,126,052
21,349,000	43,238,007	75,126,052
321,249,000	543,238,007	675,126,052

9. Read 1,234,567,890. For convenience in reading a large number, it is generally pointed off by commas, into periods of three figures each, beginning at the right.

The names of the periods in the foregoing number are shown in the following:

TABLE

BILLIONS' *period*	MILLIONS' *period*			THOUSANDS' *period*			ONES' *period*		
ONES	HUNDREDS	TENS	ONES	HUNDREDS	TENS	ONES	HUNDREDS	TENS	ONES
1,	2	3	4,	5	6	7,	8	9	0

Say, "1 billion, 234 million, 567 thousand, 890."

Beginning at the left, announce the numbers represented by the figures of each period and its name, omitting the name of the ones' period.

10. Read 1,000,000,890. Omit the name of a period composed exclusively of ciphers; say "1 billion, 890."

11. Read the following numbers: 21,000,203,890; 42,363,000,890; 157,363,275,000.

Say, "21 billion, 203 thousand, 890"; "42 billion, 363 million, 890"; "157 billion, 363 million, 275 thousand."

12. Read the following numbers:

20,006,854	103,050,000	1,006,000,053
3,059,608	234,567	2,030,405,000
875,340	18,620,100	28,306,500
51,209,007	3,210,001	493,062,009

13. Read 1200.

In the mental addition of 7 hundred and 5 hundred, or in the multiplication of 4 hundred by 3, think of the result as 12 hundred.

14. Read the following numbers as hundreds: 1600, 2100, 3600, 4200, 9800.

15. Read 1286.

In adding 722 and 564, think of these numbers as "seven, twenty-two" and "five, sixty-four," respectively; and of the result as "twelve, eighty-six." Do the same in multiplying 643 by 2, etc.

16. Read the following numbers: 1920, 1776, 1492, 1898, 1917.

WRITING INTEGERS

1. Write one hundred two billion, seventy-five.

Remember that the billions' period is the fourth, and that it must be followed by three periods of three figures each. Use three ciphers to represent the missing millions' period, and three to represent the missing thousands' period. Write a cipher in the tens' place in the billions' period and another in the hundreds' place of the ones' period. Separate the periods by commas.

102,000,000,075

NOTE: In the European countries that use a comma as a decimal point, the periods are separated by a slight space: 102 000 000 075.

2. Write the following numbers:

One billion, eleven thousand, seventeen
Sixteen hundred thousand
Fourteen, nine-two
Three million, two thousand, eighty
Twenty-four million, five thousand

Further practice will be obtained through the writing of numbers dictated in the addition exercises.

NOTE: Do not use a comma in writing a number of four figures, 1890, for example, unless such number is written in a column with larger numbers.

DECIMALS

A number preceded by a decimal point (.) is called a *decimal;* .06, .2, .135, for example.

The number 123.45 is called a *mixed decimal.* It consists of the integer 123, and the decimal .45.

ONE WAY OF READING DECIMALS

To read 333.333, say "three hundred thirty-three *and* three hundred thirty-three thousandths, using "and" between the whole number and the decimal.

.5 is a one-place decimal; read it as *tenths.*

.15 .05, etc., are two-place decimals; read them as *hundredths.* .157, .036, .009, etc., are three-place decimals; read them as *thousandths.*

Read a four-place decimal as *ten-thousandths,* a five-place decimal as *hundred-thousandths,* etc.

To read 1256.02076, say, "1256 and 2076 hundred-thousandths." Read each part in the usual way; follow the integer with the word "and," and the decimal with the denomination of its right-hand figure.

TABLE

Thousands	Hundreds	Tens	Ones	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths
1	2	5	6	.	0	2	0	7	6

ORAL EXERCISE

Read the following:

217.3	21.073	.0073	12.45
.058	654.6	1.0271	.00008
3.02	83.05	350.02	296.4

The Business Way

1. Read the following: (*a*) 700.007 (*b*) 217.3, (*c*) .0073, (*d*) 12.45

When (*a*) is dictated as 700 and 7 thousandths to a clerk who does not know that "and" is used to indicate a decimal point, he may write it .707.

The business way of stating this mixed decimal, "Seven hundred, point, o.o. seven," leaves no doubt in the mind of the hearer. It is a help to a clerk who is taking down a column from dictation to begin to write the decimal without waiting to hear its denomination.

Read the integer in the usual way; say "point" to denote that what follows is a decimal; announce separately each figure of the latter, using the letter "o" to denote a cipher.

(*b*) Say "217, point, 3." (*c*) Say "Point, o, o, 7, 3." (*d*) Say "12, point, 4, 5."

2. Read the following in the business way:

3.5 12.07 6.009 15.038 .005 30.015 .0105 231.25 127 .0356 213.4 6.8702 375.0006 300.003 12856.02 1856.035 2963.12 8345.0012 23,589.064 357.00007 1363.0087 89,362.0501 58,632.001 120,615.838 39,825.36 28,345.0003

NOTE: In some Continental countries the comma is used as a decimal point, a period indicating multiplication. Observe that the period used in writing 7.05 A.M., for example, is not a decimal point.

WRITING DECIMALS

1. Write as decimals:

a 143 and 16 ten thousandths
b 375 millionths
c 14,683 and 14,683 hundred thousandths

Write (*a*) 143.0016, prefixing two ciphers to the decimal portion to indicate ten-thousandths.

Write (*b*) .000375 to make millionths, which denotes a decimal of six places.

Write (*c*) 14,683.14683. Do not use a comma in the decimal portion to divide it into periods.

2. Write the following decimals:

a 128 and 75 thousandths
b 643 millionths
c 3 and 209 hundred-thousandths
d 489 and 6 hundredths
e 25,394 and 1087 ten-thousandths
f 3 and 96 hundred-thousandths
g 286 and 3 tenths
h 57 and 395 ten-thousandths
i 283 thousandths

3. Find the sum of the foregoing.

4. Write the following:

a 237, point, 1,0,6
b 1354, point, 0,0,7,3
c 26,903, point, 0,5,0,9
d 387, point, 0,0,0,5
e 29, point, 0,0,3,0,1

5. Find the sum of the foregoing.

A person dictating numbers in a business house omits the words "hundred" and "thousand" when the omission is not likely to lead to misunderstanding. 237 he announces as "two, thirty-seven"; 1354 as "thirteen, fifty-four"; 26,903 as "twenty-six, 9, 0, 3"; etc.

6. Write the following from dictation:

a 37, point, 3,0,5
b 13, 54, point, 0,0,0,19
c 26, nine, 0, 3, point, 2,0,7,5
d 18, 3, 75, point, 6
e 2, 45, 8, 96, point, 3

7. Find the sum of the foregoing.

READING DOLLARS AND CENTS

The coin value of a pound sterling, $4,8665, may be read as 4 dollars, 86 cents, 6 mills, and 5 tenths; or as 4 dollars, 86 cents, and 65 hundredths.

The coin value of a German mark is 23.8 cents. It may be read as 23 cents, 8 mills; or as 23 and 8 tenths cents.

The business man might state the value of a pound sterling as "4, point, 8, 6, 6, 5 dollars"; no one would call it 4 dollars and eight thousand, six hundred sixty-five thousandths of a dollar.

The word "and" is frequently omitted between the number of the dollars and that of the cents.

DICTATING DOLLARS AND CENTS

In the counting room it is the duty of a clerk frequently to call off hundreds of items in dollars and cents to another clerk, who then writes them in columns to be added.

The speed with which the writing must be done makes it advisable for the "caller-off" to omit everything unnecessary, but at the same time to be careful to announce everything essential.

$129.36	In calling off the accompanying
4375.84	items, he should say: "One twenty-
9.—	nine, thirty-six. Forty-three sev-
600.75	enty-five, eighty-four. Nine 'dolls.'
12,308.09	Six hundred, seventy-five. Twelve,
	3, 0, 8, 0, 9.

It is unnecessary to employ the word "dollars" between dollars and cents, it being sufficient to use it when the item does not contain cents, in which case the final syllable is dropped to save every possible second.

When it is agreed that the last two figures represent cents, there is no need in calling off $600.75 as "Six hundred dol-

lars, seventy-five." Announced as it should be, with the omission of the word "dollars," it cannot be misunderstood. The "caller-off" must, however, follow the wishes of the transcribing clerk, who is the one held accountable for errors in the written numbers.

WRITING UNITED STATES MONEY

Fifty-nine cents may be expressed with the cents' sign, 59¢; as a fraction of a dollar, \$59/100; or as a decimal of a dollar, \$.59, or \$0.59.

Three dollars and seventy-five cents is expressed twice in a check, draft, note, etc., once as "Three $^{75}/_{100}$ Dollars," and again as \$3$^{75}/_{100}$, the cents being invariably written in both places as a fraction of a dollar. In other connections, the form is generally \$3.75, the cents being written as a decimal of a dollar.

When a number contains mills, it may be written as a three-place decimal of a dollar, or as a decimal or a fraction of a cent. Thus, 19 cents and 3 mills, the coin value of a French franc, is written \$.193, or 19.3¢, or $19^{3}/_{10}$ cents.

ORAL EXERCISES

Read the following as you would to a fellow clerk:

a	*b*	*c*
\$81.57	\$9,104.13	\$15,836.—
123.05	360.10	500.95
47.—	41,235.—	1,206.07
60.50	300.40	.88
302.64	.95	423.92
1249.01	59.84	68.01
13,100.90	310.25	135.—
.89	1,606.20	9.96

d	*e*	*f*
$200.60	$384.92	$23,456.—
89.84	1,560.47	6,989.77
1,603.06	29,879.53	584.18
172.16	8,808.08	35.45
399.80	766.67	906.50
43.—	1,493.—	1,348.—
21,065.09	35,268.45	854.49
.63	432.96	2,036.54

WRITTEN EXERCISES

1. Write the foregoing from dictation.
2. Find the sum of each.

READING FRACTIONS

To read $\frac{3}{4}$, $\frac{18}{25}$, read the numerator as a cardinal number, and the denominator as an ordinal number; to read $34\frac{7}{8}$, use "and" between the integer and the fraction.

The listener is sometimes misled when he hears such an expression as "seven hundred and seventy-nine thousandths." Having been taught that "and" indicates a mixed decimal or a mixed fraction, he takes the expression to mean $700\ \frac{79}{1000}$. He does not know that the speaker, like many other people, is unaware of the existence of such a rule. If he did, he might be undecided between $\frac{779}{1000}$, and $\frac{779}{9000}$.

To avoid misunderstandings, $700\frac{79}{1000}$ should be announced as "700 and the fraction $\frac{79}{1000}$"; $\frac{779}{1000}$ as "numerator 779, denominator 1000."

Read a complex fraction, $\frac{2\frac{3}{4}}{3\frac{1}{2}}$, for instance, as "$2\frac{3}{4}$ *over* $3\frac{1}{2}$." This form better indicates that a fraction is meant than "$2\frac{3}{4}$ divided by $3\frac{1}{2}$," which may indicate to the hearer the form "$2\frac{3}{4} \div 3\frac{1}{2}$."

ORAL EXERCISES

Read the following:

$303\frac{3}{8}$	$300\frac{3}{8}$	$300\frac{3}{1000}$	$303\frac{3}{1000}$
$300\frac{65}{1000}$	$360\frac{1}{5000}$	$365\frac{1}{1000}$	$360\frac{5}{1000}$
$1\frac{1}{2}$	$3\frac{1}{3}$	$2\frac{2}{3}$	$4\frac{1}{6}$
$2\frac{1}{4}$	$4\frac{1}{2}$	$3\frac{3}{4}$	$7\frac{1}{2}$

WRITING FRACTIONS

In making out bills, accounts, etc., bookkeepers omit the denominator in writing a mixed number containing ¼, ½, ¾. They write $2\frac{1}{4}$, $3\frac{1}{2}$, $4\frac{3}{4}$, respectively, as 2^1, 3^2, 4^3, omitting the denominator (4) and the line above it.

SIGHT EXERCISES

Give sums:

a	*b*	*c*	d
12^1	36^2	43^3	84^2
6	5^2	7^1	3^3
27^3	10^1	6	16^1
8^2	2^3	8^2	5^2

READING PER CENTS

The expressions ⅛% ¼%, and the like, are frequently stated as ⅛ of 1%, ¼ of 1%, etc., to emphasize the fact that each is a fraction of a per cent.

WRITING PER CENTS

Per cents are generally written as they are stated; $3\frac{1}{2}\%$, 4.7%, $7\frac{7}{10}\%$.

Some countries have a sign, ‰, which means thousandths, and is called "per mil." Inasmuch as many of our statistical comparisons are carried out to thousandths, the employment of the *per mil* sign would enable them in a *per cent* to dispense with the decimal point used to denote the tenths.

Baseball records are reported in thousandths, a batter's

average being given as .297, .315, etc. For the sake of uniformity, unnecessary ciphers are used; as, .300, .290, etc. These records are sometimes stated by the unthinking as 297 *per cent*, 315 *per cent*, etc. If the term *per cent* is used, it should be 29.7 %, etc. In giving a record orally, a player is said to be batting 297, 300, etc., omitting the word "thousandths."

ROMAN NUMBERS

The use of Roman numbers is limited to the numbering of the different books of a work issued in volumes; to the numbering of chapters, sections, etc., of a book; to inscriptions showing the date of the erection of a building, etc.; to the figures on the dial of a watch or clock, etc.

In some countries a Roman number is used in a date to indicate the month, VII–1–18, meaning July 1, 1918. This obviates the confusion existing in the mind of a person in this country, who does not know whether 7–1–18 means July 1, 1918, or January 7, 1918, there being no universal agreement as to the matter. Some argue that the month should precede, as is the case when it is written out; others claim that the day first, followed in the order by the month and the year, is the more logical arrangement.

A person desiring to denote the month by a number should use the Roman form; in which case 1–VII–1918, or VII–1–1918 would mean the seventh month, whether written in the first place or the second.

Roman notation employs the following characters:

I	V	X	L	C	D	M

to denote respectively

1	5	10	50	100	500	1000

The ones to 9, the tens to 90, the hundreds to 900, etc., are written as follows:

I 1	X 10	C 100	M 1000
II 2	XX 20	CC 200	MM 2000
III 3	XXX 30	CCC 300	MMM 3000
IV 4	XL 40	CD 400	$\overline{\text{IV}}$ 4000
V 5	L 50	D 500	$\overline{\text{V}}$ 5000
VI 6	LX 60	DC 600	$\overline{\text{VI}}$ 6000
VII 7	LXX 70	DCC 700	$\overline{\text{VII}}$ 7000
VIII 8	LXXX 80	DCCC 800	$\overline{\text{VIII}}$ 8000
IX 9	XC 90	CM 900	$\overline{\text{IX}}$ 9000

Note that

The value of V or of X is decreased by the value of an I immediately preceding it.

The value of L or of C is decreased by the value of an X immediately preceding it.

The value of D or of M is decreased by the value of a C immediately preceding it.

No letter is employed more than three times in expressing a number, excepting IIII on a clock face.

Thousands beyond 3000 are indicated by a line above $\overline{\text{IV}}$, $\overline{\text{V}}$, $\overline{\text{VI}}$, etc.

To write the Roman numbers between 11 and 19, inclusive; between 21 and 29, inclusive; etc.; affix the characters representing 1 to 9, respectively, to the characters denoting 10, 20, 30, etc., respectively.

ORAL EXERCISES

1. Express in Roman numbers:

a 23 *b* 34 *c* 45 *d* 56 *e* 67 *f* 78

2. State the values of:

a XCIX *b* LXXIV *c* XLVI *d* XXXVII

To write the Roman numbers between 101 and 199, inclusive; between 201 and 299, inclusive, etc., affix the character representing 1 to 99, respectively, to characters denoting 100, 200, 300, respectively.

3. Express in Roman numbers:

a 124 *b* 347 *c* 589 *d* 799 *e* 919

4. State the values of

a CCXL *b* CDXIX *c* DCCX *d* CMXCIX
e DCXV *f* DCCVI *g* CMLV *h* DCXLVII

To write the Roman numbers between 1001 and 1999, inclusive; between 2001 and 2999, inclusive, etc., affix the characters representing 1 to 999, respectively, to the characters representing 1000, 2000, 3000, etc., respectively.

5. Express in Roman numbers:

a 1234 *b* 2345 *c* 3457 *d* 2968
e 4067 *f* 5380 *g* 2799 *h* 6042

6. State the values of

a MDCLX *b* $\underline{\text{MM}}$CDL *c* $\underline{\text{MMM}}$XC
d MCMXC *e* $\overline{\text{VII}}$DCC *f* $\overline{\text{IX}}$CMXL
g CMLIX *h* MCDLX *i* DCCCXC

"ROUND" NUMBERS

Written Exercises

1. The following is the average quantity of cotton produced annually in the United States during a 10-year period preceding the last census, expressed in thousands of bales: Texas, 2838; Georgia, 1513; Mississippi, 1370; Alabama, 1155; North Carolina, 931; Arkansas, 811; Louisiana, 717; South Carolina,

547; Oklahoma, 540; Tennessee, 310; Florida, 62; Missouri, 38; Virginia, 16. Find the total in thousands of bales.

2. How much smaller is it than the 1917 crop of 12,000,000 bales?

3. In a year the sugar production in millions of pounds was as follows: Beet Sugar, 989½; Cane Sugar, 437½; Maple Sugar, 12. Find the total weight in millions of pounds.

4. Complete the following statistics of cotton production, and find the totals; (*e*), (*f*), (*g*), and (*h*).

States	Thousands of bales (*a*)	Millions of pounds (*b*)	Millions of pounds (*c*)	Millions of pounds (*d*)
Texas	2838	141.9	142	142
Georgia	1513	75.65	75½	76
Miss'ssippi	1370	68.5	68½	68
Alabama	1155	57.75	57½	58
etc.	etc.	etc.	etc.	etc.
Totals	(*e*)	(*f*)	(*g*)	(*h*)

Take (*a*) from Example 1.

In (*b*) assume the weight of a bale as 500 pounds. Divide (*a*) by 2, and point off 1 decimal.

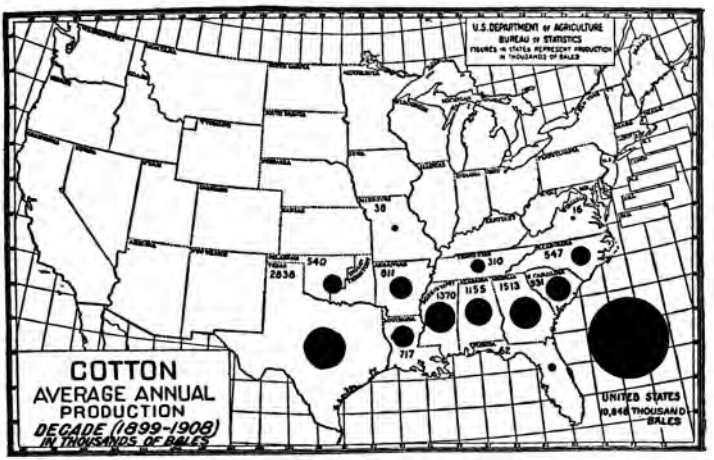
In (*c*), rewrite (*b*), rejecting decimals to .25, substituting ½ for these from .26 to .75, and increasing the integer by 1 when the decimal is greater than .75.

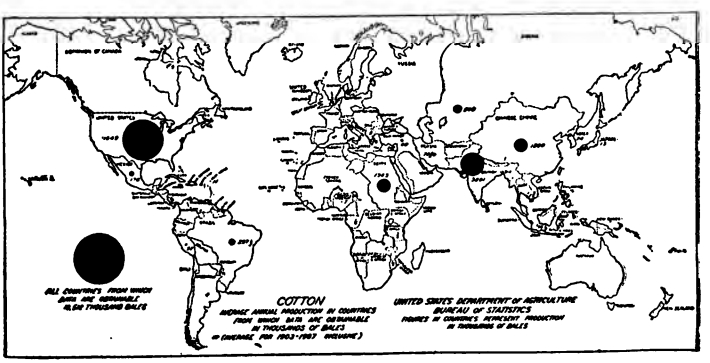
In (*d*) use integers exclusively, rejecting decimals in (*b*) to .5, and increasing the integer by 1 when the decimal is .5 or more.

In selling a quantity of cotton, every bale must be weighed, its net weight ascertained, etc. In the newspaper reports of sales, the number of bales is sufficient. In production statistics, the number of thousands of bales answers every purpose.

REPRESENTING QUANTITIES GRAPHICALLY

The first map shows the cotton belt of the United States with relative production of the cotton states. The next map shows the cotton production of the world.



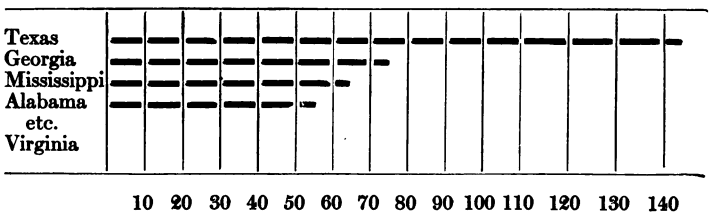


STRAIGHT LINE GRAPHS

WRITTEN EXERCISES

1. From the table of the average production of cotton in the United States, complete the following graph:

Average Annual Cotton Production for Ten Years



Each perpendicular space represents 10 million of pounds. The line for Texas extends a trifle beyond the line for 140 millions; that for Georgia, a trifle more than one-half the distance between 70 and 80 millions, etc.

2. The accompanying summary shows the effects of irrigation on two fields of oats and two of wheat.

Irrigation	Production per Acre	
	Oats	Wheat
None	425 lb.	540 lb.
13 in. 10 in.	1424 "	1200 "
20 " 18 "	2060 "	1950 "

a Find the production per acre in bushels, taking 32 pounds of oats to the bushel and 60 pounds of wheat.

b Make a graph based upon the number of pounds of each to the acre raised under the several conditions.

Pounds of Grain per Acre	Oats			Wheat		
	Inches of Water Applied					
	0	13	20	0	10	18
2000						
1800						
1600						
1400						
1200						
1000						
800						
600						
400						
200						
0						
	I	II	III	IV	V	VI

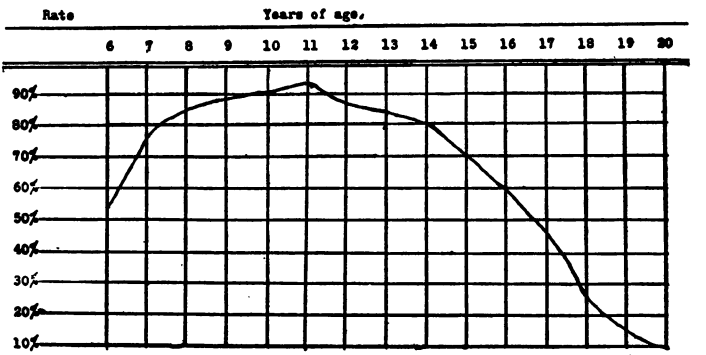
c At 96 cents a bushel for oats and $2.10 a bushel for wheat, find the value of each crop (I to VI).

d Make a graph based upon the value of each crop an acre.

BROKEN-LINE GRAPHS

The following graph shows the per cent of those 6 to 20 years of age in a certain city, who are attending school.

In this graph the horizontal lines indicate the per cents and the vertical lines the ages. On each of the latter the proper per cent is noted by a point, and a line is drawn from one point to the next.



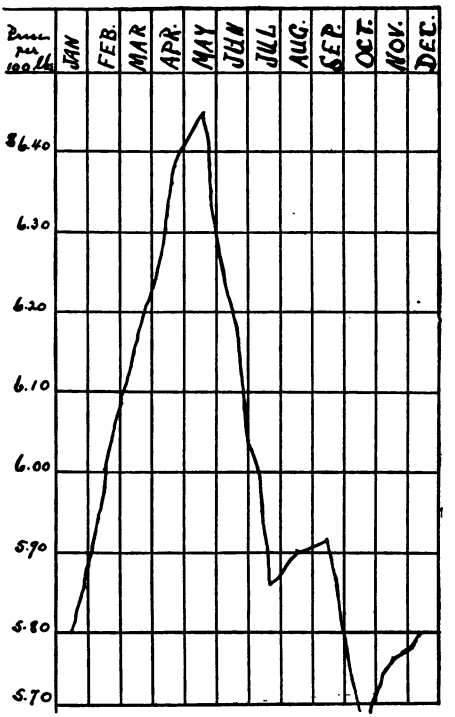
ORAL EXERCISES

1. About what per cent of the children (*a*) 6 years of age are in school? (*b*) 7 years? (*c*) 8 years? (*d*) 11 years? (*e*) 14 years?

2. What age shows the largest per cent of pupils in school?

3. About what per cent of persons 19 years of age are not in school?

The average selling price of cattle by months during 10 years is shown in the next graph. The price for each month is found in the center of the monthly space.



ORAL EXERCISES

1. Give the average price for each month.

2. (*a*) In what month did cattle average the lowest price? (*b*) What was the price? (*c*) In what month did they average the highest? (*d*) What was this price?

MAKING A BROKEN-LINE GRAPH

In making a broken-line graph, use cross-ruled paper, if it is at hand, thus saving time in laying out the necessary vertical lines.

WRITTEN EXERCISE

Make a graph showing the average prices for 12 months, as follows: \$5.92, \$6.08, \$6.36, \$6.47, \$6.44, \$6.22 \$5.63, \$5.88, \$5.66, \$5.84, \$5.80, \$6.02.

READING METERS

Every housekeeper should read the gas meter at the time it is read by the agent of the company, and should keep a record of the readings and the date in order to be able to check up the bill when it is rendered.

Dials of a Gas Meter

The small dials at the top are ignored in reading the meter, being used merely to test.

When the hand makes a complete revolution of the dial marked "100 thousand," it indicates the consumption of 100,000 cubic feet of gas. The passage of the hand over each of the ten divisions, shows, therefore, the consumption of $\frac{1}{10}$ of that quantity, or 10,000 cubic feet. Since the hand in this dial has passed eight divisions, it shows a consumption of 80,000 cubic feet.

The hand of this first dial moves from left to right, that of the second dial from right to left. For this reason the order of the dial numbers is reversed. As each division of the second dial indicates $\frac{1}{10}$ of 10,000 cubic feet, this dial shows a consumption of 4,000 cubic feet, the hand having passed four divisions.

It is in the reading of this dial that mistakes are made by the beginner, who forgets that the motion of this hand is from right to left, and that it has not reached the 5th division.

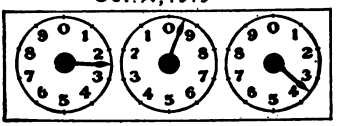
The hand of the last dial moves from left to right. As it has passed two divisions, the consumption is over 200 cubic feet. The excess is ignored, being left for the next reading.

The consumption recorded above is 84,200 cubic feet, which indicates the quantity of gas that passed through the meter since its installation.

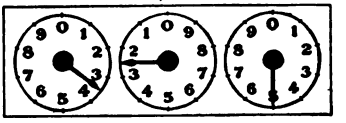
ORAL EXERCISES

1. Read the following dials, which show Mr. Hanlon's meter records for six successive months.

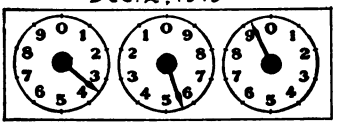
Oct. 2, 1919



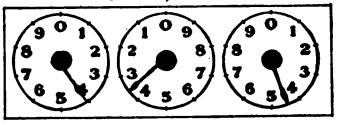
Nov. 1, 1919



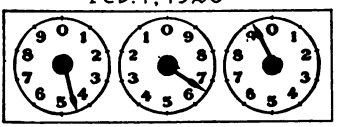
Dec. 2, 1919



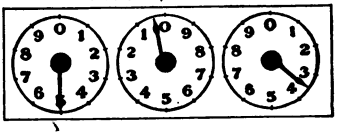
Jan. 2, 1920



Feb. 1, 1920



Mch. 3, 1920



2. Give the difference (*a*) between the reading of December and that of January. (*b*) Between the reading of January and that of February.

A Gas Bill

Jay A. Lindon
4235 Fairfax Road

To KINGS COUNTY LIGHT COMPANY, Dr.
384 Midwood Ave.

Telephone, Sunset 2700

Index of Meter, Sept 2, 1919.	26,300 cu. ft.		
" " " Aug. 2, 1919.	23,500 " "		
" " " To supply of	2,800 " " @ 90¢		2.52
Discount of 1¢ per 100 cu. ft.			.28
			2.24

Received Payment...Sep. 25,...1919.
per...Frank Regan..., for the Company

Mrs. Lindon compares this bill with the memorandum of her meter readings. This she keeps in the following form:

GAS CONSUMPTION

Date	Reading	Cu. ft. Used	Cost at 90¢ per 1000 cu. ft.	Discount	Net	Bill paid
1919						
Aug. 2	23,500	2400	2 16	24	1 92	VIII–18–19
Sep. 2	26,300	2800	2 52	28	2 24	IX–25–19
Oct. 2	30,000					
Nov. 1	34,100					
Dec. 2	38,300					
1920						
Jan. 2	42,500					
Feb. 1	43,500					
Mar. 3	46,700					

WRITTEN EXERCISES

1. Complete the foregoing memorandum, inserting (*a*) the quantity of gas used each month, (*b*) the gross cost, (*c*) the discounts, and (*d*) the net amount.

2. Find (*a*) the totals for the eight months. (*b*) The average consumption a month.

3. The daily allowance of gas to a room of an army officer on post duty is 50 cubic feet from Sep. 1 to Apr. 30, inclusive, and 30 cubic feet from May 1 to Aug. 31, inclusive. Find the yearly allowance for the lights of three rooms at 80 cents a thousand cubic feet.

4. If an ordinary open-flame burner uses 5 cubic feet an hour, what is the cost of an hour's light at the rate of $1 a 1000 cubic feet?

5. Mr. Jones formerly used 2 open-flame burners, each consuming 5 cubic feet of gas an hour. He replaced both by a single mantle burner, using 4 cubic feet of gas an hour. What did he save in a year by the change if he used gas for 1000 hours, at $1 a 1000 cubic feet, and paid 25 cents each for three mantles for the new burner?

SELLING ELECTRIC CURRENT

The unit for measuring electrical work is the *watt-hour* or the *kilowatt-hour*, the latter being 1000 watt-hours.

A 25-watt electric lamp will use 25 watt-hours of energy per hour, or a kilowatt-hour in 40 hours.

The amount used is recorded by a meter having dials similar to those employed on a gas meter.

One form of electric meter shows the number of "*kilowatt-hours*" on four dials similar to those employed on the gas meter.

MARKING GOODS

For the guidance of salespeople in large stores, the selling price of each article is marked on it, gen-

erally in plain figures. For the information of the manager, etc., the cost is also noted.

In giving the latter, letters are used instead of figures, in order that it may not be disclosed to the customer.

In selecting the letters, a word or combination of words containing at least ten letters is employed. "Frank White," for instance.

Each letter in these two words represents the figure written under it.

F	R	A	N	K		W	H	I	T	E
1	2	3	4	5		6	7	8	9	0

The number 375 is written *AHK;* 429, *NRT;* etc.

THE REPEATER

In writing a number having two consecutive figures alike, an eleventh letter is used, which is not contained in the 10-letter key.

Thus, $1.12 is written *FDR*, the second 1 being represented by the repeater, *D;* $3.99 is written *ATD*.

WRITTEN EXERCISES

1. Using "Frank White" as the key, and "D" as the repeater, express by letters:

a $4.41	*b* $.39	*c* $3.22	*d* $1.76	*e* $12.84
f 5.98	*g* .97	*h* 6.04	*i* 3.21	*j* 22.05

2. Using "Quick Reason" as the key, and taking *N*, the last letter, as the repeater, express the following by letters:

a $3.00	*b* $.27	*c* $8.04	*d* $3.95	*e* $20.08
f 4.32	*g* .55	*h* 9.99	*i* 4.68	*j* 12.23

Sometimes when a price is below $1, a dealer prefers to use three letters, and to use four when it is below $10. In such a case, he uses a letter which is to be disregarded. For instance, he represents $.99 by *BSN*, or *SNB*, with the key "Quick Reason," using *B* as the letter to be omitted.

ORAL EXERCISES

Read the following, for which the key is "Quick Reason," and in which *B* is superfluous.

a KON	*b CNK*	*c BAO*	*d UKO*	*e BSRI*	*f IC*
g IAN	*h EAE*	*i SBR*	*j QIE*	*k SNBO*	*l KN*

If it is desired to mark the cost by the use of letters, as well as the selling price, a different key is used for each.

With "Frank White" as the key for the cost, and "Quick Reason" as the key for the selling price, with *D* for the repeater in the former, and *N* in the latter and *B* for the superfluous letter, read the following, the upper line representing the cost and the lower the selling price.

a *BID* / *QNU*	*b* *FHK* / *UON*	*c* *ADE* / *CEK*	*d* *FRTA* / *QKIO*	*e* *WIA* / *ASU*

WRITTEN EXERCISES

1. Using the last two keys for the cost and the selling price, respectively, express each in letters,

increasing the cost by the given rate to obtain the selling price.

a	Cost $1.80,	selling price,		35 %	advance
b	Cost 9.20,	"	"	20 %	"
c	Cost 8.40,	"	"	33⅓ %	"
d	Cost 2.40,	"	"	37½ %	"
e	Cost 4.96,	"	"	43¾ %	"

2. Find the per cent of advance on goods marked as follows:

a *VTW* / *QUO*	*b* *RAFE* / *IOAO*	*c* *FKE* / *QEK*	*d* *ACD* / *CAK*	*e* *VTIN* / *QCER*

CHAPTER TWO

PROPERTIES OF NUMBERS

COMPOSITE NUMBERS

A number that exactly contains a number other than itself or 1, is called a *composite number*.

4, 6, 8, 9, 10, etc., are composite numbers.

An exact divisor of a number is called a *factor* of the number.

2 is a factor of 4, of 6, of 8, of 10, etc.
3 " " " " 6, of 9, of 12, etc.
5 " " " " 10, of 15, of 20, etc.

ORAL EXERCISES

1. Give the factors of

a 6	*b* 10	*c* 14	*d* 15	*e* 21	*f* 22	*g* 26
h 33	*i* 34	*j* 35	*k* 38	*l* 39	*m* 46	*n* 51
o 55	*p* 57	*q* 58	*r* 65	*s* 69	*t* 77	*u* 85
v 87	*w* 91	*x* 95	*y* 115	*z* 119		

2. Give the two equal factors of

a 4 *b* 9 *c* 25 *d* 49 *e* 121 *f* 169

PRIME NUMBERS

A number that is exactly divisible only by itself and 1 is called a *prime number*.

1, 2, 3, 5, 7, etc., are prime numbers.

ORAL EXERCISES

1. Give the prime numbers between

a 10 and 30 *b* 30 and 50 *c* 50 and 70
d 70 and 90 *e* 90 and 110 *f* 110 and 130

2. Give the prime factors of:

a 30 *b* 42 *c* 78 *d* 66 *e* 70 *f* 102 *g* 110 *h* 114

3. Give all of the exact divisors of each of the foregoing numbers.

A number that is a factor of each of two or more numbers is called a *common factor* of these numbers.

2 is a common factor of 8 and 14, 3 is a common factor of 15 and **24**, etc.

Two or more numbers that have no common factor are said to be *prime to each other;* 4 and 9, for instance; 6 and **25**; etc.

ORAL EXERCISES

1. Give a common factor of:

a 30 and 63 *b* 22 and 56 *c* 25 and 70 *d* 21 and 56

2. Give two common factors of:

a 30 and 42 *b* 42 and 66 *c* 42 and 70 *d* 66 and 105

3. Give the largest common factor of:

a 18 and 24 *b* 14 and 56 *c* 12 and 30 *d* 52 and 91
e 26 and 65 *f* 34 and 85 *g* 21 and 93 *h* 37 and 74
i 69 and 92 *j* 34 and 51 *k* 68 and 85 *l* 65 and 91

4. Express each of the following fractions in its lowest terms by dividing its numerator and its de-

nominator by their largest common factor (*greatest common divisor*).

a $^{82}/_{123}$	b $^{93}/_{124}$	c $^{62}/_{155}$	d $^{65}/_{78}$
e $^{49}/_{63}$	f $^{76}/_{95}$	g $^{35}/_{45}$	h $^{39}/_{104}$

A number that is divisible by another number is called a *multiple* of the latter. 4, 6, 8, 16, etc., are multiples of 2. 6, 24, 42, 96, etc., are multiples of 6.

A number that is a multiple of two or more numbers is called a *common multiple* of these numbers. 6, 18, 24, etc., are common multiples of 2 and 3. 24, 36, 72, etc., are common multiples of 4 and 6.

The smallest number that is a multiple of two or more numbers is called their *least common multiple.* (L. C. M.)

The least common multiple of two or more prime numbers is the continued product of these numbers.

The L. C. M. of 2, 3, and 7 is $2 \times 3 \times 7$.

The least common multiple of two or more numbers *prime to each other* is also their continued product.

The L. C. M. of 4 and 9 is 4×9; of 6 and 25 is 6×25; of 5, 8, and 9 is $5 \times 8 \times 9$; of 8 and 9 is 8×9.

In finding the least common multiple of several numbers, omit the consideration of any number that is a factor of any other one.

To find the L. C. M. of 4, 6, and 8, omit 4 since a multiple of 8 is necessarily a multiple of 4. In determining the L. C. M. of 6 and 8, consider the successive multiples of 8 (16 and 24) until one is found that is a multiple of 6.

ORAL EXERCISES

1. Give the least common multiple of each of the following:

a 4 and 6	*b* 4, 6, and 8	*c* 6 and 9
d 9 and 12	*e* 2, 3, and 5	*f* 4 and 14
g 5 and 24	*h* 6, 8, and 10	*i* 12 and 16
j 4 and 9	*k* 5, 8, and 9	*l* 6 and 15

2. Give the least common multiple of the denominators of the following fractions (least common denominator):

a $\frac{3}{4}$ and $\frac{5}{6}$	*b* $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$	*c* $\frac{3}{4}$ and $\frac{7}{10}$
d $\frac{1}{6}$ and $\frac{7}{8}$	*e* $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{7}{8}$	*f* $\frac{5}{8}$ and $\frac{7}{12}$
g $\frac{7}{8}$ and $\frac{5}{6}$	*h* $\frac{1}{6}$, $\frac{2}{9}$, and $\frac{5}{18}$	*i* $\frac{7}{10}$ and $\frac{4}{25}$

DIVISIBILITY OF NUMBERS

2 is a factor of any number whose last figure is a cipher or is divisible by 2.

5 is a factor of any number whose last figure is a cipher or a 5.

3 is a factor of any number the sum of whose digits, is divisible by 3.

11 is a factor of any number having the sum of its odd digits equal to the sum of its even digits.

The powers of the foregoing prime numbers are factors as follows:

4 (2×2) is a factor of any number whose last two figures are ciphers, or indicate a number divisible by 4.

8 ($2 \times 2 \times 2$) is a factor of any number whose last three figures are ciphers, or indicate a number divisible by 8.

25 (5×5) is a factor of any number whose last two figures are ciphers, or indicate a number divisible by 25.

125 ($5 \times 5 \times 5$) is a factor of any number whose last three figures are ciphers or indicate a number divisible by 125.

9 (3×3) is a factor of any number the sum of whose digits is divisible by 9.

ORAL EXERCISES

1. Of the numbers having terminal figures as follows, state which are divisible by 2, 5, 4, 25, 8, 125:

a ...164	*b* ...265	*c* ...000	*d* ...248
e ...200	*f* ...475	*g* ...625	*h* ...260

2. State which of the following are divisible by 3; by 9:

a 2345	*b* 5432	*c* 4533	*d* 3546
e 4653	*f* 6345	*g* 3543	*h* 5334
i 5423	*j* 3534	*k* 4365	*l* 3456

A number that is divisible by 2 and by 3 is divisible by 6.

3. (*a*) Which of the numbers in the last example are divisible by 2? (*b*) Which are divisible by 6?

A number that is divisible by 4 and by 9 is divisible by 36.

4. Which of the foregoing are divisible (*a*) by 4? (*b*) By 36?

A number that is divisible by two or more numbers prime to each other is divisible by their product.

Thus 72, 108, 144, etc., being divisible by 4 and by 9, are divisible by 36. Although 48 and 132 are divisible by 3 and by 12, they are not divisible by 3 times 12.

CHAPTER THREE

REDUCTIONS

DRILLS

Every arithmetic period should begin with a three-minute drill of one kind or other. Besides serving to "warm-up" the pupils for the work to follow, it is useful as a review of preceding topics, and as tending to develop greater facility in making arithmetical combinations.

SIGHT EXERCISES

In sight exercises the examples should be in the view of the pupils — in their textbooks, on the blackboards, etc. Those which are given here may be used in several ways. One day oral answers may be required, the class standing, each taking his seat when he announces his result. When there is time after the completion of one round of the class, the pupils again stand, and the drill is continued until time is announced by the pupil designated to act as scorer.

When a pupil gives an incorrect result, the teacher says nothing; but no answers to subsequent questions are considered until some pupil corrects the error previously made. The next pupil answers the question following the one just answered correctly, etc.

The scorer notes on the blackboard the number of questions answered and the number of errors made.

When the same set of examples is taken up again, the result should show an improvement.

WRITTEN ANSWERS

Another way to use some of these exercises is to have all of the pupils write on slips their answers to the designated questions. The answers are then announced, and each pupil checks his correct ones and notes their number. The scorer should ascertain and record the class average.

REDUCING FRACTIONS

CHANGING AN IMPROPER FRACTION TO A MIXED NUMBER

SIGHT EXERCISES

1. How many dollars are there in 85 quarter dollars?

PROCESS

$$\frac{\$85}{4} = \$21\tfrac{1}{4}. \quad \text{Ans.}$$

Change the improper fraction $\frac{85}{4}$ to a mixed number by dividing 85 by 4.

2. Express as a mixed number:

a $\frac{97}{8}$	b $\frac{71}{11}$	c $\frac{46}{15}$	d $\frac{89}{12}$	e $\frac{62}{5}$
f $\frac{87}{20}$	g $\frac{58}{9}$	h $\frac{85}{14}$	i $\frac{60}{7}$	j $\frac{91}{6}$
k $\frac{73}{16}$	l $\frac{80}{13}$	m $\frac{97}{30}$	n $\frac{75}{4}$	o $\frac{41}{18}$
p $\frac{88}{21}$	q $\frac{99}{32}$	r $\frac{69}{31}$	s $\frac{60}{27}$	t $\frac{90}{41}$
u $\frac{77}{24}$	v $\frac{97}{23}$	w $\frac{98}{25}$	x $\frac{93}{42}$	y $\frac{73}{22}$

CHANGING A MIXED NUMBER TO AN IMPROPER FRACTION

3. How many quarters are there in $24¾?

PROCESS

$24¾ = $99/4. Ans.

Think 96 (4 times 24), 99 (adding 3). Make 99 the numerator of the improper fraction, and 4 its denominator.

4. Change to an improper fraction:

a 12⅜	*b* 14⅞	*c* 15⅔	*d* 6½
e 13⅔	*f* 30¾	*g* 40⅝	*h* 2⅝
i 21¼	*j* 19½	*k* 17¼	*l* 5⅗
m 25⅓	*n* 31⅔	*o* 20⅞	*p* 4½
q 22¼	*r* 24½	*s* 26½	*t* 8⅝
u 11⅞	*v* 18⅓	*w* 16⅒	*x* 3⅜

EXPRESSING A FRACTION IN LOWEST TERMS

5. What fraction of a yard is (*a*) 27 inches? (*b*) 24 inches? (*c*) 32 inches?

PROCESS

(*a*) 27 in. = 27/36 yd. = ¾ yd. Ans.
(*b*) 24 in. = 24/36 yd. = ⅔ yd. Ans.
(*c*) 32 in. = 32/36 yd. = ⅞ yd. Ans.
(*a*) Reduce 27/36 by dividing both terms by 9.
(*b*) Reduce 24/36 by dividing both terms by 12.
(*c*) Reduce 32/36 by dividing both terms by 4.

A pupil who does not see at once that 9 is the greatest common divisor of 27 and 36 may first reduce 27/36 to 3/12 by dividing each term by the common factor, 3.

Before announcing 3/12 as the answer, he should be expected to note that it is further reducible to ¼, 9 and 12 having 3 as a common factor.

6. Express in lowest terms:

a $^{18}/_{27}$	b $^{25}/_{30}$	c $^{16}/_{36}$	d $^{15}/_{50}$	e $^{36}/_{100}$
f $^{50}/_{75}$	g $^{48}/_{56}$	h $^{18}/_{24}$	i $^{53}/_{40}$	j $^{45}/_{60}$
k $^{35}/_{49}$	l $^{20}/_{75}$	m $^{28}/_{35}$	n $^{56}/_{84}$	o $^{36}/_{80}$
p $^{60}/_{75}$	q $^{36}/_{60}$	r $^{40}/_{75}$	s $^{27}/_{60}$	t $^{24}/_{100}$
u $^{36}/_{48}$	v $^{40}/_{72}$	w $^{36}/_{54}$	x $^{48}/_{72}$	y $^{75}/_{90}$

7. Give the greatest common factor of:

a 18 and 27	b 25 and 60	c 16 and 36
d 36 and 48	e 40 and 72	f 20 and 75
g 48 and 72	h 36 and 54	i 15 and 50
j 75 and 90	k 36 and 60	l 20 and 75

8. What is the greatest common factor of 57 and 95?

PROCESS

A pupil that does not see at once that 19 is a common factor of 57 and 95, should note that both 57 and 95 are composite numbers. Obtaining 3 and 19 as the factors of 57, he tests 19 as the divisor of 95.

9. Express in lowest terms:

a $^{26}/_{39}$	b $^{57}/_{95}$	c $^{39}/_{91}$	d $^{29}/_{87}$	e $^{46}/_{69}$
f $^{34}/_{51}$	g $^{69}/_{92}$	h $^{52}/_{65}$	i $^{51}/_{68}$	j $^{57}/_{76}$

SIMPLIFYING A COMPLEX FRACTION

10. What fraction of a rod (16½ feet) is (*a*) 12½ ft.? (*b*) 9⅜ ft.?

PROCESS

Express each as a complex fraction of a rod by writing $16\frac{1}{2}$ as the denominator. Simplify the complex fractions.

(*a*) $\frac{12\frac{1}{2}}{16\frac{1}{2}}$ ft. $= \frac{25}{33}$ rd. Ans.

(*b*) $\frac{9\frac{2}{3}}{16\frac{1}{2}}$ ft. $= \frac{58}{99}$ rd. Ans.

(*a*) Multiply both terms of the complex fraction by 2. (*b*) Multiply both terms by 6, the least common denominator of $\frac{1}{2}$ and $\frac{1}{3}$.

11. Express as a simple fraction:

a $\frac{1\frac{1}{2}}{4}$	*b* $\frac{2\frac{1}{3}}{3}$	*c* $\frac{3\frac{3}{4}}{7}$	*d* $\frac{4\frac{1}{5}}{8}$	*e* $\frac{5\frac{2}{3}}{9}$
f $\frac{4}{5\frac{1}{2}}$	*g* $\frac{5}{7\frac{1}{3}}$	*h* $\frac{6}{8\frac{3}{4}}$	*i* $\frac{7}{9\frac{2}{3}}$	*j* $\frac{8}{10\frac{1}{5}}$
k $\frac{2\frac{1}{2}}{3\frac{1}{2}}$	*l* $\frac{3\frac{1}{3}}{4\frac{1}{3}}$	*m* $\frac{1\frac{3}{4}}{2\frac{1}{4}}$	*n* $\frac{5\frac{5}{6}}{6\frac{1}{6}}$	*o* $\frac{4\frac{4}{5}}{7\frac{1}{5}}$
p $\frac{3\frac{1}{2}}{4\frac{1}{4}}$	*q* $\frac{1\frac{2}{5}}{2\frac{3}{10}}$	*r* $\frac{1\frac{1}{4}}{2\frac{1}{8}}$	*s* $\frac{2\frac{1}{3}}{3\frac{1}{6}}$	*t* $\frac{3\frac{1}{2}}{4\frac{1}{8}}$
u $\frac{3\frac{1}{2}}{4\frac{1}{3}}$	*v* $\frac{2\frac{1}{3}}{3\frac{1}{4}}$	*w* $\frac{1\frac{1}{2}}{2\frac{1}{5}}$	*x* $\frac{3\frac{1}{4}}{4\frac{1}{10}}$	*y* $\frac{1\frac{1}{3}}{2\frac{1}{5}}$

EXPRESSING A COMMON FRACTION AS A DECIMAL

12. What decimal of a pound is an ounce?

PROCESS

1 oz. $= \frac{1}{16}$ lb. $= .0625$ lb.

Think of $\frac{1}{16}$ as $\frac{1}{4}$ of $\frac{1}{4}$, which is $\frac{1}{4}$ of .25. This is .06¼, or .0625.

13. Express as a decimal:

a $\frac{1}{2}$	b $\frac{1}{4}$	c $\frac{1}{5}$	d $\frac{1}{8}$	e $\frac{3}{4}$
f $\frac{2}{5}$	g $\frac{3}{8}$	h $\frac{1}{25}$	i $\frac{3}{5}$	j $\frac{5}{8}$
k $\frac{7}{8}$	l $\frac{4}{5}$	m $\frac{1}{50}$	n $\frac{3}{25}$	o $\frac{1}{40}$
p $\frac{7}{25}$	q $\frac{7}{50}$	r $\frac{7}{20}$	s $\frac{1}{80}$	t $\frac{4}{25}$
u $\frac{1}{20}$	v $\frac{8}{25}$	w $\frac{9}{50}$	x $\frac{3}{50}$	y $\frac{3}{20}$

CHANGING A DECIMAL TO A COMMON FRACTION

14. Change to a common fraction, lowest terms: (a) .87. (b) .124. (c) .0275.

PROCESS

(a) $.87 = {}^{87}\!/_{100}$, Ans.

(b) $.124 = {}^{124}\!/_{1000} = {}^{31}\!/_{250}$, Ans.

(c) $.0275 = {}^{275}\!/_{10000} = {}^{55}\!/_{2000} = {}^{11}\!/_{400}$, Ans.

(a) The common fraction ${}^{87}\!/_{100}$ is expressed in lowest terms since 87 is divisible by neither 2 nor 5.

(b) Express as a common fraction and reduce to lowest terms by dividing both terms by 4.

(c) If you do not observe that both terms are divisible by 25, divide twice by 5.

15. Express as a common fraction — lowest terms:

a .5	b .25	c .33	d .125	e .008
f .8	g .45	h .05	i .025	j .012
k .6	l .06	m .08	n .375	o .037
p .7	q .32	r .44	s .625	t .045
u .4	v .09	w .56	x .875	y .168

DENOMINATE NUMBERS — LOWER TERMS

16. A vessel made a trip from Liverpool to New York in 5 days 12 hours. How many hours were consumed in making the trip?

PROCESS

5 da. 12 hr. = 132 hr., Ans.
Think 120 hr. (5 times 24 hr.) 132 hr. (adding 12 hr.)

17. Change to hours:

a 2 da. 13 hr.	*b* 3 da. 20 hr.	*c* 4 da. 16 hr.
d 5 da. 18 hr.	*e* 6 da. 11 hr.	*f* 7 da. 10 hr.
g 8 da. 12 hr.	*h* 9 da. 15 hr.	*i* 8 da. 16 hr.

18. Change to ounces:

a 4 lb. 15 oz.	*b* 5 lb. 14 oz.	*c* 10 lb. 8 oz.
d 6 lb. 13 oz.	*e* 7 lb. 12 oz.	*f* 20 lb. 5 oz.
g 8 lb. 11 oz.	*h* 9 lb. 10 oz.	*i* 30 lb. 7 oz.

19. Change to quarts:

a 13 gal. 1 qt.	*b* 15 gal. 2 qt.	*c* 17 gal. 3 qt.
d 19 gal. 3 qt.	*e* 21 gal. 2 qt.	*f* 23 gal. 1 qt.
g 25 gal. 1 qt.	*h* 31 gal. 2 qt.	*i* 42 gal. 3 qt.

20. Change to months:

a 7 yr. 10 mo.	*b* 11 yr. 8 mo.	*c* 20 yr. 9 mo.
d 8 yr. 11 mo.	*e* 12 yr. 7 mo.	*f* 25 yr. 7 mo.
g 9 yr. 10 mo.	*h* 13 yr. 6 mo.	*i* 30 yr. 3 mo.

21. Change to pecks:

a 12 bu. 3 pk.	*b* 32 bu. 1 pk.	*c* 24 bu. 3 pk.
d 22 bu. 1 pk.	*e* 41 bu. 2 pk.	*f* 52 bu. 2 pk.
g 15 bu. 2 pk.	*h* 18 bu. 3 pk.	*i* 61 bu. 1 pk.

22. Change to quarts:

a 13 pk. 7 qt.	*b* 21 pk. 4 qt.	*c* 41 pk. 1 qt.
d 14 pk. 6 qt.	*e* 22 pk. 3 qt.	*f* 51 pk. 2 qt.
g 15 pk. 5 qt.	*h* 31 pk. 2 qt.	*i* 61 pk. 3 qt.

23. Change to inches:

a 11 ft. 6 in.	*b* 21 ft. 3 in.	*c* 13 ft. 5 in.
d 25 ft. 9 in.	*e* 15 ft. 8 in.	*f* 22 ft. 7 in.
g 31 ft. 2 in.	*h* 33 ft. 1 in.	*i* 12 ft. 4 in.

24. Change to feet:

a 33 yd. 1 ft.	*b* 25 yd. 1 ft.	*c* 43 yd. 2 ft.
d 22 yd. 2 ft.	*e* 53 yd. 2 ft.	*f* 23 yd. 1 ft.
g 42 yd. 1 ft.	*h* 24 yd. 1 ft.	*i* 32 yd. 2 ft.

25. Change to yards (1 rd. = 5½ yd.):

a 4 rd. 1 yd.	*b* 12 rd. 4 yd.	*c* 14 rd. 3 yd.
d 6 rd. 2 yd.	*e* 10 rd. 5 yd.	*f* 22 rd. 2 yd.
g 8 rd. 3 yd.	*h* 20 rd. 4 yd.	*i* 30 rd. 1 yd.

26. A strip of embroidery measured ⅚ yd. What was its length in feet and inches?

PROCESS

⅚ yd. = ⅚ times 3 ft. = ⅚ ft. = 2½ ft. = 2 ft. 6 in. Ans.

Change ⅚ yd. to feet by multiplying by 3. Change ½ ft. to inches by multiplying by 12.

27. Change to compound denominate numbers of lower denominations:

a 5/12 yd.	*b* 7/12 yd.	*c* ¾ yd.	*d* ½ yd.
e ⅙ wk.	*f* ⅓ wk.	*g* ⅔ wk.	*h* ⅚ wk.
i ⅛ yr.	*j* ⅛ yr.	*k* ⅛ yr.	*l* 1/10 yr.
m ⅕ da.	*n* ⅖ da.	*o* ⅗ da.	*p* ⅘ da.

EXPRESSING A FRACTION OF A DOLLAR AS CENTS

28. When silk is sold at $\frac{7}{8}$ a yard, what is the price in cents?

PROCESS

$\frac{7}{8}$ = 700¢ ÷ 8 = 87½ cents, Ans.

Do not perform this division unnecessarily. The pupil should know 12½ cents as $\frac{1}{8}$, and multiply 12½¢ by 7.

29. Change to cents:

a $\frac{1}{3}$	*b* $\frac{5}{8}$	*c* $\frac{1}{16}$	*d* $\frac{1}{15}$
e $\frac{1}{6}$	*f* $\frac{2}{3}$	*g* $\frac{1}{20}$	*h* $\frac{7}{30}$
i $\frac{1}{8}$	*j* $\frac{3}{5}$	*k* $\frac{1}{30}$	*l* $\frac{8}{50}$
m $\frac{1}{5}$	*n* $\frac{1}{9}$	*o* $\frac{1}{40}$	*p* $\frac{6}{25}$
q $\frac{1}{6}$	*r* $\frac{4}{5}$	*s* $\frac{1}{50}$	*t* $\frac{3}{40}$
u $\frac{3}{8}$	*v* $\frac{5}{6}$	*w* $\frac{1}{25}$	*x* $\frac{1}{60}$

HIGHER TERMS

30. What fraction of a dollar is 6⅔ cents?

PROCESS

$$6\tfrac{2}{3}¢ = \frac{\$6\frac{2}{3}}{100} = \frac{\$20}{300} = \$\tfrac{1}{15}, \text{ Ans.}$$

If you do not surely recall the fraction, express 6⅔ cents as a complex fraction of a dollar. Reduce this to a simple fraction by multiplying both terms by 3. Express this fraction in lowest terms.

31. Express as a fraction of a dollar:

a $1\frac{1}{4}¢$	*b* $12\frac{1}{2}¢$	*c* $37\frac{1}{2}¢$	*d* 35¢	*e* $87\frac{1}{2}¢$
f $2\frac{1}{2}¢$	*g* $16\frac{2}{3}¢$	*h* $83\frac{1}{3}¢$	*i* 85¢	*j* $11\frac{1}{9}¢$
k $3\frac{1}{3}¢$	*l* $18\frac{3}{4}¢$	*m* $31\frac{1}{4}¢$	*n* 95¢	*o* $14\frac{2}{7}¢$
p $6\frac{1}{4}¢$	*q* $33\frac{1}{3}¢$	*r* $62\frac{1}{2}¢$	*s* 60¢	*t* $22\frac{2}{9}¢$
u $8\frac{1}{3}¢$	*v* $43\frac{3}{4}¢$	*w* $68\frac{3}{4}¢$	*x* 65¢	*y* $28\frac{4}{7}¢$

32. Interest for 144 days is due on a loan. What part of a year's interest is due?

PROCESS

144 da. = $\frac{144}{360}$ yr. = $\frac{2}{5}$ yr. Ans. Express the time as the fraction of a year of 360 days. Reduce this fraction to lowest terms.

33. Express as a fraction of a year:

a 80 da.	*b* 108 da.	*c* 120 da.	*d* 180 da.
e 72 da.	*f* 144 da.	*g* 135 da.	*h* 216 da.
i 60 da.	*j* 225 da.	*k* 285 da.	*l* 215 da.
m 45 da.	*n* 200 da.	*o* 160 da.	*p* 252 da.
q 40 da.	*r* 320 da.	*s* 324 da.	*t* 280 da.

34. What fraction of a year is 10 months 15 days?

PROCESS

10 mo., 15 da. = $10\frac{1}{2}$ mos. = $\frac{10\frac{1}{2}}{12}$ yr. = $\frac{21}{24}$ yr. = $\frac{7}{8}$ yr.; or change 10 mo., 15 da. to 315 da., or $\frac{315}{360}$ yr. Reduce this fraction to $\frac{63}{72}$, then to $\frac{7}{8}$.

35. Express as a fraction of a year:

a 1 mo. 10 da.	*b* 9 mo. 10 da.	*c* 6 mo. 20 da.
d 4 mo. 20 da.	*e* 2 mo. 12 da.	*f* 4 mo. 15 da.
g 1 mo. 15 da.	*h* 3 mo. 18 da.	*i* 3 mo. 20 da.
j 4 mo. 24 da.	*k* 7 mo. 15 da.	*l* 9 mo. 45 da.
m 9 mo. 18 da.	*n* 1 mo. 20 da.	*o* 4 mo. 12 da.

36. A plot of ground contains 600 square rods. How many acres and square rods does it contain?

PROCESS

600 sq. rd. = 3 A. 120 sq. rd. Ans.

Divide 600 sq. rd. by 160 sq. rd., which gives a quotient of 3 and a remainder of 120 sq. rd.

37. Change to acres and square rods:

a 197 sq. rd.	*b* 325 sq. rd.	*c* 1681 sq. rd.
d 968 sq. rd.	*e* 487 sq. rd.	*f* 3207 sq. rd.
g 360 sq. rd.	*h* 645 sq. rd.	*i* 4809 sq. rd.

38. Change to pounds and ounces:

a 57 oz.	*b* 73 oz.	*c* 100 oz.	*d* 164 oz.
e 68 oz.	*f* 84 oz.	*g* 120 oz.	*h* 329 oz.
i 97 oz.	*j* 45 oz.	*k* 110 oz.	*l* 485 oz.

39. Change to feet and inches:

a 99 in.	*b* 35 in.	*c* 79 in.	*d* 110 in.
e 88 in.	*f* 57 in.	*g* 93 in.	*h* 137 in.
i 46 in.	*j* 90 in.	*k* 63 in.	*l* 150 in.

40. Change to days and hours:

a 35 hr.	*b* 42 hr.	*c* 53 hr.	*d* 100 hr.
e 60 hr.	*f* 75 hr.	*g* 80 hr.	*h* 121 hr.
i 95 hr.	*j* 90 hr.	*k* 97 hr.	*l* 250 hr.

41. Change to years and months:

a 98 mo.	*b* 87 mo.	*c* 45 mo.	*d* 109 mo.
e 38 mo.	*f* 56 mo.	*g* 90 mo.	*h* 123 mo.
i 77 mo.	*j* 92 mo.	*k* 66 mo.	*l* 100 mo.

42. Change to months and days:

a 72 da.	*b* 87 da.	*c* 196 da.	*d* 164 da.
e 45 da.	*f* 96 da.	*g* 215 da.	*h* 265 da.
i 58 da.	*j* 69 da.	*k* 257 da.	*l* 338 da.

43. Change to pecks and quarts:

a 100 qt.	*b* 121 qt.	*c* 169 qt.	*d* 180 qt.
e 201 qt.	*f* 150 qt.	*g* 243 qt.	*h* 325 qt.
i 165 qt.	*j* 281 qt.	*k* 401 qt.	*l* 487 qt.

44. Change to bushels and pecks:

a 245 pk.	*b* 50 pk.	*c* 63 pk.	*d* 89 pk.
e 127 pk.	*f* 97 pk.	*g* 75 pk.	*h* 91 pk.
i 169 pk.	*j* 85 pk.	*k* 59 pk.	*l* 77 pk.

45. Change to gallons and quarts:

a 317 qt.	*b* 89 qt.	*c* 98 qt.	*d* 51 qt.
e 285 qt.	*f* 57 qt.	*g* 77 qt.	*h* 66 qt.
i 174 qt.	*j* 94 qt.	*k* 83 qt.	*l* 71 qt.

46. Change to yards and feet:

a 50 yd.	*b* 97 yd.	*c* 29 yd.	*d* 154 yd.
e 88 yd.	*f* 70 yd.	*g* 46 yd.	*h* 218 yd.
i 82 yd.	*j* 52 yd.	*k* 80 yd.	*l* 163 yd.
m 40 yd.	*n* 49 yd.	*o* 95 yd.	*p* 241 yd.

OMITTING "SIDE" CALCULATIONS

To show the pupil that he can dispense with many figures he has been accustomed to use, a feature should

be made of exercises in which the pupil writes only the answers to examples from the blackboard or the textbook. These exercises should be more difficult than the regular "sight" exercises.

REDUCING FRACTIONS

Only answers to be written

Write answers directly from the book:

1. How many twenty-fourths are there in $223\frac{17}{24}$?

PROCESS

$223\frac{17}{24} = \frac{5369}{24}$ Ans.

Multiply 223 by 24 and "add in" 17.
Say 72 (24 times 3), 89 (adding 17); write 9.
Say 48 (24 times 2), 56 (carrying 8); write 6.
Say 48 (24 times 2), 53 (carrying 5); write 53.

2. Change to an improper fraction:

a $27\frac{7}{8}$	*b* $32\frac{13}{16}$	*c* $36\frac{1}{12}$	*d* $22\frac{13}{16}$
e $45\frac{5}{9}$	*f* $21\frac{5}{24}$	*g* $33\frac{13}{32}$	*h* $48\frac{5}{12}$
i $36\frac{3}{8}$	*j* $11\frac{11}{18}$	*k* $12\frac{8}{15}$	*l* $64\frac{3}{11}$
m $54\frac{4}{7}$	*n* $22\frac{6}{25}$	*o* $39\frac{7}{10}$	*p* $11\frac{5}{36}$

3. How many 18th's are there in $9\frac{13}{18}$?

PROCESS

$9\frac{13}{18} = \frac{175}{18}$ Ans.

Multiply 18 by 9, and "add in" 13.
Think 72 (9 times 8), 85 (adding 13); write 5.
Think 9 (9 times 1), 17 (carrying 8); write 17.

4. Change to an improper fraction:

a $8\frac{7}{32}$	*b* $9\frac{15}{16}$	*c* $7\frac{7}{24}$	*d* $8\frac{3}{19}$
e $7\frac{5}{36}$	*f* $8\frac{11}{27}$	*g* $9\frac{7}{15}$	*h* $6\frac{11}{48}$
i $9\frac{19}{25}$	*j* $8\frac{11}{28}$	*k* $4\frac{9}{49}$	*l* $3\frac{29}{64}$
m $7\frac{8}{33}$	*n* $8\frac{11}{28}$	*o* $9\frac{7}{22}$	*p* $6\frac{13}{75}$

5. How many 84ths are there in $7\frac{65}{84}$?

PROCESS

$7\frac{65}{84} = \frac{653}{84}$ Ans.

Instead of adding 65 to 7 times 4, add only 5, adding 6 (tens), the next partial product, to 8.

Think 28 (7 times 4), 33 (adding 5); write 3.

Think 56 (7 times 8), 59 (carrying 3), 65 (adding 6); write 65.

6. Change to an improper fraction:

a $3\frac{37}{48}$	*b* $4\frac{29}{36}$	*c* $7\frac{38}{45}$	*d* $5\frac{57}{64}$
e $6\frac{25}{28}$	*f* $8\frac{25}{27}$	*g* $9\frac{25}{44}$	*h* $8\frac{35}{48}$
i $4\frac{39}{56}$	*j* $5\frac{47}{63}$	*k* $6\frac{19}{36}$	*l* $7\frac{29}{45}$
m $5\frac{23}{25}$	*n* $6\frac{25}{39}$	*o* $7\frac{59}{66}$	*p* $8\frac{31}{38}$

7. Change $\frac{653}{84}$ to a mixed number.

PROCESS

$\frac{653}{84} = 7\frac{?}{84}$. First write 7 for the integral part of the result, then write 84 as the denominator of the fractional part. Obtain the numerator of the latter, thus:

Think 28 (7 times 4) and *5* (writing 5) are 33.
Think 56 (7 times 8), 59 (carrying 3)
and *6* (writing 6) are 65. Ans. $7\frac{65}{84}$

This method is the only one known to millions of European pupils, who are never taught to write partial products in long division. They subtract 7 times 84 from 653 by finding what number added to 28 will make the next higher number ending in 3, which is 5, etc., reversing the process used in Ex. 5, to change $7\frac{65}{84}$ to an improper fraction.

8. Change to mixed numbers:

a $\frac{191}{24}$	*b* $\frac{281}{36}$	*c* $\frac{157}{16}$	*d* $\frac{142}{15}$
e $\frac{161}{18}$	*f* $\frac{271}{32}$	*g* $\frac{238}{27}$	*h* $\frac{407}{48}$
i $\frac{356}{45}$	*j* $\frac{513}{56}$	*k* $\frac{437}{72}$	*l* $\frac{280}{33}$
m $\frac{211}{28}$	*n* $\frac{296}{55}$	*o* $\frac{297}{35}$	*p* $\frac{505}{66}$

DENOMINATE NUMBERS

9. How many ounces are there in a package weighing 23 pounds 11 ounces?

PROCESS

Change 23 lb. to oz. by multiplying 16 oz. by 23, and "adding in" 11 oz. Use 16 as the multiplier.
Think 48 (16 times 3), 59 (adding in 11); write 9.
Think 32 (16 times 2), 37 (carrying 5); write 37.
Ans. 379 oz.

10. Change to numbers of the lower denomination:

a 27 bu. 5 pk.	*b* 32 lb. 13 oz.	*c* 21 da. 5 hr.
d 48 yr. 5 mo.	*e* 38 ft. 11 in.	*f* 43 yd. 2 ft.
g 29 gal. 2 qt.	*h* 11 mo. 27 da.	*i* 25 pk. 6 qt.
j 16 wk. 6 da.	*k* 15 yr. 11 mo.	*l* 16 rd. 2 yd.
m 38 ft. 7 in.	*n* 22 lb. 11 oz.	*o* 36 yr. 7 mo.

11. A certain journey requires 179 hours. How many days and hours does it require?

PROCESS

179 hr. = 7 da. 11 hr. Ans.

The integral part of the quotient of 179 hr. divided by 24 hr. is 7; write 7 (da.). To obtain the number of hours remaining, deduct 7 times 24 from 179.

Think 28 (7 times 4), and *1* (writing 1) are 29.

Think 14 (7 times 2), 16 (carrying 1), and *1* (writing 1) are 11.

12. Change to compound numbers — two denominations:

a 191 hr.	*b* 157 oz.	*c* 257 in.
d 223 pk.	*e* 354 da.	*f* 235 mo.
g 389 in.	*h* 197 ft.	*i* 200 hr.
j 180 oz.	*k* 225 pk.	*l* 205 oz.
m 475 da.	*n* 195 mo.	*o* 157 hr.
p 365 da.	*q* 365 *s.*	*r* 364 far.

WRITTEN EXERCISES

REDUCING FRACTIONS

1. When steel bars are worth $43.20 per ton, how much can be bought for $16.20?

PROCESS

$16.20 ÷ 43.20 = $^{16}\!/_{32}$ = $^{8}\!/_{16}$ = $^{1}\!/_{4}$ = $^{3}\!/_{8}$ (T.) Ans.

Change the original fraction to $^{16}\!/_{32}$ by rejecting the dollar signs, the decimal points, and the terminal ciphers of both terms.

Divide both terms of $^{16}\!/_{32}$ by 2, since they are even numbers. Divide both terms of $^{8}\!/_{16}$ by 9, since the sum of the digits in each is 9. Divide both terms of $^{1}\!/_{4}$ by 3. To complete the answer, write T. in a parenthesis.

2. Reduce to lowest terms:

a $^{192}/_{288}$	b $^{168}/_{192}$	c $^{216}/_{576}$	d $^{180}/_{288}$
e $^{336}/_{442}$	f $^{330}/_{396}$	g $^{192}/_{240}$	h $^{270}/_{486}$
i $^{108}/_{180}$	j $^{135}/_{210}$	k $^{104}/_{244}$	l $^{105}/_{160}$
m $^{128}/_{160}$	n $^{245}/_{280}$	o $^{180}/_{216}$	p $^{147}/_{392}$
q $^{280}/_{315}$	r $^{292}/_{672}$	s $^{225}/_{360}$	t $^{117}/_{208}$

3. A woman paid \$3.22 for a piece of velvet, at the rate of \$5.22 per yard. What fraction of a yard did she buy?

After 322/552 is reduced to 161/276, a common divisor of 161 and 276 is not readily determinable by inspection, 161 not being divisible by 2 or by 3, which are factors of 276. Employ the following method:

PROCESS

$^{322}/_{552} = {}^{161}/_{276} = {}^{7}/_{12}$ (yd.), Ans.

	161	276
Divide 276 by 161. Omit the quotient, 1; write only the remainder, 115.	46	115
	0	23

Divide 161 by 115, writing only the remainder, 46.

Divide 115 by 46; omitting the quotient, 2. Obtain the remainder, 23, by thinking 12 (twice 6), and 3 (writing 3), are 15; 8 (twice 4), 9 (carrying 1) and 2 (writing 2) are 11.

Divide 46 by 23. Since there is no remainder, 23 is a factor of 46; it is, therefore, a common factor of 161 and 276.

Divide both terms of the fraction by 23, the G. C. D.

4. Express in lowest terms:

Reduce as far as possible by dividing both terms by 2, 3, 5, etc., before using the foregoing method of obtaining the greatest common divisor.

a $^{369}/_{492}$	*b* $^{296}/_{814}$	*c* $^{348}/_{620}$	*d* $^{342}/_{912}$
e $^{425}/_{765}$	*f* $^{195}/_{612}$	*g* $^{552}/_{966}$	*h* $^{435}/_{725}$
i $^{415}/_{664}$	*j* $^{252}/_{344}$	*k* $^{395}/_{474}$	*l* $^{376}/_{470}$
m $^{424}/_{742}$	*n* $^{549}/_{732}$	*o* $^{536}/_{670}$	*p* $^{438}/_{657}$

5. A farmer has 231 acres of land under cultivation. There are 112 acres of corn, 78 of wheat, 22 of rye, and 19 of oats. Find for each of the foregoing (*a*) the fraction it constitutes of the total, and (*b*) the decimal (4 places).

PROCESS

				(*a*)	(*b*)
Corn	112	acres,	$^{112}/_{231}$ =	$^{16}/_{33}$ =	.4848 +
Wheat	78	"	$^{78}/_{231}$ =	$^{26}/_{77}$ =	.3377 −
Rye	22	"	$^{22}/_{231}$ =	$^{2}/_{21}$ =	.0952 +
Oats	19	"	$^{19}/_{231}$ =	$^{19}/_{231}$ =	.0823 −
Total	231	"	$^{231}/_{231}$	1	1.0000

Express each item as a fraction, making its number of acres the numerator, and 231 its denominator.

For (*a*), reduce each fraction to its lowest terms.

For (*b*), change each fraction to a decimal by dividing its numerator by its denominator.

```
      .4848 +
33)16.0
    2 80
      160
```

Place a decimal point after 16 and annex a cipher. Place a decimal point in the quotient immediately over the one in the dividend. When the second remainder is found to be 16, the same as the original dividend,

discontinue the division, as the quotient will be 48484848 ad infinitum.

Write a plus sign after the fourth decimal to show that the next figure is less than 5.

In this division the fifth figure is greater than 5. Write the answer, therefore, as .3377 –, the minus sign indicating that the result is greater than .33765.

```
      .33766
77)26.0
    2 90
      590
       510
        48
```

```
      .09523
21)2.00
     110
       50
        8
```

Place two ciphers after the decimal point in this dividend. Follow the decimal point in the quotient with a cipher preceding 9, the first significant figure. Write a plus sign after 2 to show that the next figure is less than 5.

Since the fifth figure is more than 5, write the answer as .0823 –

```
      .08225
231)19.00
       520
        580
        118
```

6. Express as 4 place decimals:

a $\frac{5}{33}$	*b* $\frac{62}{77}$	*c* $\frac{8}{21}$	*d* $\frac{23}{231}$
e $\frac{17}{39}$	*f* $\frac{11}{42}$	*g* $\frac{19}{24}$	*h* $\frac{38}{117}$

Since the denominator of a decimal is a power of 10, the prime factors of which are 2 and 5, a fraction expressed in lowest terms is not reducible to a *pure*

decimal, unless its denominator is a power of 2 (2, 4, 8, 16, etc.); a power of 5 (5, 25, 125, 625, etc.); or a product of a power of 2 and a power of 5 (10, 20, 40, etc.); when the fraction is expressed in lowest terms.

7. There are 128 cubic feet in a cord of wood. What decimal of a cord is 5 cubic feet?

PROCESS

$$\frac{5}{128} = \frac{.625}{16} = \frac{.078125}{2} = .0390625 \text{ (cord)} \quad \text{Ans.}$$

Divide both terms of the fraction $\frac{5}{128}$ by 8, remembering that $5 \div 8 = .625$. Divide both terms of the new fraction $.625/16$ by 8. The new numerator becomes $.078\frac{1}{2}$. At this point substitute 125 for $\frac{1}{8}$ without completing the division.

8. Change to decimals (carrying out as many places as may be necessary to give the exact value):

a $\frac{7}{16}$	*b* $\frac{9}{32}$	*c* $\frac{19}{64}$	*d* $\frac{103}{128}$	*e* $\frac{11}{256}$
f $\frac{13}{16}$	*g* $\frac{3}{32}$	*h* $\frac{7}{64}$	*i* $\frac{13}{128}$	*j* $\frac{129}{256}$

9. Change to decimals: (*a*) $\frac{37}{125}$ (*b*) $\frac{54}{3125}$

PROCESS

(*a*) $\frac{37}{125} = \frac{296}{1000} = .296$, Ans.

(*b*) $\frac{54}{3125} = \frac{432}{25000} = \frac{1328}{100000} = .01328$, Ans.

In (*a*) change the denominator to 1000 by multiplying it by 8. Multiply the numerator by 8. Write the resulting fraction in decimal form.

In (*b*) multiply both terms of the fraction by 8, and of the resulting fraction by 4.

10. Change to decimals:

a $^{16}/_{125}$	b $^{37}/_{625}$	c $^{432}/_{3125}$
d $^{109}/_{125}$	e $^{348}/_{625}$	f $^{2134}/_{3125}$

11. What decimal of a ton of 2000 pounds is 163 pounds?

PROCESS

$$\frac{.163/}{2000} = .0815 \text{ (T.) Ans.}$$

Divide the denominator by 1000 by canceling the three ciphers; divide the numerator by 1000 by pointing off three decimal places. Then divide .163 by 2. Be careful to point off the quotient properly. Write the decimal point. Think 2 into 1 does not go; write a cipher. Think 2 into 16 goes 8 times; write 8, etc.

12. Change to decimals:

a $^{33}/_{400}$	b $^{17}/_{80}$	c $^{95}/_{800}$	d $^{123}/_{200}$
e $^{143}/_{800}$	f $^{33}/_{40}$	g $^{117}/_{400}$	h $^{21}/_{800}$
i $^{7}/_{200}$	j $^{3}/_{80}$	k $^{369}/_{800}$	l $^{17}/_{500}$

13. The distance between two houses is 281 rods. What fraction of a mile (320 rods) are they apart?

PROCESS

$$\frac{28.1\not{\emptyset}}{32\not{\emptyset}} = \frac{7.025}{8} = .878125 \text{ (mi.) Ans.}$$

Cancel the cipher in the denominator, thus dividing it by 10. Divide the numerator by 10 by pointing off one decimal place. Reduce further by dividing both terms by 4; then by 8.

CHECK

Multiply .878⅛ by 320, by first multiplying it by 8, then by 4, then by 10.

14. Change to decimals:

a $^{19}/_{160}$	b $^{131}/_{320}$	c $^{29}/_{640}$	d $^{107}/_{160}$
e $^{53}/_{320}$	f $^{449}/_{640}$	g $^{141}/_{160}$	h $^{289}/_{320}$

15. Change to four-place decimals. First multiply both terms by 2:

a $^{16}/_{35}$	b $^{23}/_{45}$	c $^{18}/_{55}$	d $^{6}/_{35}$
e $^{11}/_{45}$	f $^{9}/_{55}$	g $^{29}/_{35}$	h $^{31}/_{45}$

REDUCING DECIMALS

WRITTEN EXERCISES

1. Of the 400 employees in a store, .3125 are men, .3 are women, .29 are boys, and the remainder are girls. Find (*a*) the corresponding fraction for each, and (*b*) the number of each class of employees.

PROCESS

			(*a*)	(*b*)
Men	.3125	= $^{3125}/_{10000}$	= $^{5}/_{16}$	= 125
Women	.3	=	= $^{3}/_{10}$	= 120
Boys	.29	=	= $^{29}/_{100}$	= 116
Girls	.0975	= $^{975}/_{10000}$	= $^{39}/_{400}$	= 39
	1.0000		$^{400}/_{400}$	400

First find the decimal of the girl employees. Express each as a common fraction, and reduce the latter to lowest terms.

TEST

Test results (*a*) by employing the fractions to ascertain the items of (*b*). If these total 400, the answers to both questions are correct.

NOTE: A common fraction, with its denominator a power of 10, cannot be reduced unless its numerator is an even number, or ends in 5.

2. Express as common fractions or mixed numbers. Write answers directly from the book.

a .008	*b* .075	*c* 6.0125	*d* 84.0075
e .179	*f* .004	*g* 3.0044	*h* 63.0648
i .084	*j* .175	*k* 8.0025	*l* 57.0005
m .165	*n* .005	*o* 7.0365	*p* 70.0125
q .006	*r* .395	*s* 9.3284	*t* 25.3125
u .316	*v* .002	*w* 5.3248	*x* 40.0375

3. Change the following complex decimals to common fractions, lowest terms:

a .03⅓ *b* .8⅓ *c* .006⅔

PROCESS

a $.03\frac{1}{3} = 3\frac{1}{3}/100 = \frac{10}{300} = \frac{1}{30}$ Ans.
b $.8\frac{4}{7} = 8\frac{4}{7}/10 = \frac{60}{70} = \frac{6}{7}$ Ans.
c $.006\frac{2}{3} = 6\frac{2}{3}/1000 = \frac{20}{3000} = \frac{1}{150}$ Ans.

Write each as a complex fraction. Change to a simple fraction by multiplying both terms by the denominator of the fraction in the numerator. Reduce the simple fraction to lowest terms.

4. Express as common fractions or mixed numbers:

a $1.83\frac{1}{3}$	*b* $.85\frac{5}{7}$	*c* $13.42\frac{2}{9}$	*d* $.742\frac{2}{7}$
e $3.51\frac{3}{17}$	*f* $.08\frac{1}{3}$	*g* $54.23\frac{1}{13}$	*h* $.384\frac{4}{11}$
i $5.06\frac{2}{13}$	*j* $.54\frac{4}{11}$	*k* $77.06\frac{2}{3}$	*l* $.210\frac{10}{19}$

DENOMINATE NUMBERS

Weighing and Measuring

There is a growing tendency in the business world to substitute weighing for measuring. The farmer disposes of cabbages in large quantities by the ton; olive oil and milk are sold by the pound; the number of bushels in a given quantity of grain is determined by its weight. All kinds of vegetables are retailed by the pound. (For Tables see pp. 459.)

REDUCING DENOMINATE NUMBERS

Changing to Lower Denominations

WRITTEN EXERCISES

1. A vessel took 14 weeks, 6 days, 18 hours to make a trip to the East Indies. How many hours were required to make the trip?

PROCESS

Change 14 wk. 6 da. to days by multiplying 7 da. by 14, and "adding in" 6 da. However, use 7 as a multiplier, but do not write it.

14 wk. 6 da. 18 hr.
104 da.
Ans. 2514 hr.

Think 28 (7 times 4), 34 ("adding in" 6); write 4 da. under 6 da.

Think 7 (7 times 1), 10 (carrying 3); write 10.

Change 104 da. 18 hr. by multiplying 24 hr. by 104, and "adding in" 18 hr. However, use 24 as a multiplier, but do not write it.

Think 96 (24 times 4), 104 (carrying 8); write 4 hr. under 8 hr., dropping a line.

Think 0 (24 times 0), 10 (carrying 10), 11 ("adding in" 1); write 1.

Think 24 (24 times 1), 25 (carrying 1); write 25.

CHECK

See the reduction of 2514 hr. to days, weeks, and hours, p. 205.

2. Change to hours:

a 15 wk.	*b* 12 wk. 6 da.	*c* 15 wk. 5 da. 15 hr.
d 35 da.	*e* 21 da. 5 hr.	*f* 11 wk. 3 da. 21 hr.
g 22 wk	*h* 33 wk. 9 hr.	*i* 13 wk. 9 da. 18 hr.

3. Change to ounces:

a 24 lb. 15 oz.	*b* 35 lb. 14 oz.	*c* 26 lb. 8 oz.
d 34 lb. 13 oz.	*e* 53 lb. 12 oz.	*f* 55 lb. 5 oz.
g 42 lb. 11 oz.	*h* 45 lb. 10 oz.	*i* 43 lb. 7 oz.

4. Change to quarts:

a 19 bu. 3 pk. 7 qt.	*b* 37 bu. 1 pk. 4 qt.
c 28 bu. 2 pk. 5 qt.	*d* 26 bu. 3 pk. 3 qt.
e 35 bu. 1 pk. 6 qt.	*f* 18 bu. 2 pk. 2 qt.

5. Change to hours (*a*) 38 wk. (*b*) 29 wk. 9 hr.

PROCESS

(*a*) 35 wk.
245 da.
Ans. 5880 hr.

Either change to hours directly by multiplying 168 hr. by 35, or employ the factors 7 (da.) and 24 (hr.) as shown above.

Insert 0 da. (the missing denomination).

(*b*) 29 wk. 0 da. 9 hr.
203 da.
Ans. 4881 hr.

CHECK

If the result in (*a*) is obtained in the manner shown above, check it by multiplying 168 hr. by 35.

Check (*b*) by multiplying 168 hr. by 29, "adding in" 9 hr.

6. Change to inches:

a 19 yd.	*b* 27 yd. 11 in.	*c* 33 yd. 2 ft. 9 in.
d 38 yd.	*e* 37 ft. 10 in.	*f* 28 yd. 1 ft. 6 in.
g 45 yd.	*h* 68 ft. 11 in.	*i* 43 yd. 2 ft. 7 in.

7. Change to feet:

a 14 rd.	*b* 14 rd. 4 yd.	*c* 16 rd. 4 yd. 2 ft.
d 22 rd.	*e* 24 rd. 5 yd.	*f* 42 rd. 3 yd. 1 ft.
g 30 rd.	*h* 32 rd. 3 yd.	*i* 54 rd. 2 yd. 2 ft.

8. Change to pence:

a £19	*b* £3 18s	*c* £13 4s 6d
d £24	*e* £7 16s	*f* £25 9s 8d
g £37	*h* £9 15s	*i* £53 6s 7d

9. A field contains 85 acres 137 sq. rd.; how many sq. rd. are there in the field?

PROCESS

160
65 A. 137 sq. rd.
10,537 sq. rd. Ans.

Write, if necessary, 160 (the number of square rods to the acre) as a help. Use it as the multiplier.

Think 0 (6 times 65), 7 (adding in 7); write 7 sq. rd.

Think 80 (16 times 5), 83 (adding in 3); write 3.

Think 96 (16 times 6), 104 (carrying 8), 105 (adding in 1); write 105.

CHECK

Check by dividing 10,537 by 160, using short division.

10. Change to square rods:

a 16 A. 127 sq. rd. *b* 43 A. 84 sq. rd.
c 23 A. 109 sq. rd. *d* 54 A. 96 sq. rd.
e 32 A. 132 sq. rd. *f* 62 A. 75 sq. rd.

11. Change to days: (*a*) 2 years 7 months 12 days. (*b*) 3 years 11 months 21 days.

PROCESS

(*a*)		(*b*)	
2 yr.	= 720 da.	3 yr.	= 1080 da.
7 mo.	= 210 "	11 mo.	= 3330 "
12 da.	= 12 "	21 da.	= 21 "
Ans.	942 da.	Ans.	1431 da.

In reducing the following, employ either method, using the other one as a check.

12. Change to days:

a 2 yr. 7 mo. 19 da.	*b* 3 yr. 10 mo. 28 da.
c 4 yr. 8 mo. 24 da.	*d* 5 yr. 11 mo. 17 da.
e 6 yr. 9 mo. 13 da.	*f* 7 yr. 10 mo. 24 da.

CHANGING TO HIGHER DENOMINATIONS

1. A certain quantity of coal was consumed in a factory in 2514 hours. How many weeks, days, and hours did it last?

PROCESS

24 hr.)2514 hr.
7 (da.) 104 (da.) 18 hr.
Ans. 14 (wk.) 6 (da.) 18 hr.

Divide 2514 hr. by 24 hr. The quotient is 104 (the number of days), and 18 hr. remaining.

Divide 104 da. by 7 da. The quotient is 14 (the number of weeks), and 6 da. remaining.

To avoid the appearance of a concrete quotient with concrete divisor and dividend, write *da.* and *wk.* in parentheses.

Bring down 18 hr. the first remainder. Insert the proper denominations.

2. Change to weeks, days, and hours:

a 1500 hr.	*b* 2759 hr.	*c* 1699 hr.	*d* 2508 hr.
e 3007 hr.	*f* 1594 hr.	*g* 2306 hr.	*h* 3240 hr.

3. Change to pounds and ounces:

a 375 oz.	*b* 495 oz.	*c* 1629 oz.	*d* 1746 oz.
e 594 oz.	*f* 687 oz.	*g* 2345 oz.	*h* 3369 oz.

4. Change to years, months, and days:

a 1984 da.	*b* 1763 da.	*c* 2746 da.	*d* 1195 da.
e 3265 da.	*f* 1429 da.	*g* 1876 da.	*h* 2344 da.

5. Change to bushels, pecks, and quarts:

a 695 qt.	*b* 879 qt.	*c* 1015 qt.	*d* 1244 qt.
e 389 qt.	*f* 467 qt.	*g* 1137 qt.	*h* 1878 qt.

6. Change to yards, feet, and inches:

a 690 in.	*b* 798 in.	*c* 1246 in.	*d* 1095 in.
e 587 in.	*f* 937 in.	*g* 1315 in.	*h* 1457 in.

7. Change to pounds sterling, shillings, and pence:

a 698 d.	*b* 884 d.	*c* 1847 d.	*d* 2358 d.
e 987 d.	*f* 576 d.	*g* 3015 d.	*h* 4444 d.

CHAPTER FOUR

SIGNS AND OPERATIONS

The following diagram shows the several arithmetical operations with their signs, the names of the terms, etc.

Operation	Expression	16 is called	2 is called	Result is called
Addition	$16 + 2 = 18$	Addend	Addend	Sum
Subtraction	$16 - 2 = 14$	Minuend	Subtrahend	Difference, or Remainder
Multiplication	16×2 or $16 \cdot 2 = 32$	Factor, or Multiplicand	Factor, or Multiplier	Product
Division	$16 \div 2 = 8$	Dividend	Divisor	Quotient
Ratio	$16 : 2 = 8 : 1$	Antecedent	Consequent	Ratio
Involution	$16^2 = 256$	Base	Exponent	Power
Evolution	$\sqrt[2]{16} = 4$	Base	Index	Root

The expression 16×2 is read "16 times 2," or "16 multiplied by 2." When one of the terms is concrete, it is generally read as the multiplicand; thus $\$16 \times 4$ is generally stated "$16 multiplied by 4." A person desiring to use the word "times" reads it "4 times $16," regardless of the order of the terms in the expression.

In arithmetical subtraction, the larger number is considered the minuend in such an example as "Find the difference between 316 and 500."

That 16 is to be divided by 2 may also be indicated by $\frac{16}{2}$, or $2\overline{)16}$.

SIGHT EXERCISES

1. Give the value of each:

a $75 + 19 + 25$	*b* $75 + 25 + 19$	*c* $19 + 25 + 75$
d $25 \times 6\frac{1}{2} \times 4$	*e* $25 \times 4 \times 6\frac{1}{2}$	*f* $4 \times 6\frac{1}{2} \times 25$

It will be noted that the values of *a*, *b*, and *c* are the same, showing that addends may be taken in any order; that the values of *d*, *e*, and *f* are the same, showing that factors may be taken in any order.

2. Give the values of the following:

a $16\frac{2}{3} \times 8 \times 6$	*b* $12\frac{1}{2} \times 3\frac{1}{2} \times 8$	*c*. $33\frac{1}{3} \times 5 \times 6$

PRECEDENCE OF SIGNS

It is agreed among mathematicians that when an expression contains a multiplication ($\times$) or a division ($\div$) sign, and also one of addition ($+$) or subtraction ($-$), the product or quotient must first be found.

Thus, *a* $12 \times 8 + 2$ means $96 + 2$
b $40 + 8 \times 2$ " $40 + 16$
c $18 \div 6 - 2$ " $3 - 2$
d $30 - 18 \div 9$ " $30 - 2$

To avoid misleading a person unacquainted with this convention, such quantities should be placed in a parenthesis — $(12 \times 8) + 2$

a $(12 \times 8) - 2$	*b* $40 - (8 \times 2)$
c $(18 \div 6) + 2$	*d* $30 + (18 \div 9)$

A better plan for the last two would be to write them thus:

$c\ \frac{1}{8} + 2 \qquad d\ 30 + \frac{1}{8}$

SIGNS OF AGGREGATION

A *parenthesis* denotes that the value of an expression contained in it is to be taken as a whole in performing an operation.

A *vinculum*, which is a horizontal line over an expression, has the same meaning as a parenthesis.

In dictating an expression containing a parenthesis or a vinculum, care must be taken to avoid misleading the hearer. Thus, it is difficult to distinguish between $(3 \times 8) - 4$, announced as 3 times 8, minus 4, and $3 \times (8 - 4)$, announced as three times, 8 minus 4, notwithstanding the difference in the pauses.

A better plan would be to dictate the first "From 3 times 8, take 4," and the second "3 times the difference between 8 and 4."

SIGHT EXERCISES

1. Read the following:

$a\ (9 \times 5) - (6 \div 3) - 10 \qquad b\ 24 - 9 - (24 \div 4)$

$c\ 9 \times (5 - 3) - 16 + (6 - 2) \qquad d\ (8 \div 4) + (3 \times 5) - 2$

$e\ \frac{14 + 10}{8} - \frac{3 + 7}{5} \qquad f\ \frac{8 + 16}{5 - 1} + 23 - 2$

Notice, in *e* and *f*, the line between two expressions has the effect of a vinculum.

2. Give the value of each.

3. Which of the foregoing marks of aggregation could be omitted?

INDICATING OPERATIONS

PREPARATORY EXERCISES

State the operations needed to obtain each of the following:

1. After spending $1.50 for a hat, and 30 cents for a necktie, John has 5 cents left. How much had he at first?

2. How much would Mary have left out of $1.50 after spending 30 cents for ribbon and 5 cents for pins?

3. A drover sold 150 sheep to one farmer, and 30 sheep to another. How much did he receive in all at $5 a head?

4. How many cubic feet of water are there in a rectangular pond 150 feet long and 30 feet wide when the water is 5 feet deep?

5. A man had a farm of 150 acres. How many acres would he have after he had sold 30 acres to Mr. A and 5 acres to Mr. B?

6. A grocer had 150 pounds of sugar. After selling 30 pounds, he put up the remainder in 5-pound packages. How many packages were there?

7. A woman had $150 in the bank. How much would she have in the bank, exclusive of interest, after making 30 weekly deposits of $5 each?

8. At the opening day of school it had 150 pupils. During the month 30 pupils were admitted, and 5 left. How many pupils belonged to the school at the end of the month?

9. A planter raised 150 bales of cotton in 5 fields of 30 acres each. How many bales did he average to the acre?

10. Five children picked 150 quarts of blackberries and 30 quarts of blueberries. How many quarts of berries did each child pick on the average?

11. A farmer had 150 pigs. He kept 30 and sold the others at \$5 each. What did he receive for those he sold?

SIGHT EXERCISES

Combine numbers inclosed in a parenthesis before combining them with the other number.

1. Give answers:

a $150 + 30 + 5$	*b* $(150 + 30) + 5$	*c* $150 + (30 + 5)$
d $150 \times 30 \times 5$	*e* $(150 \times 30) \times 5$	*f* $150 \times (30 \times 5)$

2. What answers are the same (*a*) in the first line? (*b*) In the second?

3. Give answers:

a $150 - 30 - 5$	*b* $(150 - 30) - 5$	*c* $150 - (30 - 5)$
d $150 \div 30 \div 5$	*e* $(150 \div 30) \div 5$	*f* $150 \div (30 \div 5)$

4. What two expressions in example 3 are equivalent to each other (*a*) in the first line? (*b*) In the second?

5. What parentheses could be removed without affecting the result in example 3?

In the following examples treat two numbers connected by the sign of multiplication or of division as if they were inclosed in a parenthesis.

6. Give answers:

a $150 + 30 \times 5$	*b* $150 \times 30 + 5$	*c* $150 \times (30 + 5)$
d $(150 + 30) \times 5$	*e* $150 - 30 \times 5$	*f* $150 \times 30 - 5$
g $150 \times (30 - 5)$	*h* $(150 + 30) \div 5$	*i* $150 + 30 \div 5$
j $150 \div 30 + 5$	*k* $150 \div (30 + 5)$	*l* $(150 - 30) \times 5$
m $150 - 30 \div 5$	*n* $150 \div 30 - 5$	*o* $150 \div (30 - 5)$
p $(150 - 30) \div 5$	*q* $150 - 30 - 5$	*r* $150 - (30 - 5)$

When an expression is composed of numbers connected by signs of addition and subtraction (+ and −) exclusively, the result may be obtained by commencing at the right and making the combinations in the order in which they are given.

1. In determining the value of the expression $20 - 12 + 16 - 8 - 4$ the successive steps may be 8 (20 − 12); 24 (adding 16); 16 (subtracting 8); 12 (subtracting 4). Ans. 12.

The same result is obtained when these numbers are taken in any other order, such as

a $20 - 8 - 4 + 16 - 12$
b $16 - 8 - 4 + 20 - 12$
etc.

As an algebraic expression the order may be

$$-12 - 8 + 20 - 4 + 16$$

In dictating (1) say

20 minus 12 plus 16 minus 8 minus 4

In evaluating an expression of this kind, it is customary, however, to combine the addends, and from their sum to deduct the sum of the subtrahends.

This may be expressed in the following form:

$$20 + 16 - (12 + 8 + 4)$$

the parenthesis indicating that the value of the expression within the parenthesis is first to be ascertained.

Note the change of the signs prefixed to 8 and 4 when they are placed within the parenthesis.

2. An expression containing numbers connected by signs of multiplication and division ($\times$ and $\div$) may be evaluated in the same way. Thus in

$$16 \times 4 \div 8 \times 10 \div 5$$

the successive steps may be

64 (16×4), 8 (dividing by 8), 80 (multiplying by 10), 16 (dividing by 5). Ans. 16.

The same result is obtained by making the successive operations in any order. The following are examples:

a $16 \div 8 \times 10 \times 4 \div 5$
b $10 \div 8 \times 16 \div 5 \times 4$
etc., etc.

In practice, however, divide the product of the multipliers by the product of the divisors, thus

c $16 \times 10 \times 4 \div (8 \times 5)$,

writing it as shown below:

d $$\frac{16 \times 10 \times 4}{8 \times 5}$$

Then, shorten the work by cancellation.

Note the change of the sign preceding the 5 when it is placed in the parenthesis in *c* and below the line in *d*.

CHAPTER FIVE

ADDITION

COUNTING EXERCISES

Count by 6's, beginning (*a*) with 6. (*b*) With 7. (*c*) With 8. (*d*) With 9. (*e*) With 16. (*f*) With 17.

2. Count by 7's, beginning (*a*) with 7. (*b*) With 8. (*c*) With 9. (*d*) With 17. (*e*) With 19. (*f*) With 27.

3. Count by 8's, beginning (*a*) with 8. (*b*) With 9. (*c*) With 18. (*d*) With 19. (*e*) With 28. (*f*) With 29. (*g*) With 22. (*h*) With 23.

4. Count by 9's, beginning (*a*) with 9. (*b*) With 19. (*c*) With 29. (*d*) With 21. (*e*) With 22. (*f*) With 23. (*g*) With 24. (*h*) With 25. (*i*) With 26.

ORAL DRILLS

One type of the daily "warming-up" drill is a counting exercise.

The teacher announces "Counting Drill." The class stands. A scorer at the blackboard records the time. The teacher says "By 6's." Successive pupils say 12, 18, 24, 30, etc., each taking his seat as he answers. When 102 is reached, the teacher says "7," and the pupil whose turn it is to answer says "13," and the others continue 20, 27, etc. to 103.

The scorer's duty is to call "time" at the expiration of 3 minutes, and to note the number of combinations that have been correctly made in the period. Each

result should be compared with previous ones to determine the rate of progress made by the class in rapid work.

SIGHT DRILLS

Give the total of each column:

a	*b*	*c*	*d*
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
66,666	77,777	88,888	99,999
98,765	56,789	97,586	45,678

Drills similar to the foregoing should be written in large figures on manila paper for occasional use.

The foregoing sight drills contain the same combinations as occur in the counting exercises. The numbers are, however, in sight, which is of great help to many pupils; it is also the way in which most of their adding is done.

MENTAL WORK

Some drills should be employed in which the numbers are not in sight. Answers to these combinations may be given orally by individuals at one recitation. At another, answers may be written by all the pupils.

1. Miss Bollin spent \$1.67 for muslins and 75¢ for gloves. Find the total.

Think two, thirty-seven (1.67 + .70); two, forty-two (adding .05). Give the answer as two, forty-two.

If the answer is to be written, first write 2 42, then insert the decimal point and prefix the dollar sign.

2. Give sums:

a 39¢ + 46¢	*b* 47¢ + 84¢	*c* $1.59 + $.33
d 27¢ + 38¢	*e* 18¢ + 96¢	*f* 4.17 + .67
g 63¢ + 17¢	*h* 75¢ + 38¢	*i* 6.36 + .45
j 45¢ + 25¢	*k* 37¢ + 63¢	*l* 8.67 + .28
m 54¢ + 36¢	*n* 95¢ + 78¢	*o* 3.78 + .16
p 18¢ + 68¢	*q* 28¢ + 84¢	*r* 7.45 + .39
s 73¢ + 19¢	*t* 57¢ + 58¢	*u* 5.80 + .18
v 81¢ + 13¢	*w* 67¢ + 87¢	*x* 2.28 + .56

3. Carl Hall has two farms, one containing 368 acres and the other containing 475 acres. How many acres are there in both?

PROCESS

Think 768 (368 + 400), 838 (adding 70), 943 (adding 5). 943 A Ans.

4. Give sums:

a 459 + 83	*b* 659 + 183	*c* 378 + 659
d 684 + 17	*e* 198 + 247	*f* 456 + 548
g 852 + 49	*h* 484 + 176	*i* 737 + 837
j 275 + 29	*k* 145 + 693	*l* 295 + 926
m 729 + 95	*n* 838 + 129	*o* 816 + 495
p 369 + 38	*q* 134 + 568	*r* 648 + 372
s 546 + 57	*t* 356 + 155	*u* 189 + 818
v 138 + 68	*w* 119 + 777	*x* 576 + 654

5. Edward traveled 268 miles on Monday and 197 miles on Tuesday. How far did he go in the two days?

METHOD

Since 197 is 3 less than 200, deduct 3 from the sum of 200 and 268. Think 468 (268 + 200), 465 (subtracting 3). 465 mi. Ans.

6. Give sums:

a 347 + 99	*b* 568 + 399	*c* 784 + 499
d 568 + 98	*e* 245 + 698	*f* 426 + 899
g 848 + 97	*h* 785 + 197	*i* 189 + 999
j 289 + 96	*k* 318 + 596	*l* 538 + 698
m 627 + 39	*n* 609 + 299	*o* 616 + 597
p 465 + 29	*q* 177 + 598	*r* 256 + 898
s 719 + 59	*t* 644 + 297	*u* 838 + 299
v 153 + 49	*w* 437 + 496	*x* 347 + 998

ORAL PROBLEMS

1. Dr. Bragg paid $975 for a car and $98 for additional equipment. What did he spend in all?

2. Find the total amount of the bill for a $75 graphophone and $19 worth of records.

3. A farmer had last year 265 acres in wheat. This year his wheat acreage is 69 acres larger. How many acres has he in wheat this year?

4. Before the war a certain grade of paper was sold at $95 a ton. During the war the price increased $38. Find the later price.

5. A pile of wood contains 78 cubic feet more than

a cord, which is 128 cubic feet. How many cubic feet are there in the pile?

6. Newton is 654 miles from Chicago, and La Junta is 370 miles beyond Newton. How far is La Junta from Chicago?

7. A man has $675 in the bank. How much will he have in the bank after he deposits $175?

8. In 1917 a man's salary was $1575. In 1918 he was paid $250 more. What was his salary in 1918?

9. A man born in 1843 died at the age of 69. In what year did he die?

10. The capacity of a shoe factory was 465 pairs a day. By better management this was increased by 78 pairs. What was its later capacity?

11. An employee was paid a monthly salary of $285 and certain commissions. How much did he receive in all in a month during which his commission amounted to $178?

WRITTEN EXERCISES

ADDITION

1. A farmer's account book shows the following expenses an acre in connection with his potato crop:

Plowing	$3.47	Spray Material	$1.23
Fertilizing	6.50	Spraying, twice	.25
Disking	1.37	Digging	1.83
Harrowing	.52	Picking up	2.70
Seed	13.12	Sacks	4.05
Cutting seed	1.75	Sewing & Loading	.45
Planting	1.05	Transporting	1.35
1st Cultivating	.57	Interest	15.—
Cultivating, 4 times	4.20	Taxes	3.10

a Find the total expenses an acre.

PROCESS

$3.47	Write the items in a column. Beginning
6.50	at the bottom, think 10, etc., . . . 61
1.37	and write 1, carrying 6; think 7, 10, etc.
.52	When the total is found by adding up-
etc.	ward, cover the answer with a piece of
1.35	paper and write on the latter the column
4.05	totals found by adding downward.
15.	Think 14, 16, 18, etc.
3.10	Carrying 6, think 10, 15, 18, 23, etc.

Compare the two results. If they agree, the addition may be taken as correct.

In adding aloud follow a similar procedure. Ignore the cipher in the first column (and in all others) and announce 10, the sum of the first two significant figures, without mentioning five and five. When the final total 61 is announced write 1 and carry 6 without saying anything about it. That 6 has been carried is shown by its combination with 1 of the second column to make 7.

NOTE: Omit superfluous words and figures.

b If the gross receipts were $95.20 an acre, what was the profit on 130 acres?

c. What was the value of the land an acre if the rate of interest paid was 6%?

2. The following is an itemized statement of the expense account of Fleming & Co. for a month:

Salaries	$2304.75
Labor	409.50
Traveling expenses	45.83
Taxes	29.60

Insurance	18.24
Office supplies	118.66
Advertising	265.40
Telegrams	463.27
Telephone	245.18
Postage	295.87
Light, heat, etc.	162.—
Painting and repairs	230.—
Cartage	56.84
Rent	375.—
Miscellaneous	46.89

What is the total for the month?

When the addends are numerous and composed of large numbers, write the total of each column alongside, then write the figures in the footing. In checking the result cover the side totals as well as the footing. On a strip covering the side totals, write the new ones. Compare the two. See that the footing agrees with the second set of side totals.

In case of disagreement between two results, the side totals render it unnecessary to go back more than a column to make sure of the number to be carried to the column in which a discrepancy exists.

3. Find sums. Test answers.

a	*b*	*c*	*d*
97,864	124,756	32,785	37,694
7,987	325,675	137,393	82,969
2,767	39,248	145,358	130,402
89,574	7,878	72,364	69,735
32,478	17,669	8,442	77,496
6,724	347,896	83,739	84,968
5,978	73,059	321,452	198,695
86,456	8,877	37,242	6,956
59,472	56,893	836	92,729
8,769	6,425	45,878	234,919
68,245	447	8,384	268,948
7,988	8,348	48,927	17,963
47,747	82,720	3,229	25,698
8,486	53,587	16,279	8,778
748	2,352	131,832	8,969
69	178	4,075	576

e	*f*	*g*	*h*
$6,837.42	$53,819.37	468.987	5,628.3478
924.85	16,445.86	37.5384	353.68
6,193.74	2,437.75	1,428.3	1,753.0809
33,448.93	827.54	926.74	4,736.249
217.68	54,394.68	8,394.8945	37,459.08
437.56	37.25	7,534.3	3,485.6052
5,827.38	876.34	48,269.057	4,796.804
5,672.84	6,786.91	2,736.8052	6,248.72
54,984.93	827.36	8,548.291	645.0783
847.62	42,345.89	3.0787	486.57
9,382.49	2,651.48	65.45	3,467.343
3,483.57	753.43	382.345	25,895.8
37,896.82	48,269.27	37.0006	28,378.56
6,438.75	36,854.82	826.623	243.93
52,417.24	91.76	4,327.14	7,284.075
432.66	3,824.53	2,557.8346	762.88
13.89	826.62	68,349.05	8,234.9
4.75	34,327.14	7,654.345	28,351.0402
23.64	2,557.83	23.68	3.006

4. The following are the receipts for a week in the specified departments:

	Dry Goods	Millinery	Notions	Shoes	Total
Monday	$1,928.75	$346.42	$289.85	$358.77	(*e*)
Tuesday	1,056.34	275.98	305.64	323.84	(*f*)
Wednesday	1,328.69	304.69	316.38	336.91	(*g*)
Thursday	1,046.78	236.77	337.49	305.83	(*h*)
Friday	984.67	251.09	250.08	298.64	(*i*)
Saturday	2,345.56	546.57	375.97	475.86	(*j*)
Totals	(*a*)	(*b*)	(*c*)	(*d*)	(*k*)

Find the total for each department (*a*) to (*d*). For each day, (*e*) to (*j*). The grand total for the week (*k*).

NOTE: Check by comparing the grand total, (*k*), found by adding the daily totals, (*e*) to (*j*), with that found by adding the department totals (*a*) to (*d*).

5. From the following, find the cost to the government of the outfit of an infantry private for clothing and shelter:

1 bedsack	$0.98	2 flannel shirts @ $3.64	
3 blankets @ 6.25		2 pr. shoes " 5.10	
1 waist belt	.25	5 " stockings " .30	
2 pr. breeches @ 4.45		4 identification tags @ .00½	
2 service coats " 7.60		3 undershirts @ .50	
1 hat cord	.08	4 " " 1.22	
3 pr. drawers @ .50		1 overcoat	14.92
3 " " " 1.62⅔		5 tent pins @ .04	
1 " gloves	.61	1 " pole	.26
1 hat	1.70	1 poncho	3.55
2 pr. shoe laces @ .02½		1 shelter tent	2.95
1 " leggings	1.05		

6. Find the cost to a midshipman of the following articles with which he must provide himself upon his admission to the Naval Academy:

1 white cap and anchor	$2.45	1 box soap	.30
1 dress jacket	20.78	1 hair brush	.65
1 blouse	15.22	stationery	1.75
1 pr. dress trousers	11.83	12 white handkerchiefs @ .20	
1 " service "	6.68	1 pr. suspenders	.40
1 overcoat	26.98	4 suits pajamas @ .70	
1 reefer	12.18	1 tooth brush	.18
1 mackintosh	11.50	thread and needles	.75
1 cap cover	.24	brush and blacking	.50
2 pr. leggings @ $.70		nail brush	.50
1 parade cap	3.10	6 pillow cases @ .13	
1 mug	.07	name plate	.15
1 soap box	.18	2 bedspreads @ 1.25	
1 laundry book	.25	1 slop jar	1.—
1 pr. blankets	3.75	2 spatter cloths @ .50	

1 pair overshoes	.83	1 hair pillow	.75
2 " high shoes @ 4.80		1 rug	.75
8 white shirts " .50		1 hair mattress	4.85
12 collars " .10		1 broom	.35
2 white blouses " 4.—		3 khaki blouses @ 1.67	
12 pr. cuffs " .18⅓		4 " shirts " 2.30	
12 " socks " .20		1 " belt	.17
8 towels " .20		1 waste paper basket	.65
1 shaving outfit	2.65	3 white hats @ .35	
12 pr. drawers @ .40		1 jackknife	.25
12 undershirts @ .36		2 lanyards @ .12	
1 hand glass	1.15	6 sheets @ .65	
1 blue sweater	3.15	hammock clews	.50
2 " jerseys @ 2.—		1 pr. bathing trunks	.15
1 pr. white shoes	1.80	3 pr. white gloves @ .40	
1 requisition book	.40	1 trousers hanger	.30
1 pass book	.30	6 coat hangers @ .06	
3 stencils @ .25		1 strong box	1.60
1 basin and pitcher	.90	1 pr. ear protectors	.20
1 pr. gymnasium slippers	.87	2 manuals @ .41½	
1 whisk broom	.17	1 pr. collar anchors	.75
1 coarse comb	.12	2 clothes bags @ .25	

7. Add horizontally and vertically:

$$
\begin{array}{rcrcrl}
2{,}259{,}969 & + & 313{,}225 & + & 2{,}835{,}546 & = (a) \\
30{,}631{,}114 & + & 4{,}624{,}231 & + & 67{,}384{,}012 & = (b) \\
15{,}192{,}362 & + & 3{,}657{,}641 & + & 32{,}702{,}416 & = (c) \\
1{,}241{,}410 & + & 132{,}640 & + & 2{,}421{,}798 & = (d) \\
650{,}599 & + & 220{,}299 & + & 1{,}334{,}004 & = (e) \\
2{,}336{,}043 & + & 156{,}708 & + & 3{,}271{,}787 & = (f) \\
62{,}997{,}808 & + & 8{,}444{,}473 & + & 127{,}914{,}369 & = (g) \\
14{,}070{,}829 & + & 1{,}160{,}278 & + & 23{,}466{,}950 & = (h) \\
19{,}380{,}698 & + & 1{,}822{,}756 & + & 32{,}610{,}057 & = (i) \\
\underline{27{,}796{,}815} & + & \underline{6{,}799{,}875} & + & \underline{42{,}621{,}617} & = \underline{(j)} \\
(k) & + & (l) & + & (m) & = (n)
\end{array}
$$

ADDING FRACTIONS

DRILL EXERCISES

1. Give answers rapidly.

a	½	*b*	½	*c*	½	*d*	½	*e*	½	*f*	½	*g*	¼
	+¼		+⅛		+¾		+⅜		+⅝		+⅞		+⅛
h	¼	*i*	¼	*j*	¼	*k*	¾	*l*	¾	*m*	¾	*n*	⅝
	+⅜		+⅝		+⅞		+⅜		+⅝		+⅞		+⅞
o	⅜	*p*	⅝	*q*	⅞	*r*	⅜	*s*	¾	*t*	⅝	*u*	¾
	+⅜		+⅝		+⅞		+⅝		+⅞		+⅝		+¾

2. Give sums.

a	½	*b*	¼	*c*	½	*d*	⅓	*e*	½	*f*	⅓	*g*	⅙
	+⅓		+⅓		+⅙		+⅙		+⅚		+⅚		+⅙
h	⅚	*i*	⅙	*j*	½	*k*	¾	*l*	¾	*m*	¾	*n*	¾
	+⅚		+⅔		+⅔		+⅓		+⅙		+⅔		+⅚

ORAL PROBLEMS

1. Last year a farmer's crop of wheat averaged 21¼ bushels to the acre. This year's was 1⅛ bushels greater. What is the average to the acre this year?

METHOD	
Think 22¾ (21¾ + 1), 22⅞ (adding ⅛)	22⅞ bu. Ans.

2. Before the war copper brought 12¼¢ a pound. A few months later the price was increased 3½¢. What was the new price?

3. One pile of wood contained 2⅞ cords, another contained 1⅝ cords. How much wood was there in the two piles?

4. Pohick is 23/10 miles from Seminary. Falls

Church is $8\frac{3}{10}$ miles farther. How far is Falls Church from Seminary?

5. Two pieces of silk contained $18\frac{1}{3}$ and $10\frac{1}{6}$ yards, respectively. How many yards were there in both?

6. After $7\frac{1}{8}$ pounds of butter were sold from a tub it contained $48\frac{1}{2}$ pounds. How many pounds did it contain originally?

7. A man worked $8\frac{1}{5}$ hours on Monday and $7\frac{1}{2}$ hours on Tuesday. How many hours did he work on both days?

8. A woman bought $7\frac{3}{8}$ pounds of beef and $3\frac{1}{8}$ pounds of veal. How many pounds of both did she buy?

9. A girl's expenses for a week were \$8¾, and her savings were \$2¼. What did she earn?

10. The distance between the first plant in a row and the last is 87 feet. These plants are each $1\frac{1}{2}$ feet from the end of the row.

(*a*) How long is the row? (*b*) How many plants 3 feet apart are there in the row?

SIGHT EXERCISES

When the mixed numbers are in the view of the pupils, those who desire to begin the work by adding the fractions may be permitted to do so.

When the answers to the following are to be written, the result should be obtained by the pupil before he begins to write.

1. How many acres are there in two fields, one containing $40\frac{3}{4}$ acres and the other containing $7\frac{1}{2}$ acres.

METHOD	
Think $47\frac{3}{4}$ ($40\frac{3}{4}+7$), $48\frac{1}{4}$ (adding $\frac{1}{2}$).	$48\frac{1}{4}$ A. Ans.

2. Give sums:

a $17\frac{1}{2}$ + $\frac{3}{4}$	*b* $18\frac{2}{3}$ + $\frac{2}{3}$	*c* $19\frac{5}{8}$ + $\frac{1}{2}$	*d* $20\frac{5}{6}$ + $\frac{5}{6}$
e $15\frac{1}{2}$ + $1\frac{3}{4}$	*f* $16\frac{2}{3}$ + $2\frac{2}{3}$	*g* $20\frac{7}{8}$ + $3\frac{1}{2}$	*h* $21\frac{5}{6}$ + $4\frac{2}{3}$
i $15\frac{3}{4}$ + $10\frac{3}{4}$	*j* $16\frac{5}{8}$ + $10\frac{5}{8}$	*k* $20\frac{3}{8}$ + $10\frac{3}{8}$	*l* $21\frac{7}{8}$ + $10\frac{7}{8}$
m $15\frac{3}{5}$ + $12\frac{3}{5}$	*n* $16\frac{6}{9}$ + $11\frac{6}{9}$	*o* $20\frac{4}{7}$ + $15\frac{4}{7}$	*p* $21\frac{5}{7}$ + $15\frac{5}{7}$

3. Give sums:

a $10\frac{5}{6}$ + $5\frac{5}{6}$	*b* $11\frac{3}{7}$ + $4\frac{3}{7}$	*c* $12\frac{2}{9}$ + $5\frac{2}{9}$	*d* $13\frac{4}{9}$ + $6\frac{4}{9}$
e $20\frac{1}{6}$ + $5\frac{1}{6}$	*f* $21\frac{1}{4}$ + $4\frac{1}{4}$	*g* $22\frac{1}{8}$ + $5\frac{1}{8}$	*h* $23\frac{2}{9}$ + $6\frac{2}{9}$
i $30\frac{3}{5}$ + $1\frac{3}{5}$	*j* $31\frac{4}{7}$ + $2\frac{4}{7}$	*k* $32\frac{2}{5}$ + $3\frac{2}{5}$	*l* $33\frac{3}{9}$ + $4\frac{3}{9}$
m $41\frac{1}{2}$ + $2\frac{1}{4}$	*n* $42\frac{1}{4}$ + $3\frac{1}{8}$	*o* $43\frac{1}{2}$ + $4\frac{1}{8}$	*p* $44\frac{1}{4}$ + $5\frac{3}{8}$
q $52\frac{1}{2}$ + $5\frac{3}{4}$	*r* $53\frac{1}{2}$ + $3\frac{5}{8}$	*s* $54\frac{1}{2}$ + $2\frac{7}{8}$	*t* $55\frac{1}{4}$ + $1\frac{7}{8}$
u $65\frac{1}{3}$ + $1\frac{1}{6}$	*v* $64\frac{2}{3}$ + $2\frac{2}{3}$	*w* $63\frac{5}{6}$ + $3\frac{5}{6}$	*x* $62\frac{5}{8}$ + $4\frac{5}{8}$

WRITTEN EXERCISES

In wholesale dry-goods houses $12\frac{1}{4}$ is written 12^1, $13\frac{1}{2}$ is written 13^2, $14\frac{3}{4}$ is written 14^3, the denominator, 4, being omitted.

1. Add the following. Write answers directly from the book:

a	*b*	*c*	*d*	*e*
14^{1} yd.	7^{2} yd.	31^{2} yd.	11^{1} yd.	2^{3} yd.
8^{3}	16^{3}	40^{2}	18^{2}	34^{3}
36^{2}	25^{2}	3^{1}	6^{3}	28^{3}
5^{1}	4^{3}	19^{3}	33^{2}	10^{2}
17^{1}	21^{1}	22^{3}	20^{1}	1^{2}
9^{2}	37^{2}	39^{2}	26^{3}	38^{2}

Check each result by adding downward, if the first result is obtained by adding upward.

In writing pounds and ounces, grocers sometimes use small figures to express ounces, writing the latter as fractions of a pound, but omitting 16, the denominator.

2. Add the following, writing answers directly from the book:

a	*b*	*c*	*d*	*e*
38^{1} lb.	21^{15} lb.	19^{2} lb.	6^{14} lb.	8^{3} lb.
261^{1} "	22^{5} "	3^{8} "	28^{4} "	14^{13} "
39^{7} "	20^{8} "	5^{6} "	34^{6} "	7^{7} "
371^{4} "	1^{4} "	36^{3} "	18^{9} "	31^{10} "
91^{3} "	10^{12} "	25^{2} "	40^{12} "	11^{11} "
17^{5} "	33^{9} "	3^{10} "	16^{15} "	2^{1} "

Fractions in business are generally limited to halves, quarters, eighths, sixteenths, etc.

3. Add. Write answers directly from the book.

a	*b*	*c*	*d*
$16\frac{1}{2}$	$23\frac{1}{2}$	$25\frac{1}{2}$	$9\frac{1}{2}$
$8\frac{1}{4}$	$9\frac{3}{4}$	$42\frac{3}{8}$	$18\frac{3}{4}$
$23\frac{1}{8}$	$42\frac{3}{8}$	$42\frac{5}{8}$	$6\frac{7}{8}$
$59\frac{1}{16}$	$6\frac{1}{16}$	$7\frac{3}{16}$	$17\frac{1}{16}$

e	*f*	*g*	*h*
$23\frac{1}{4}$	$37\frac{3}{4}$	$48\frac{1}{4}$	$50\frac{3}{4}$
$7\frac{1}{8}$	$8\frac{1}{8}$	$9\frac{3}{8}$	$10\frac{3}{8}$
$12\frac{1}{16}$	$12\frac{5}{16}$	$13\frac{5}{16}$	$14\frac{3}{16}$
$9\frac{1}{32}$	$8\frac{1}{32}$	$7\frac{3}{32}$	$6\frac{3}{32}$

4. Find the total weight of six pieces of meat weighing, respectively, 16½ lb., 8¾ lb., 9⅞ lb., 11⅜ lb., and 14⁳⁄₁₆ lb.

PROCESS

16½ lb.	8
8¾	12
9⅞	14
11⅜	6
14⁳⁄₁₆	3
Ans. 60¹¹⁄₁₆ lb.	⁴³⁄₁₆ = 2¹¹⁄₁₆

Write the addends in a column and draw a perpendicular line on the right to separate the new numerators from the original fractions. In the second column, write 16, the least common denominator, on a line below the last addend in the second column, and write over this the sum of the new numerators when found. Write the new numerators alongside the corresponding fractions. Write 43, their sum, over 16 previously written. Reduce ⁴³⁄₁₆ to 2¹¹⁄₁₆. Write ¹¹⁄₁₆ under the original fractions, and carry 2 to the whole numbers.

A Shorter Method

A pupil that notes that the sum of ⅞ and ⅜ is 1¼, which makes 2 when united with ¾, has left only two fractions to combine, ½ and ³⁄₁₆, whose sum is ¹¹⁄₁₆, which he writes. He then carries 2 to the whole numbers.

Use this method to check the sum of the fractions.

In adding mixed numbers containing such fractions, for example, as ⁵⁄₁₆, ½, ⅛, ³⁄₁₆, ¾, and ⅞, accountants frequently rearrange the addends, especially in testing a result, to bring the fractions together in this order: ⅛, ⅞; ⁵⁄₁₆, ³⁄₁₆, ½; ¾.

Even when combinations making 1 are not possible, as in ½, ⅞, ⅚, ⅑, ¾, ⅓, they rearrange the addends in some such way as this: ½, ⅞, ¾; ⅚, ⅑, ⅓; combining the first three mentally into ⅞, or 2⅛, and the next three into ²⁸⁄₁₈ or 1⁵⁄₁₈, etc.

5. Add:

a	*b*	*c*	*d*
$36\frac{5}{16}$	$28\frac{3}{8}$	$86\frac{1}{2}$	$125\frac{1}{2}$
$8\frac{1}{2}$	$86\frac{1}{2}$	$8\frac{3}{4}$	$20\frac{9}{16}$
$93\frac{1}{8}$	$7\frac{5}{16}$	$95\frac{1}{8}$	$354\frac{3}{4}$
$27\frac{3}{16}$	$20\frac{3}{4}$	$5\frac{7}{16}$	$68\frac{3}{8}$
$45\frac{3}{4}$	$9\frac{5}{18}$	$66\frac{7}{8}$	$98\frac{3}{4}$
$8\frac{7}{8}$	$53\frac{1}{4}$	$8\frac{13}{16}$	$7\frac{11}{16}$

6. Find the sum of $3\frac{1}{2}$ days, $5\frac{1}{3}$ days, $7\frac{1}{4}$ days, $9\frac{1}{6}$ days, $11\frac{1}{8}$ days, $13\frac{1}{12}$ days.

PROCESS

In finding the least common multiple of the denominators of these fractions, omit from consideration 2, which is a factor of 4; 3, a factor of 6; 4, a factor of 8; 6, a factor of 12. Find the least common multiple of 8 and 12, by considering multiples of 12, beginning with 24. As this is a multiple of 8, it is the least common denominator.

7. Add the following:

a	*b*	*c*	*d*
$5\frac{1}{2}$	$75\frac{1}{2}$	$33\frac{1}{3}$	$432\frac{1}{2}$
$7\frac{2}{3}$	$9\frac{2}{5}$	$80\frac{2}{9}$	$83\frac{7}{8}$
$9\frac{1}{4}$	$23\frac{3}{4}$	$7\frac{1}{16}$	$157\frac{5}{16}$
$11\frac{5}{6}$	$8\frac{7}{10}$	$36\frac{7}{12}$	$28\frac{2}{9}$
$13\frac{1}{8}$	$13\frac{5}{8}$	$5\frac{3}{4}$	$7\frac{3}{4}$
$15\frac{5}{12}$	$8\frac{1}{16}$	$29\frac{1}{2}$	$18\frac{1}{3}$

8. A person made purchases of pencils as follows: $2\frac{1}{2}$ gross, $3\frac{7}{8}$ gross, $\frac{5}{6}$ gross, $8\frac{1}{3}$ gross, $\frac{3}{4}$ gross, and $9\frac{5}{12}$ gross. How many gross did he buy in all?

PROCESS

2½ gross	After rejecting denominators that are multiples of others, there remain the following:
3⅞ "	
⅚ "	
8⅓ "	
¾ "	
9⅝ "	
/72	L. C. M. = 4 × 2 × 9 = 72

$$4)\ 8 - 9 - 12$$
$$2 - 9 - 3$$

If you do not notice that 8 and 9 are *prime to each other*, which makes 72 their least common multiple, and that 72 is also a multiple of 12, find the least common multiple of 8, 9, and 12 by writing these numbers in a line. Then divide by 4, which is a common factor of 8 and 12. Write under 8 and 12 their quotients, bringing down 9. Cancel 3, which is a factor of 9.

Since 2 and 9, the remaining numbers, are prime to each other; that is, since they have no common factor, multiply their product by the divisor 4, which gives 72, the least common multiple.

9. How many pens are there in 2½ gross, 4⅓ gross, 6¼ gross, ⅙ gross, 5⅛ gross, 1⅑ gross, 3$\frac{1}{16}$ gross, 7$\frac{1}{18}$ gross, $\frac{1}{24}$ gross, 9$\frac{1}{36}$ gross, and 8$\frac{1}{72}$ gross?

10. Add the following:

a	*b*	*c*	*d*
18$\frac{7}{12}$	6⅓	22⅞	125$\frac{5}{16}$
9$\frac{5}{24}$	27⅞	5⅝	14½
16⅜	85⅑	16½	8⅗
7$\frac{9}{16}$	8⅑	30⅚	$\frac{9}{10}$
20¾	26⅝	8⅔	27¾
8⅓	1⅔	29$\frac{5}{24}$	3⅛

11. (*a*) Express in years and a fraction the sum of ¼ year, ⅜ year, ⅛ year, 7/10 year, and 5/12 year. (*b*) Change each to days (taking 360 days to year), and find their sum.

12. A machine consists of four parts, which are manufactured from steel "blooms," weighing 290 pounds each. A bloom will make either 7 of one part, 9 of the second, 20 of the third, or 25 of the fourth. (*a*) Express the weight of each part as a mixed number, and find their sum. (*b*) Express each as a mixed decimal and find their sum. (*c*) Change the fractional part of (*a*) to a 2-place decimal.

PROCESS

(*a*)		(*b*)
41⅞ lb.	Since there is no factor	41.4286 lb
32⅔ "	common to any two of	32.2222 "
14½ "	the denominators the	14.5 "
11⅘ "	L. C. D. is their continued	11.6 "
lb.	/630 product, $2 \times 5 \times 7 \times 9$	

13. (*a*) Find the sum of 1½, 2⅓, 3¼, 4⅕, 5⅙, 6⅐, 7⅛, 8⅑ and 9/10.

PROCESS

Find the least common multiple of 6, 7, 8, 9 and 10, rejecting the others.

(*b*) Give the answer as a mixed decimal, two places.
(*c*) Change the fractions to four-place decimals,

add the numbers as mixed decimals, and express the result as a mixed two-place decimal.

PROCESS

1½	.5
2⅓	.3333
etc.	etc.
5⅙	.1667
6⅐	.1429
etc.	etc.

Write the decimal equivalent alongside. Increase the fourth place of the decimal equivalent of ⅙ and of ⅐ since the next figure in each is greater than 5.

17. Add the following, changing the fractions to decimals. Give answer as a mixed decimal.

$1\frac{1}{2} + 2\frac{2}{3} + 3\frac{3}{4} + 4\frac{4}{5} + 5\frac{5}{6} + 6\frac{6}{7} + 7\frac{7}{8} + 8\frac{8}{9} + 9\frac{9}{10}$

ADDING COMPOUND NUMBERS

SIGHT EXERCISES

1. How many pounds and ounces are there in two pieces of meat, one of which weighs 5 pounds 10 ounces and the other 3 pounds 8 ounces?

METHOD

Think 8 lb. 10 oz. (5lb. 10 oz. + 3 lb.),
8 lb. 18 oz. (adding 8 oz.), 9 lb. 2 oz. (reducing)
Ans. 9 lb. 2 oz.

2. Add:

a	3 lb. 10 oz.	*b*	4 lb. 9 oz.	*c*	6 lb. 10 oz.
	+2 lb. 6 oz.		+4 lb. 9 oz.		+2 lb. 11 oz.

d 4 yd. 2 ft.
+3 yd. 1 ft.

e 5 yd. 2 ft.
+6 yd. 2 ft.

f 9 yd. 1 ft.
+7 yd. 1 ft.

g 6 gal. 1 qt.
+2 gal. 2 qt.

h 7 gal. 1 qt.
+3 gal. 3 qt.

i 8 gal. 3 qt.
+1 gal. 3 qt.

j 4 bu. 2 pk.
+4 bu. 2 pk.

k 5 bu. 2 pk.
+3 bu. 3 pk.

l 3 bu. 2 pk.
+2 bu. 3 pk.

m £5 10s
+£6 10s

n £8 12s
+£9 12s

o £6 18s
+£2 10s

p 6 ft. 3 in.
+4 ft. 9 in.

q 8 ft. 10 in.
+1 ft. 10 in.

r 9 ft. 8 in.
+2 ft. 7 in.

WRITTEN EXERCISES

Add the following. Write answers from the book:

a £24 16s 3d
8 9 6
15 10 10

b 32 yd. 1 ft. 10 in.
5 2 6
18 1 7

c 16 gal. 2 qt. 1 pt.
35 3
9 1 1
4 1

d 62 bu. 1 pk. 6 qt.
5 3 4
24 2
6 1 3

e 43 lb. 8 oz.
18 10
6 5

f 8 ft. 10 in.
63 5
8 9

g 2 mi. 90 rd.
10 120
3 84

CHAPTER SIX

SUBTRACTION

PREPARATORY EXERCISES

Making Change

1. What change does a clerk hand a person who gives a $20 bill to pay for articles amounting to $16.85?

METHOD

The clerk hands a nickel, saying "sixteen, ninety"; a dime, saying "seventeen dollars"; a dollar, saying "eighteen dollars"; and a 2-dollar bill, saying "twenty dollars."

He gives $5¢ + 10¢ + \$1 + \$2 = \$3.15$

2. State the denominations of the money used to make change from $1 tendered in payment for purchases amounting to the sum specified below. State also, in each case, the total amount given in change.

a 3¢	*b* 76¢	*c* 89¢	*d* 45¢	*e* 23¢
f 7¢	*g* 18¢	*h* 51¢	*i* 29¢	*j* 34¢
k 4¢	*l* 94¢	*m* 62¢	*n* 82¢	*o* 79¢

WRITTEN EXERCISES

1. A man earned $1800 during the year. He spent $1475.35. How much did he save?

PROCESS

\$1800.— To subtract, begin with 1475.35
− 1475.35 (the subtrahend). Think 5 and 5
\$324.65 Ans. (writing 5) are 10. Think 4 (carrying 1) and 6 (writing 6) are 10. Think 6 (carrying 1) and 4 (writing 4) are 10. Think 8 (carrying 1) and 2 (writing 2) are 10. Think 5 (carrying 1) and 3 (writing 3) are 8.

CHECK

Cover \$1800 (the minuend) and add \$324.65 (the remainder) to \$1475.35.

2. Find remainders. Check.

a 345.1	*b* 473	*c* 1016.82
− 57.064	− 389.49	− 893.9

3. Subtract without rearranging.

a − 29.86	*b* − 146.5	*c* − 383.47
157.328	212.17	1000.
d \$9245.18	*e* 16,059	*f* − 764.58
− 264.83	− 1,088	1113.2
g − \$321.69	*h* 10,193.8	*i* 12,657
1523.07	− 654.95	− 9,879
j 101,087	*k* − 93,847	*l* \$1364.57
− 65,564	104,305	− 890.09
m − 172,654	*n* − 18.25975	*o* 293,647
200,001	106.0005	− 188,898

4. The following is a statement of Mr. Pallen's account with Gaston and Carroll at the close of business Jul. 31, 1920.

MR. C. PALLEN
2562 Georgetown Boulevard

TUCSON, ARIZ., Aug. 1, 1920

In Account with GASTON and CARROLL

Jul.	6	To Mdse.	273	46	
	9	" "	58	95	
	11	" "	187	84	
	15	" "	36	92	
	18	" "	375	14	
	20	" "	263	88	
	29	" "	95	44	(*a*)
		Cr.			
Jul.	12	By Mdse.	256	40	
	18	" Cash	100	—	
	22	" Mdse.	310	89	
	29	" "	63	75	
	30	" Cash	100	—	(*b*)
		Balance due			(*c*)

Copy the foregoing statement inserting at (*a*) the sum of the debits, at (*b*) the sum of the credits, and at (*c*) the balance due Gaston and Carroll.

5. A farmer's receipts and expenditures, respectively, for the year are shown in the following table:

	Receipts		Expenditures		Balance
January	$187.43	–	$138.98	=	$48.45
February	156.14	–	125.47	=	(*c*)
March	195.80	–	156.—	=	(*d*)
April	163.44	–	135.29	=	(*e*)
May	201.59	–	163.88	=	(*f*)
June	198.65	–	148.77	=	(*g*)
July	302.88	–	205.93	=	(*h*)
August	356.93	–	216.84	=	(*i*)
September	298.67	–	225.98	=	(*j*)
October	215.42	–	160.56	=	(*k*)
November	198.68	–	123.15	=	(*l*)
December	125.94	–	112.68	=	(*m*)
Totals	(*a*)	–	(*b*)	=	(*n*)

Find (*a*) his receipts for the year. (*b*) His expenditures. (*c* to *m*) His monthly balances. (*n*) The balance at the end of the year.

Find (*n*) by adding the last column. Check by covering (*n*) and writing on the paper the difference between (*a*) and (*b*).

ORAL DRILLS

1. Anna had 85¢. How much will she have after spending 49¢?

From 85¢ take 40¢, then take 9¢.
Do not follow the method used in your Written Exercises in subtraction.

2. Give remainders.

a 65 − 27	*b* 91 − 62	*c* 84 − 36	*d* 73 − 56
e 54 − 16	*f* 86 − 47	*g* 70 − 29	*h* 82 − 17
i 93 − 65	*j* 40 − 24	*k* 52 − 18	*l* 63 − 24

3. Miss Bruen paid $3.42 for muslin and gloves. The gloves cost $1.75. What did the muslin cost?

Think $2.42 (deducting $1), $1.72 (deducting 70¢), $1.67 (deducting 5¢). Ans., $1.67

4. Give remainders:

a 121 − 75	*b* 190 − 175	*c* 253 − 164
d 137 − 94	*e* 183 − 116	*f* 270 − 195
g 110 − 26	*h* 174 − 138	*i* 265 − 187
j 142 − 88	*k* 162 − 125	*l* 210 − 173
m 246 − 77	*n* 395 − 316	*o* 321 − 135
p 315 − 68	*q* 572 − 544	*r* 432 − 146
s 420 − 37	*t* 783 − 717	*u* 511 − 154
v 511 − 54	*w* 964 − 909	*x* 613 − 165

5. From a crop of 1216 bushels of corn, Mr. Popkins sold 658 bushels. How many bushels has he left?

Think six, sixteen (deducting six hundred); five, sixty-six (deducting fifty); five, fifty-eight (deducting 8). Ans. 558 bu.

COMBINING ADDITION AND SUBTRACTION

WRITTEN EXERCISES

1. A dealer had 1000 bushels of oats. How many bushels would he have after he had sold 154 bushels, 368 bushels, and 87 bushels?

PROCESS

From	1000	bu.
Take	154	"
	368	"
	87	"
Ans.	391	"

Beginning with the last subtrahend, think 15 (7+8), 19 (adding 4), and *1* (writing 1) are 20. Think 10 (carrying 2), 16 (adding 6), 21 (adding 5), and *9* (writing 9) are 30. Think 6 (carrying 3) 7 (adding 1), and *3* (writing 3) are 10.

CHECK

Cover 1000 with a piece of paper. On this write the sum of 391 and the three subtrahends.

2. Write answers to the following directly from the book.

	(*a*)	(*b*)	(*c*)	(*d*)
From	1000	1234	3256	5167
	159	216	1038	369
Take	87	157	887	2588
	355	99	95	1269

3. Give the value of each of the following:

a 756 − (184 + 95 + 367)
b 1239 − (257 + 388 + 86)
c 2000 − (1234 + 277 + 95)
d 4430 − (1234 + 345 + 68)
e 3754 − (2345 + 456 + 77)
f 5473 − (3456 + 567 + 84)

4. Write answers to the following directly from the book or blackboard:

a \$10.50 − (\$2.75 + \$.89 + \$3.—)
b 26.43 − (9.50 + .75 + 1.28)
c 35.19 − (7.63 + .67 + 2.29)
d 43.26 − (8.79 + .93 + 3.14)
e 50.20 − (6.28 + .52 + 5.67)

5. Supply missing items (*a*) to (*i*):

\$137.86 + \$75.93 + \$288.79 = (*e*)
(*a*) + 168.76 + 45.63 = (*f*)
289.65 + (*b*) + 195.84 = (*g*)
48.76 + 253.92 + (*c*) = (*h*)
123.45 + 88.87 + 216.77 = (*i*)
\$819.30 + \$916.— + \$989.98 = (*d*)

6. A merchant's cash account shows the following receipts and payments for eleven months, and the totals for the year.

	Receipts	Payments	Balance
January	\$4,748.56	\$3,949.82	\$798.74
February	4,294.87	3,870.89	(*a*)
March	4,655.18	4,327.65	(*b*)
April	4,693.25	4,784.57	(*c*)
May	4,705.93	4,259.85	(*d*)
June	4,456.88	4,078.68	(*e*)
July	4,327.65	3,963.26	(*f*)
August	4,278.58	3,859.85	(*g*)
September	4,683.95	3,965.78	(*h*)
October	4,727.53	4,218.65	(*i*)
November	4,515.78	3,887.79	(*j*)
December	(*k*)	(*l*)	(*m*)
	\$54,837.63	\$48,684.13	(*n*)

Find the balances for the eleven months (*a*) to (*j*), the receipts for December (*k*), the payments for December (*l*), December's balance (*m*), the balance at the end of the year (*n*).

The following table shows the sums appropriated for a year for the specified items, also the expenditures for ⅝ year:

Items	Appropriations	Expenditures	Balance	Per Cent remaining
Telephone	$150	80.25	(*a*)	A
Repairs	1600	1267.80	(*b*)	B
Equipment	1500	1350.—	(*c*)	C
Supplies Manual Training	600	244.68	(*d*)	D
Janitors' Supplies	500	366.90	(*e*)	E
Domestic Science Supplies	400	241.40	(*f*)	F
Printing	500	317.10	(*g*)	G
Water, Light, Gas	550	239.80	(*h*)	H
Fuel	2075	726.25	(*i*)	I
Books	500	434.60	(*j*)	J
Helpers	1000	441.52	(*k*)	K
Janitors' Salaries	4000	2114.—	(*l*)	L
Drawing Supplies	350	173.95	(*m*)	M
Athletics	300	274.56	(*n*)	N
Miscellaneous	1525	969.90	(*o*)	O
Science Supplies	800	507.36	(*p*)	P
Incidentals	250	184.62	(*q*)	Q
Music	300	105.30	(*r*)	R
	I	II	III	IV

I. Find the total amount appropriated. II. The expenditures. III. The balance remaining of each appropriation, (*a*) to (*r*). IV. The per cent each balance is of the sum appropriated, (A to R). (Carry out to two decimal places.)

TAKING ONE NUMBER FROM THE SUM OF TWO OR MORE NUMBERS

PREPARATORY EXERCISES

1. A boy who had \$1.75 earned 50 cents and spent 98 cents. How much had he then?

METHOD

\$1.75
\+ .50
− 1.—
\+ .02
\$1.27 Ans.

His balance is found by taking \$1 from the sum of \$1.75 and \$0.50, and adding 2¢ to the remainder.

In practice the \$1 is not written, but the deduction is made nevertheless.

\$1.75
.50
.02
\$1.27

THE COMPLEMENT OF A NUMBER

The 2 thus added is called the *complement* of 98, the complement of a number being the difference between it and a unit of the next higher order.

Thus, the complement of 9 is 1 (10 minus 9), of 79 is 21 (100 minus 79), of 675 is 325 (1000 minus 675).

To find the complement of 783.951 take 7, 8, 3, 9 and 5 from 9; and 1 from 10, writing the successive remainders from left to right.

2. A girl who had \$1.75 received 50¢ from her aunt, and then spent \$1.88. How much had she left?

METHOD

$1.75	Use the complement of $1.88,
.50	which is $8.12. Add the three
8.12	numbers. Before writing the total
$ Ans.	of the last column, deduct $10.

WRITTEN EXERCISES

1. At the beginning of work in the morning, the factory had on hand 475 tons of steel. During the day 350 tons were made and 587 tons were sold. How many tons remained?

PROCESS

475 T	Write the subtrahend in the regu-
+ 350 "	lar way, but use its complement,
− 587 "	413.

Think 3 (10 − 7), 8 (adding 5); write 8.
Think 1 (9 − 8), 6 (adding 5), 13 (adding 7); write 3.
Think 4 (9 − 5), 5 (carrying 1), 8 (adding 3) 12 (adding 4); write 2, omitting the 1.

2. There were in a warehouse on Monday morning 649 barrels of flour. During the week 488 barrels were received and 574 were withdrawn. How many remained in the warehouse at the end of the week?

PROCESS

649 bbl.
+ 488 "
− 574 "

Write 574 bbl. but use its complement, 426. Begin at the bottom so that you will be less likely to overlook the fact that you are dealing with the complement.

3. Give the value of 746 + 184 + 95 − 367.

PROCESS

Beginning with 367, and using its complement, think 3 (10 − 7), 8, 12, 18; write 8. Carrying 1, think 4 (adding the complement, 9 − 6), 13, 21, 25; write 5. Carrying 2, think 8 (adding the complement, 9 − 3), 9, 16; write 6. Omit the 1. Ans. 658.

4. Write the answers to the following directly from the book or the blackboard.

a \$4.50 + \$2.75 − \$1.89.
b \$12.60 + \$8.50 − \$10.89.
c 1875 + 387 + 96 − 448.
d 2015 + 86 + 250 − 1234.
e 1887 + 2460 + 329 − 2563.
f 2065 + 1265 + 157 − 4257.
g 3217 + 3087 + 234 − 3628.
h 4382 + 2342 + 689 − 1367.
i 3562 + 4056 + 408 − 6924.
j 2341 + 6027 + 824 − 5833.
k 1766 + 5150 + 569 − 3426.
l 2598 + 3006 + 736 − 4762.

WRITTEN EXERCISES

The following is a statement of the exports and imports of each business day for three weeks, beginning Jul. 17.

Copy this statement, and complete it by inserting for each day its excess (*a*) of exports or (*b*) of imports, (*c*) the total exports for three weeks, (*d*) the total imports, and (*e*) the net excess of the exports.

		Exports	Imports	Excess Exports	Excess Imports
Jul.	17	7,728,468	4,601,395	3,127,073	
	18	12,558,896	2,902,051	etc.	
	19	8,440,177	4,322,549	(*a*)	
	20	8,701,825	2,751,883		
	21	6,263,634	2,394,123		
	22	9,172,369	839,442		
	24	10,748,034	5,472,260		
	25	10,082,167	5,048,748		
	26	3,568,542	3,711,062		142,520
	27	1,043,375	2,800,355		etc.
	28	1,964,560	4,385,087		(*b*)
	29	4,865,135	3,704,030		
	31	8,815,609	3,791,273		
Aug.	1	20,128,921	5,864,208		
	2	14,178,438	4,503,504		
	3	6,580,265	6,776,233		
	4	5,271,135	5,105,781		
	5	1,471,401	2,737,628		
Totals		(*c*)	(*d*)	(*e*)	

GROSS WEIGHT, TARE, NET WEIGHT

The *gross weight* of merchandise includes the weight of the barrel, tub, wagon, etc. The *tare* is the weight of the covering, wagon, etc. The *net weight*, which is the difference between the two former, is the weight of the merchandise.

The buyer weighs the packages when he receives them and compares the weight of each with that marked on the package. When the package is emptied,

he weighs it and compares the weight with the marked one.

1. Find (*a*) the total gross weight. (*b*) The tare. (*c*) The total net weight of the following purchase of sugar, 15 barrels.

327 – 20	336 – 18	340 – 21	335 – 17	339 – 20
332 – 18	332 – 19	337 – 19	336 – 18	340 – 21
327 – 17	330 – 19	331 – 18	329 – 17	328 – 17

2. The following are the gross weights and the tares of 12 tubs of lard. Find (*a*) the total gross weight, (*b*) the total tare, and (*c*) the total net weight.

74 – 15	70 – 14	71 – 16	60 – 13	70 – 15	68 – 13
68 – 13	71 – 14	70 – 15	72 – 16	73 – 14	69 – 14

3. From the following data find (*a*) the total gross, (*b*) the total tare, (*c*) the total net of 18 loads of coal, and (*d*) its value at $7.50 a ton of 2000 pounds.

Gross	Tare	Gross	Tare	Gross	Tare
4764	1236	4756	1216	4912	1248
4588	1232	4972	1232	4636	1272
4648	1240	4568	1244	4592	1284
4720	1248	4884	1312	4756	1268
4936	1264	4728	1296	4872	1236
4652	1256	4632	1272	4928	1308

In the vicinity of a market to which farmers bring their produce, there is generally a public scale in charge of a sworn weigher. When a farmer sells a load of hay, he drives it on the scales. The weigher enters the weight of the load and wagon, and when

the hay is removed, he weighs the wagon. To the farmer he gives a statement in the following form:

CERTIFICATE OF WEIGHT

	Lawrence, Michigan, Aug. 30, 1920
LOAD OF... Hay.....	GROSS WEIGHT....3250 lb.....
OWNER... John Ziegler	TARE " ....1130 " ....
SOLD TO Dwight Braman	NET " ....2120 " ....
	Samuel Goldstone
	City Weigher.

4. Find the value of the foregoing load of hay at $1.35 a hundred pounds.

5. During the day Mr. Goldstone issued certificates for loads of hay weighing as follows. Find the value of each at the price specified.

	Gross	Tare	Rate per 100		Gross	Tare	Rate per 100
a	3325	1165	$1.25	*b*	3575	1525	$1.40
c	3430	1210	$1.30	*d*	3360	1240	$1.35
e	3245	1085	$1.15	*f*	3410	1150	$1.20

SUBTRACTING FRACTIONS

DRILL EXERCISES

1. Give answers rapidly:

a $\frac{1}{2} - \frac{1}{4}$	*b* $\frac{1}{2} - \frac{1}{8}$	*c* $\frac{1}{2} - \frac{3}{8}$	*d* $\frac{1}{2} - \frac{1}{6}$	*e* $\frac{3}{4} - \frac{1}{2}$	*f* $\frac{3}{4} - \frac{3}{8}$	*g* $\frac{3}{4} - \frac{5}{8}$
h $\frac{1}{2} - \frac{1}{3}$	*i* $\frac{2}{3} - \frac{1}{2}$	*j* $\frac{1}{2} - \frac{1}{6}$	*k* $\frac{5}{6} - \frac{1}{2}$	*l* $\frac{5}{6} - \frac{1}{3}$	*m* $\frac{5}{6} - \frac{2}{3}$	*n* $\frac{1}{3} - \frac{1}{4}$

2. Give remainders:

a $1 - \frac{1}{2}$	*b* $1 - \frac{1}{4}$	*c* $1 - \frac{1}{8}$	*d* $1 - \frac{3}{8}$	*e* $1 - \frac{5}{8}$	*f* $1 - \frac{3}{4}$	*g* $1 - \frac{7}{8}$
h $1\frac{3}{4} - \frac{1}{2}$	*i* $1\frac{1}{4} - \frac{1}{2}$	*j* $1\frac{1}{4} - \frac{3}{4}$	*k* $1\frac{1}{8} - \frac{1}{2}$	*l* $1\frac{1}{2} - \frac{1}{2}$	*m* $1\frac{1}{8} - \frac{1}{4}$	*n* $1\frac{1}{8} - \frac{3}{4}$
o $1\frac{1}{3} - \frac{1}{2}$	*p* $1\frac{1}{2} - \frac{5}{6}$	*q* $1\frac{1}{3} - \frac{5}{6}$	*r* $1\frac{2}{3}$ $6\frac{5}{6}$	*s* $1\frac{1}{2} - \frac{2}{3}$	*t* $1\frac{1}{6} - \frac{1}{2}$	*u* $1\frac{1}{6} - \frac{1}{3}$

3. (*a*) From $1\frac{19}{25}$ take $\frac{23}{25}$. (*b*) From $1\frac{11}{18}$ take $\frac{17}{18}$.

METHOD

(*a*) Think $\frac{2}{25}$ $(1 - \frac{23}{25})$, $\frac{21}{25}$ (adding $\frac{19}{25}$). Ans. $\frac{21}{25}$.

(*b*) Think $\frac{1}{18}$ $(1 - \frac{17}{18})$, $\frac{12}{18}$ (adding $\frac{11}{18}$). Reduce $\frac{12}{18}$ to $\frac{2}{3}$. Ans. $\frac{2}{3}$.

4. Subtract:

a $1\frac{19}{25} - \frac{23}{25}$	*b* $1\frac{17}{25} - \frac{21}{25}$	*c* $1\frac{15}{28} - \frac{23}{28}$	*d* $1\frac{13}{28} - \frac{25}{28}$	*e* $1\frac{25}{36} - \frac{31}{36}$
f $1\frac{19}{32} - \frac{31}{32}$	*g* $1\frac{17}{35} - \frac{29}{35}$	*h* $1\frac{16}{27} - \frac{25}{27}$	*i* $1\frac{19}{24} - \frac{23}{24}$	*j* $1\frac{13}{18} - \frac{17}{18}$

WRITTEN EXERCISES

1. A dealer's stock of buttons at the beginning of the week was $905\frac{5}{8}$ gross; at the end of the week it was $356\frac{13}{18}$ gross. If none were bought during the week, how many gross were sold?

A LONG METHOD

905 ⅞ gross	4	22
356 13/18 "	13	
548½ gross	9/18 = ½	

After writing the numbers and drawing the vertical line, as in addition, write 18 as the denominator of the fraction in the remainder, it being the L. C. D. of the other fractions. Change ⅞ to 9/18 writing 4, the new numerator, to the right of the vertical line. Write 13, the numerator of the fraction in the subtrahend, under 9. Since 13/18 is greater than 9/18, increase the latter by 18/18, writing 22, the increased numerator, to the right of the 4. Subtracting 13 from 22, write 9 over 18, making the fraction 9/18 and reduce it to ½. Write ½ under the original fractions, increase 356 by 1, and complete the work.

NOTE: The foregoing method gives all of the steps. The pupil should omit as many of them as possible.

2. Write answers from the book if possible:

a 192¼	*b* 385½	*c* 270⅖	*d* 452⅞
− 29⅞	− 137 7/10	− 28 7/20	− 216 9/24
e 563 5/12	*f* 771⅔	*g* 682 3/10	*h* 843⅜
− 338¾	− 97 11/12	− 129⅕	− 65⅞

3. Subtract:

a 158 5/12	*b* 862⅜	*c* 329⅗	*d* 613⅓
− 99 7/15	− 593 7/12	− 98 3/10	− 267⅝
e 261⅞	*f* 706⅞	*g* 510¼	*h* 432 1/10
− 104⅘	− 69 11/12	− 238⅓	− 75 7/20

SIGHT EXERCISES

1. From a field of 48¼ acres, 7¾ acres are sold. How many acres are left?

PROCESS

Think 41¼ acres (48¼ − 7), 40½ (deducting ¾).
Ans. 40½ A.

2. Give remainders:

a 20¼ − ¾	*b* 16⅓ − ⅔	*c* 20⅜ − ⅝	*d* 17⅙ − ⅚
e 20¼ − 19½	*f* 17⅓ − 16⅔	*g* 21⅛ − 20⅝	*h* 18⅜ − 17⅞
i 20⅛ − 9½	*j* 18⅛ − 9¼	*k* 31⅛ − 9½	*l* 40⅛ − 9⅞
m 20⅜ − 16¾	*n* 18⅜ − 12½	*o* 31½ − 25⅙	*p* 40½ − 30⅓

ORAL PROBLEMS

1. Last year a farmer's crop of wheat averaged 21¼ bushels to the acre. This year it averaged 22⅞ bushels. What is the increase?

PROCESS

Think 1⅞ (22⅞ − 21), 1⅛ (deducting ¾).
Ans. 1⅛ bu.

2. Since the war copper has been sold at 15¾ a pound, an increase of 3½¢. What was the former price?

3. Two piles of wood contain 4⅞ cords. One contains 2½ cords; how many cords does the other contain?

4. A man starts for Falls Church, 32 miles away. How far has he to go after traveling 23⅞ miles?

5. From a piece of cloth containing 41¼ yards 18⅜ yards were sold. How many yards remain?

6. A tub of butter weighs, with the tub, 56 pounds. The tub weighs 7½ pounds. What is the weight of the butter?

7. A man worked on Monday 8½ hours, on Tuesday 1¼ hours less. How many hours did he work on Tuesday?

8. A woman bought 12¾ pounds of meat, 8¼ pounds being beef and the rest mutton. How many pounds of mutton were there?

9. A girl earned $11¼ a week. What did she spend if her savings were $3½?

10. In a row 20 feet long, the distance between the end plants is 18½ feet. How many feet are there between each end plant and the end of the row if the two distances are equal?

SUBTRACTING COMPOUND NUMBERS

WRITTEN EXERCISES

1. A farmer has 32 bushels, 3 pecks, 7 quarts of seed. He requires 36 bushels, 1 peck, 4 quarts. How much is he short?

METHOD

32 bu. 2 pk. 7 qt.
36 bu. 1 pk. 4 qt.

The question is to find the quantity by which 32 bu. 2 pk. 7 qt. must be increased to make 37 bu. 1 pk. 4 qt.

Write these quantities as shown above. Under 7 qt. write the number of quarts, which added to 7 qt. will make 12 qt. (1 pk. + 4 qt.) Carry 1 pk. to 2 pk., making 3 pk. In this column write the number of pecks which added to 3 pk. will make 5 pk. (1 bu. + 1 pk.). Carry 1 bu. to 32 bu. In this column write the number of bushels which added to 33 bu. will make 36 bu. Cover the last line with a piece of paper. On this write the sum of the two addends, adding upwards. Compare this result with the original sum, 36 bu., etc.

In the following examples use the same method without changing the arrangement of the quantities.

2. Subtract. Write answers directly from the text book.

a £40 3s. 6d.
− £19 16s. 10d.

b 63 yd. 1 ft. 2 in.
− 48 yd. 2 ft. 8 in.

c 93 gal. 2 qt.
− 44 gal. 3 qt. 1 pt.

d 74 bu. 2 pk. 3 qt.
− 28 bu. 2 pk. 6 qt.

SIGHT EXERCISES

1. (*a*) From 9 lb. take 3 lb. 9 oz. (*b*) From 8 bu. 2 pk. take 5 bu. 3 pk.

METHOD

(*a*) Think 6 lb. (9 lb. − 3 lb.), 5 lb. 7 oz. (deducting 9 oz.). 5 lb. 7 oz. Ans.

(*b*) Think 3 bu. 2 pk. (8 bu. 2 pk. − 5 bu.), 2 bu. 3 pk. (deducting 3 pk.) 2 bu. 3 pk. Ans.

2. Give answers:

a £18 6s.	*b* 9s. 6d.	*c* 4 lb.
− £13 10s.	− 5s. 9d.	− 1 lb. 7 oz.
d 25 bu. 1 pk.	*e* 5 pk. 2 qt.	*f* 8 gal. 1 qt.
− 8 bu. 3 pk.	− 1 pk. 7 qt.	− 2 gal 3 qt.

TIME BETWEEN DATES

PREPARATORY EXERCISES

1. If a man begins work at the opening hour of May 5, and finishes at the closing hour (*a*) of May 6, (*b*) of May 21, how many days has he worked?

2. An importer receives some bales numbered consecutively from 52 to 73. How many bales are there?

3. How many fence posts 8 feet apart will be needed for a strip of fence (*a*) 8 feet long? (*b*) 16 feet? (*c*) 80 feet? (*d*) 160 feet?

4. If there are 21 fence posts 8 feet apart, what is the distance between the first and the last?

The difference between two dates, say March 1 and March 31, is 31 days if both days are included, 29 days if both days are excluded, and 30 days if one is included and the other is excluded.

When both days are included, the time is stated as March 1 to March 31, inclusive; when both days are excluded, it is stated as March 1 to March 31, exclusive; when one is

included and the other is excluded, the time is stated merely as March 1 to March 31.

In some places, however, interest for 2 days is charged on money borrowed on March 1, and repaid on March 2, both days being included. Ascertain the practice prevalent in your locality. In the following examples, include only 1 day.

Time less than a Year

When dates are less than a year apart, the time between them is usually found in days.

WRITTEN EXERCISES

1. How many days' interest is due on a loan made Jul. 3, 1919, and paid May 16, 1920?

METHOD

Jul.	28	days
Aug.	31	"
Sep.	30	"
Oct.	31	"
Nov.	30	"
Dec.	31	"
Jan.	31	"
Feb.	29	"
Mar.	31	"
Apr.	30	"
May	16	"
Total	318	days

Write the time remaining in July by deducting 3 days from 31 days, thus excluding July 3. Write the number of days in each of the other months to April, inclusive, remembering that 1920 is a leap year. For May, write the number of days expressed by the date, thereby including May 16.

A method of checking is to take the time as 10 months 13 days, which would make 313 days if each month had 30 days. Adding 6 extra days for July, August, October, December, January, and March, and deducting 1 for February makes 5 days more than 313.

2. Find the time between:

a	Dec. 28, 1919	and	Jan. 16, 1920
b	Mar. 19, 1920	"	Feb. 29, 1920
c	Jan. 22, 1919	"	May 12, 1919
d	Aug. 17, 1918	"	Jun. 12, 1919
e	Jan. 31, 1919	"	Aug. 24, 1919
f	May 29, 1918	"	Mar. 22, 1919
g	Feb. 23, 1920	"	Jun. 18, 1920
h	Jul. 25, 1919	"	Apr. 20, 1920
i	Sep. 27, 1918	"	Jul. 15, 1919
j	Apr. 30, 1919	"	Sep. 21, 1919

Bankers use a table to ascertain the time between two dates.

WRITTEN EXERCISES

1. Find the time that has elapsed between May 15, 1917, the date on which a note was drawn, and Jan. 3, 1920, the date on which it was paid.

In this case, also, the practice varies. Some states require that first the whole number of years be taken (2 years from May 15, 1917, to May 15, 1919); then the whole number of months (8 months from May 15, 1919, to Dec. 15, 1919); finally, the number of days from Dec. 15, 1919, to Jan. 3, 1920, viz. 19 days. The more common practice is the one given below, which assumes that each month contains 30 days.

PROCESS

1920 − 1 − 3
1917 − 5 − 15

Write 1920, first month, third day, as the minuend; and 1917, fifth month, fifteenth day, as the subtrahend. Find the difference by the method given for subtracting compound numbers (compound subtraction).

2. Find the difference in time between

a	Apr. 17, 1916	and	Sep. 10, 1919
b	Sep. 30, 1917	"	Jul. 12, 1920
c	Jul. 28, 1914	"	Apr. 15, 1918
d	Nov. 19, 1916	"	Jun. 11, 1919
e	Feb. 17, 1915	"	Aug. 15, 1920
f	May 22, 1918	"	Mar. 20, 1921
g	Jan. 16, 1919	"	May 12, 1922
h	Oct. 20, 1916	"	Feb. 18, 1920
i	Aug. 25, 1918	"	Mar. 22, 1921
j	Jun. 18, 1919	"	Jan. 13, 1922

SIGHT EXERCISES

Find the number of days that elapsed between the planting and the first picking of the following:

a	Beans,	planted	May 12,	first	picking	Aug. 10
b	Beets	"	Apr. 15,	"	"	Jun. 15
c	Corn	"	May 5,	"	"	Aug. 1
d	Melons	"	May 15,	"	"	Aug. 20
e	Peas	"	Apr. 5,	"	"	Jun. 10
f	Tomato,	"	May 1,	"	"	Aug. 1
g	Squash,	"	May 25,	"	"	Sep. 1
h	Radish	"	Apr. 1,	"	"	May 10
i	Onion,	"	Apr. 10,	"	"	Aug. 15
j	Leek,	"	Apr. 15,	"	"	Aug. 15

CHAPTER SEVEN

SPECIAL TESTS

AVOIDING MISTAKES

Legible Figures

"Blind" figures are one source of error. Learn to make figures that are easily read, and to write each figure in its proper place.

Do not make a correction by writing a second figure on top of the first one. Draw your pen through the original figure, write the correct one, and assume the responsibility for the change by affixing your initials.

Do not erase anything in a business document. Erasures beget suspicion at times. Make a necessary change in the manner suggested above.

TESTING A RESULT

In testing any result, look first at its reasonableness. The product of 37 by 27 should be less than 1200 (40×30). It should be more than 925 (37×25).

Count the number of figures in a result. The product of 2×4 contains 1 figure, that of 4×3 contains 2; the product of 23×30 contains 3 figures, that of 40×25 contains 4, etc., — the number of figures in a product of two factors being equal to the total number of figures in the two factors or 1 less. In the latter case examine the product.

A pupil in multiplying 316 by 307 might, by mistake, place the first figure of the second partial product in the tens' place instead of in the hundreds', obtaining the incorrect result 11,692. When he finds that this product has five figures, one fewer than the total of the two factors, he should observe that a 5-figure product must be greater than 90,000.

Error

$$\begin{array}{r} 316 \\ \times\ 307 \\ \hline 2212 \\ 948 \\ \hline 11{,}692 \end{array}$$

Correct

$$\begin{array}{r} 316 \\ \times\ 307 \\ \hline 2212 \\ 948 \\ \hline 97{,}012 \end{array}$$

Of course, if he makes the test by using 316 as the multiplier, he will discover his mistake. This will not be the case if he applies the test of "casting out 9's."

In testing the product of 43×67, of 274×689, or of any other two factors containing the same number of figures, obtain a second product by reversing the factors.

$$\begin{array}{rr} 43 & 67 \\ \times\ 67 & \times\ 43 \end{array} \qquad \begin{array}{rr} 274 & 689 \\ \times\ 689 & \times\ 274 \end{array}$$

When you obtain the product of 72×28769 by multiplying by 72, test it by using 8 and 9, the factors of 72.

$$\begin{array}{r} 28769 \\ 72 \\ \hline 57538 \\ 201383 \\ \hline 2071368 \end{array}$$

(*a*) 28769×72

(*b*) 258,921 — 9 times (*a*)

2,071,368 — 8 " (*b*)

If you obtain the product originally by using 9 and 8 as the factors, find the second product by changing the order to 8 and 9.

The product of 73×34568, when first obtained by the method shown at the left, is tested by the one shown at the right.

(*a*)	34,568 × 73		(*c*)	34,568 × 73
(*b*)	103,704	3 times (*a*)	(*d*)	276,544 8 times (*c*)
	2,523,464	70 times (*a*) + (*b*)		2,523,464 9 times (*d*) + (*c*)

Explanations of these abbreviated processes will be given later.

"CASTING OUT 9's"

It has been seen that a number is divisible by 9 if the sum of the digits is divisible by 9. By the "excess" of 9's in a number is meant the remainder left when a number is divided by 9. To obtain this excess, find the sum of the digits in the number, and divide this total by 9.

The "excess" of 9's in 3460875, for instance, is the remainder left when $3 + 4 + 6 + 8 + 7 + 5$, or 33, is divided by 9; namely, 6. It is not even necessary to divide 33 by 9 to obtain this remainder, the "excess" of 33 being $3 + 3$.

In practice reject 9 when the sum of two or more digits is 9 or more.

Think 7 (3 + 4), 13 (adding 6), 4 (rejecting 9), 12 (adding 8), 3 (rejecting 9), 10 (adding 7), 1 (rejecting 9), 6 (adding 5).

This process of finding the excess of 9's in a number is called "Casting out 9's."

TESTING A SUM

A sum may be tested by comparing its excess with the sum of the excesses of the addends. In the given example the excesses of the addends are 5, 3, and 7, the sum of which is 15, of which the excess is 6. The excess of 6036 is 6.

3461	(Exc.)	5
822	"	3
1753	"	7
6036	(Exc.)	6

That is, *the excess in a sum equals the excess of the total of the excesses of the addends.*

SUBTRACTION TEST

A difference may be similarly tested. In this example, the excess of 9's in the minuend is 5, in the subtrahend, 7. Since 7 is greater than 5, the latter is increased by 9, making this excess 14. Deducting 7 gives an excess of 7. This excess equals the one in the remainder, which is 7.

3461	(Exc.)	5
1753	"	7
1708	(Exc.)	7

Another way is to add the excess in the remainder to that in the subtrahend (7+ 7), which gives 5 as the excess in the minuend (14–9).

That is, *the excess in a minuend equals the excess of the total of the excesses of the remainder and the subtrahend.*

The fact that the excess of the total of the excesses of the addends does not equal the excess of the sum, shows that the result is incorrect; but the converse is not true. If in performing the addition, the person wrote 1 in the second column, and carried 3, in-

			Error
3461	(Exc.)	5	
822	"	3	(Exc. 6)
1753	"	7	
6216	(Exc.)	6	

stead of writing 3 and carrying 1, the excess of the erroneous result would still be 6. The substitution of 31 for 13 (or any other such transposition) does not change the excess, the total of the digits being the same in 31 as in 13.

TESTING PRODUCTS

Applying this test to products heretofore found, we find that the excess in 2,071,368 equals the product of the excesses in 28,769 and 72.

$$\begin{aligned} 28{,}769 \times \quad 72 &= 2{,}071{,}368 \\ (\text{Exc.})\ 5 \times (\text{Exc.})\ 0 &= (\text{Exc.})\ 0 \end{aligned}$$

and the excess in 2,523,464 equals the product of the excesses in 34,568 and 73

$$\begin{aligned} 34{,}568 \times \quad 73 &= 2{,}523{,}464 \\ (\text{Exc.})\ 8 \times (\text{Exc.})\ 1 &= (\text{Exc.})\ 8 \end{aligned}$$

$$\begin{aligned} 316 \times \quad 307 &= \quad 97{,}012 \\ (\text{Exc.})\ 1 \times (\text{Exc.})\ 1 &= (\text{Exc.})\ 1 \end{aligned}$$

That is, *the excess in a product is equal to the excess of the product of the excesses of the two factors.*

When the excess of the product is not equal to the product of the excesses there is a mistake; but they may be equal, and the product may still be wrong.

Thus the erroneous calculation given before

$$\begin{aligned} 316 \times \quad 307 &= 11{,}692 \\ (\text{Exc.})\ 1 \times (\text{Exc.})\ 1 &= (\text{Exc.})\ 1 \end{aligned}$$

gives 1 as the excess of the product and 1 as the product of the factor excesses.

WHEN THIS TEST FAILS

The test by casting out 9's does not detect an error made by the transposition of figures, the total of the digits not being affected by the change. It does not discover, in a multiplication example, the substitution of 948 tens for 948 hundreds, since here the sum of the digits is also unchanged, their place value not being taken into consideration.

A BETTER TEST — "CASTING OUT 11'S"

A number is divisible by 11 when the sum of the odd-numbered digits is equal to the sum of the even-numbered ones.

Thus in 253 (which is 11×23), $3 + 2 = 5$; in 2574, (which is 11×234), $4 + 5 = 7 + 2$.

The excess of 11's in 254 is $(4 + 2) - 5$, which is 1. The excess of 11's in 2576 is $(6 + 5) - (7 + 2)$. To find the excess in this way begin with the ones' figure, and take from the sum of the first, third, etc., the sum of the second, fourth, etc. The excess in 2,071,368 = $(8 + 3 + 7 + 2) - (6 + 1 + 0) = 20 - 7 = 13$, the excess of which is 2. This agrees with the remainder obtained by dividing this number by 11.

$$11\overline{)2071368}$$
$$188306 \text{ Rem. } 2$$

In practice, however, find the excess of 11's by continued subtraction beginning at the left.

To find the excess of 11's in 2,071,368, take 2 from 0, the remainder from 7, the remainder from 1, the remainder from 3, the remainder from 6, the remainder

from 8, increasing each minuend by 11 when it is less than the corresponding subtrahend. The successive results are 9 (2 from 0 + 11), 9 (9 from 7 + 11), 3 (9 from 1 + 11), 0 (3 from 3), 6 (0 from 6), 2 (6 from 8), the last remainder, 2, being the excess.

TESTING BY EXCESS OF 11'S

The following is the test of the correct addition result given on a previous page. The incorrect answer, 6216, which is not detected by the excess of 9's, gives 1 as the excess of 11's, which disagrees with 8, the excess of the total of the addends' excesses.

3461	(Exc.)	7
822	"	8
1753	"	4
6036	(Exc.)	8

(Exc. 8)

Testing the two products found in multiplying 316 by 307, the following are the results by casting out 11's:

$$316 \times 307 = 97{,}012$$
$$(\text{Exc.})\ 8 \times (\text{Exc.})\ 10 = (\text{Exc.})\ 3$$

In this the excess of 80, the product of 8×10, the excesses of the factors, is 3, which agrees with the excess of 97,012 in the above answer.

The excess of 11's in the wrong answer (11,692), is 10, which shows that a mistake has been made somewhere.

SELECTING THE SAFER TEST

The fact that it is slightly easier to apply the test of casting out 9's than that of casting out 11's is no reason for the employment of the one that is less certain.

TESTS OF SUMS AND DIFFERENCES

Inasmuch as it probably takes longer to apply "excess" tests in addition and in subtraction, check these operations by the method suggested for each, testing a sum by adding "down" when the first result is obtained by adding "up,"—and testing a remainder by combining it with the subtrahend to make the minuend.

TESTING QUOTIENTS

When the division is exact, the dividend is equal to the product of the quotient by the divisor. In testing the result, apply the method shown in the multiplication example previously given.

72)2,071,368
Ans. 28,769

When there is a remainder, add the excess of the remainder to the product of the excesses of the quotient and the divisor.

21)1896
90 Rem. 6

Quotient 90	(Exc.)	2	
Divisor 21	(Exc.) ×	10	
Product of excesses =		20	the excess of which is 9
Remainder		+ 6	" " " " " " 6
Total		26	" " " " " " 4

The excess of 1896, the dividend, is also 4

DECIMAL RESULTS

In testing a result by casting out 11's, consider every figure that enters into it.

The cost of 14 yards at $1.125 per yard is given as $15.75. If the 11's are cast out of this product, the excess is found to be 2, which does not agree with the product (9) of the factor excesses.

$1.125	(Exc.) 3
× 14	(Exc.) 3
$15.750	(Exc.) 9

By taking the result obtained by the multiplication, 15.750, thus including the rejected decimal cipher, the excess is 9.

In testing a quotient, include as a part of the dividend any decimal ciphers that need to be annexed (even when not written) to produce the quotient.

$$\begin{array}{r} 14)\$15.75 \\ \hline \$1.125 \end{array}$$

When a number contains two or more terminal decimal ciphers, reject them by twos. The excess in 78.0 is the same as in 78,000.

$$\begin{array}{rl} 1.625 & \text{(Exc.) } 8 \\ \times 48 & \text{(Exc.) } 4 \\ \hline 78.000 & \text{(Exc.) } 10 \end{array}$$

SIGHT EXERCISES

1. Give the excess of 11's in each of the following products when it contains all of the figures of the result.

a 13764.83	*b* 20306.5	*c* .035007
d 20403.14	*e* 42631.4	*f* .002464
g 31060.25	*h* 56009.8	*i* .437692

2. Give the excess of 11's in each of the following products from which one (or three) terminal decimal ciphers has been dropped:

a 3674.183	*b* 563.0062	*c* .703005
d 4140.002	*e* 431.6024	*f* .040604
g 5206.301	*h* 890.0056	*i* .269473

3. Give the excess of 9's in the previous sets.

CHAPTER EIGHT

MULTIPLICATION

ONE FACTOR AN INTEGER

SIGHT DRILLS

	A	B	C	D	E	F	G	H	I	J
a	9	12	21	35	43	51	62	72	80	99
b	8	13	23	34	45	52	65	74	81	98
c	7	14	22	33	44	53	63	75	82	97
d	6	15	25	32	41	54	61	73	83	96
e	5	16	24	31	42	55	64	71	84	95

1. Multiply by 2, by 3, by 4, by 5, by 6, by 7, by 8, by 9.

Give answers rapidly (*a*) by columns. (*b*) By lines.

2. Multiply by 15: (*a*) 6, (*b*) 12, (*c*) 21, (*d*) 22, (*e*) 44, (*f*) 55

3. Multiply by 21: (*a*) 8, (*b*) 13, (*c*) 24, (*d*) 32, (*e*) 41, (*f*) 44

ORAL PROBLEMS

1. In a year of 52 weeks how much does a person earn who receives (*a*) \$6 a week? (*b*) \$7? (*c*) \$8? (*d*) \$9? (*e*) \$12? (*f*) \$15? (*g*) \$18? (*h*) \$25?

2. How much less than \$1000 a year is the income of a girl earning \$18 a week?

3. What is the weight (*a*) of 120 bushels of potatoes at 60 pounds to the bushel? (*b*) Of 80 bushels of oats at 32 pounds to the bushel?

4. How many yards are there in 12 pieces of lining averaging 45 yards to the piece?

5. How many ounces are there in 11 pounds 4 ounces?

6. Find the cost (*a*) of 84 yards of dress goods at 25 cents a yard. (*b*) Of 25 yards of silk at 88 cents a yard.

7. The sales of a small store average $35 for the six days of a week. If the sales for five days average $30, what is the amount of Saturday's sales?

8. What is the freight on 16 tons at $2¼ a ton?

9. How many square rods are there in a rectangular field 21 rods long and 16 rods wide?

10. Find the volume of a rectangular block of marble whose dimensions are 2¼ ft., by 4 ft., by 3⅓ ft.

SIGNS OF MULTIPLICATION

To indicate that one number is to be multiplied by another, a cross ($\times$) is placed between them.

The expression 3×5 is read "3 times 5," or "3 multiplied by 5."

To indicate that 3¢ is to be multiplied by 5, the expression may be written $3¢ \times 5$, or $5 \times 3¢$. In this second form it is read "5 times 3¢," in order to announce the abstract number as the multiplier. However, in finding the weight of 275 packages of sugar each containing 5 pounds, multiply 275 by 5, even if you arrange the factors in the manner shown herewith.

$$\begin{array}{r} 5 \text{ lb.} \\ \times\ 275 \\ \hline 1375 \text{ lb.} \end{array}$$

Another way to indicate multiplication is to use a period between the factors, writing it above the line,

9·15·17. The period is employed in algebraic work to avoid the use of a sign that might be mistaken for the letter x.

In the European countries that use a comma to denote a decimal point, the period employed as a sign of multiplication is written on the line.

The product of two literal numbers is indicated in algebra by writing the letters together, without an intervening sign. Thus ab means a times b (or b times a). To indicate the product of 2 times the sum of x and 4, the expression takes this form, $2(x + 4)$, no multiplication sign being employed.

The expression $16' \times 3' \times 8''$, used by mechanics in denoting dimensions is read "18 feet *by* 3 feet *by* 8 inches."

WRITTEN EXERCISES

1. (*a*) At \$1.23 a yard, find the cost of 16 yards of silk. (*b*) Find the cost of 123 yards of muslin at 16 cents a yard.

METHOD

(*a*) Since the figures of the multiplicand are small, write the answer directly from the book.

(*b*) Use 16 as the multiplier.

TEST

First write the product of \$1.23 by 4; then multiply this product by 4.

The multiplicand and the multiplier are called the FACTORS of a product.

To obtain a product, use either factor as the multiplier.

2. Write answers directly from the book.

a 16×142	*b* 311×18	*c* 16×420	*d* 322×23
e 21×231	*f* 212×14	*g* 18×333	*h* 221×41
i 32×412	*j* 612×24	*k* 24×444	*l* 202×36

Test products by writing on a second strip the successive products obtained by using the factors of the multiplier.

MULTIPLYING AND ADDING

ORAL PROBLEMS

1. How many quarters are there in 21¼?
2. What is the cost of five pounds of 8-cent sugar and 25 cents' worth of eggs?
3. How many inches are there in 10 feet 11 inches?
4. Change $9\frac{13}{20}$ to an improper fraction.

Each of the foregoing requires the finding of a product and the combination of the latter with a given number.

SIGHT EXERCISES

1. A farmer paid a debt by giving 12 tons of hay at $15 a ton and $20 in cash; how much did he owe?
2. Give the value of the following:

a $16 \cdot 25 + 59$	*b* $34 + (6 \times 50)$	*c* $83 + 12 \times 33\frac{1}{3}$
d $(9 \times 12) + 20$	*e* $(9 \times 12) + 20$	*f* $12\frac{1}{2} \times 48 + 67$

Remember that the product of the numbers connected by a sign of multiplication is to be added to the remaining number.

In *a*, add 59 to the product of 16 by 25; in *c* add 83 to the product of 12 times 33⅓, combining the numbers connected by a sign of multiplication before performing the indicated addition.

In *b* and in *e* the parenthesis is unnecessary, but its use serves to indicate to a person unaware of the effect of the multiplication sign that the numbers within it are to be combined into a single number.

WRITTEN EXERCISES

1. How many square inches are there in the surface of a steel plate containing 8 square feet 96 square inches?

PROCESS

8 sq. ft. 96 sq. in.
1248 sq. in.

There are 144 square inches in a square foot. In 8 square feet there are 8 times 144 square inches. To the product add 96 square inches.

Do this in one operation.

Think 32 (8×4), 38 ("adding-in" 6); write 8.

Think 32 (8×4), 35 (carrying 3), 44 ("adding-in" 9); write 4.

Think 8 (8×1), 12 (carrying 4); write 12.

TEST

Divide 1152 (1248 − 96) by 8.

2. Find the value (*a*) of $(9 \times 86) + 129$. (*b*) Of $237 + (7 \times 97)$.

PROCESS

a $(9 \times 86) + 129$
Ans. 903

b $237 + (7 \times 97)$
916 Ans.

Write the result under the number to be "added-in."

3. Find the value of each of the following:

a $(4 \times 38) + 95$	*b* $77 + (9 \times 83)$	*c* $(10 \times 87) + 261$
d $(5 \times 46) + 87$	*e* $55 + (8 \times 67)$	*f* $(20 \times 43) + 129$
g $(6 \times 54) + 66$	*h* $68 + (7 \times 95)$	*i* $(30 \times 52) + 104$
j $(7 \times 62) + 59$	*k* $86 + (6 \times 79)$	*l* $(40 \times 23) + 115$

SIGHT EXERCISES

1. (*a*) How many times 87 is 261? (*b*) How many times 87 is $(10 \times 87) + 261$?
2. How many times 43 is $(20 \times 43) + 129$?
3. How many times 52 is $(30 \times 52) + 104$?
4. How many times 23 is $(40 \times 23) + 115$?

Business Ways

The pupil who thinks it impossible for him to dispense with any of the crutches he learned to employ in the lower grades should at least become acquainted with the fact that there are other and shorter ways of performing operations. Even if he cannot use all of those suggested, he should occasionally try some of them.

The methods recommended are used by elementary pupils somewhere. They do not include types of combinations that have only a limited application.

MULTIPLIERS OF MORE THAN ONE FIGURE

WRITTEN EXERCISES

1. How many square rods are there in a rectangular plot 87 rods long and 43 rods wide?

PROCESS

One Way

87 (rd.)
× 43 (rd.)
261
348
3741 (sq. rd.) Ans.

Place the right-hand figure of the product by 3 under 3, of the product by 4 under 4. Combine the partial products.

Saving a Line

Write the product by 3.

Under it draw a line, and then multiply 87 by 4 (tens).

87 (rd.)
43 "
261
3741 (sq. rd.)

Instead of writing this partial product, combine it with the first partial product.

First bring down 1, the one's figure of the product by 3.

Think 28 (4 × 7), 34 (adding 6); write 4.

Think 32 (4 × 8), 35 (carrying 3), 37 ("adding in" 2); write 37. Ans. 3741 (sq. rd.)

TEST

Multiply 43 by 87.

2. Find products. Test. Try the short method.

a 94 × 86	*b* 73 × 57	*c* 63 × 84	*d* 78 × 74
e 28 × 69	*f* 94 × 96	*g* 54 × 72	*h* 86 × 36
i 46 × 54	*j* 29 × 27	*k* 87 × 93	*l* 56 × 52
m 68 × 83	*n* 48 × 43	*o* 65 × 26	*p* 39 × 79
q 37 × 67	*r* 35 × 62	*s* 42 × 42	*t* 24 × 58
u 53 × 47	*v* 89 × 38	*w* 76 × 34	*x* 32 × 82

3. Find the cost of 76 acres of land at $85 an acre.

METHOD

$$\begin{array}{r} \$85 \\ 76 \\ \hline 595 \\ \hline \$6460 \text{ Ans.} \end{array}$$

Many accountants begin multiplication with the left-hand figure.

Write the first figure of the product by 7 under 7, etc. Draw a line, multiplying by 6; think 30 (6×5); write 0.

Think 48 (6×8), 51 (carrying 3), 56 (adding 5); write 6. Carry 5 to 59 and write 64.

4. Multiply. Save a line.

a 27×34	*b* 23×37	*c* 75×43	*d* 38×92
e 53×96	*f* 54×57	*g* 45×49	*h* 67×59
i 79×27	*j* 59×98	*k* 56×74	*l* 93×58
m 64×63	*n* 47×64	*o* 82×68	*p* 97×82
q 38×59	*r* 57×47	*s* 32×49	*t* 93×84
u 28×63	*v* 72×74	*w* 67×46	*x* 59×45

Omit as many figures as you can.

aa $28 \times 35 \times 73$	*bb* $24 \times 38 \times 64$	*cc* $76 \times 45 \times$
dd $54 \times 97 \times 29$	*ee* $55 \times 58 \times 76$	*ff* $57 \times 73 \times$
gg $78 \times 26 \times 64$	*hh* $97 \times 95 \times 94$	*ii* $83 \times 69 \times$
jj $65 \times 37 \times 56$	*kk* $48 \times 63 \times 35$	*ll* $33 \times 48 \times$
mm $39 \times 58 \times 43$	*nn* $58 \times 46 \times 73$	*oo* $68 \times 74 \times$

5. Edward Regan bought 127 covers from Mr. Plumridge at $2.85 apiece. (*a*) How much was paid? (*b*) What would be the cost of 712 covers?

PROCESS

(*a*) \$2.85
× 127
1995
\$361.95 Ans.

First multiply by 7, then by 12 (tens); combine this product with the first.

Multiply by 12; then by 7 (hundreds). Bring down 20, the ones and the tens of the product by 12.

(*b*) \$2.85
712
3420
\$2019.20

Note in (*b*) that 5, the right-hand figure of the product of 7 times 5, belongs in the hundreds' place (under the 7); hence, combine it with 4 of the first partial product, to make 39; etc.

6. Find products. Try to limit the number of figures you use.

a 127 × 195	*b* 138 × 234	*c* 149 × 321
d 116 × 586	*e* 129 × 639	*f* 117 × 845
g 316 × 432	*h* 169 × 543	*i* 178 × 232
j 712 × 591	*k* 912 × 396	*l* 711 × 458
m 611 × 685	*n* 512 × 345	*o* 812 × 942
p 134 × 347	*q* 615 × 235	*r* 184 × 374
s 414 × 253	*t* 161 × 357	*u* 418 × 224
v 156 × 624	*w* 417 × 192	*x* 149 × 328

Learn to multiply numbers below 10 by numbers below 20.

aa 128 × 196 × 163	*bb* 117 × 537 × 152
cc 812 × 642 × 711	*dd* 314 × 628 × 143

7. A school used in a year 134 gross of pens. (*a*) How many pens were used? (*b*) How many pens are there in 56 gross?

METHOD

(*a*) 134 (gro.) × 144

1608 (12 × 134)

19296 (pens) (12 × 1608) Ans.

(*b*) 144 pens × 56

1008 (7 × 144)

8064 pens (8 × 1008) Ans.

In (a) use the factors 12 and 12; in (*b*) use 7 and 8.

TEST

(*a*) 134 (gro.) × 144

2144 (16 × 134)

× 9

19,296 (pens) (9 × 2144)

(*b*) 144 pens × 56

1152 (8 × 144)

8064 pens (7 × 1152) Ans.

In (*a*) use 16 and 9 as factors; in (*b*) reverse the order of the factors.

Do not write the expressions in parenthesis (12 × 134), (12 × 1608), (7 × 144), etc.

Some accountants think that they save time by the employment of the factors of the multiplier. These can frequently be used to advantage in testing a product obtained in another way.

8. Find products, using factors:

a 32 × 647	*b* 56 × 389	*c* 52 × 587	*d* 36 × 328
e 64 × 267	*f* 72 × 456	*g* 63 × 195	*h* 28 × 419
i 96 × 853	*j* 78 × 323	*k* 54 × 635	*l* 42 × 926

9. An agent bought 308 horses at an average of $209 a head. How much did they cost?

One Way

$209
308
1672
627
$64,372 Ans.

Place the right-hand figure of the product by 8 under 8, of the product by 3 under 3. Ignore the cipher in the multiplier.

The Other Way

Bring down 72. Multiply 209 by 3 and "add in" 16, the remaining figures of the first partial product.

Test by multiplying 308 by 209.

$209
308
1672
$64,372 Ans.

10. Find products:

a 709 × 805	*b* 208 × 906	*c* 307 × 709
d 406 × 608	*e* 505 × 409	*f* 609 × 209
g 708 × 507	*h* 807 × 306	*i* 906 × 408
j 706 × 409	*k* 607 × 407	*l* 306 × 603
m 405 × 907	*n* 504 × 506	*o* 704 × 407
p 609 × 208	*q* 805 × 508	*r* 809 × 908
s 906 × 305	*t* 207 × 702	*u* 908 × 607

11. Find the cost (*a*) of 213 acres of land at \$164 an acre. (*b*) Of 321 acres at \$416 an acre.

A Short Way

(*a*) \$164
×213
492
3444
\$34,932

Multiply by 3, placing the right-hand figure of the product under 3. Obtain the product by 21 (tens) by multiplying 492 by 7 (tens). Why? Place the right-hand figure of this product under 1.

(*b*) Multiply by 3 (hundreds), placing the right-hand figure of the product under 3. Obtain the product by 21 by multiplying 1248 by 7. Place the right-hand figure of the product under 1.

(*b*) \$416
× 321
1248
8736
\$133,536 Ans.

A Shorter Way

(*a*) \$164
× 213
492
\$34,932 Ans.

Bring down 2, then prefix 7 (tens) times 492 with 49 (tens) "added in."

(*b*) Combine 1248 (hundreds) with 7 times 1248.

b \$416
× 321
1248
\$133,536 Ans.

12. Find products:

a 122 × 189	*b* 123 × 279	*c* 153 × 355	*d* 246 × 497
e 212 × 918	*f* 312 × 927	*g* 315 × 535	*h* 624 × 749
i 427 × 568	*j* 567 × 364	*k* 459 × 328	*l* 287 × 546
m 742 × 856	*n* 756 × 436	*o* 945 × 832	*p* 728 × 564

13. Find the cost of 3868 pounds of Rio coffee at 12.84 cents a pound.

PROCESS

3868 $.1284 46416 $496.6512	Exc. 7 Exc. 8 Exc. 1	Exc. 1	Write 12.84¢ as dollars, four decimal places. Although it is a concrete number, use it as the multiplier.

First multiply by 12, placing the right-hand figure of the product under 2. Multiply this product by 7, placing the right-hand figure of the product under 4. During the multiplication "add in" 46416.

In giving the answer retain only the two decimal places representing cents.

TEST BY CASTING OUT 11'S

Casting out 11's, the excess in the multiplicand is 7; in the multiplier, 8; the product of these excesses is 56, of which the excess is 1. The excess of the product, 4966512, is 1.

In finding this last excess include 12, the figures canceled in giving the answer.

14. Multiply. Test by reversing the factors.

a 1177 × 4206	*b* 8407 × 1272	*c* 12,144 × 11,132
d 3612 × 1296	*e* 8811 × 1248	*f* 12,132 × 11,088

15. At an average of 765 pounds to the acre, what will be raised (*a*) on 18 acres? (*b*) On 81 acres? (*c*) On 102 acres? (*d*) On 201 acres? (*e*) On 316 acres?

PROCESS

When there is a 1 in the multiplier, write the latter alongside the multiplicand, and make it one of the partial products without writing it a second time. Do this whether you "add-in" the second partial product or not.

(*a*) 18 × 765 lb.
13,770 lb. Ans.

(*b*) 81 × 765 lb.
61,965 lb. Ans.

(*c*) 102 × 765 lb.
78,030 lb. Ans.

(*d*) 201 × 765 lb.
153,765 lb. Ans.

(*e*) 316 × 765 lb.
2295
4590
241,740 lb. Ans.

In (*e*) use 765 for the first partial product of 765 by 1 (ten). Multiply by 3 (hundred) for the second partial product. Multiply this by 2 for the third partial product, placing the right-hand figure in the ones' column.

Accustom yourself to begin with any figure of the multiplier. There are advantages at times in beginning with the left-hand figure rather than with the ones' figure.

16. Multiply:

a 879 × 17	*b* 789 × 107	*c* 978 × 316
d 786 × 71	*e* 687 × 701	*f* 876 × 613

17. What is the weight of a piece of armor plate containing 374 cubic feet, at the rate of 476 pounds to the cubic foot?

	PROCESS
476 lb. × 374 1904 3332 1428 178,024 lb. Ans.	When you require three partial products in performing a multiplication do not "add in." Check the product by reversing the factors or by casting out 11's.

18. Multiply. Test by casting out 11's:

a 379 × 286	*b* 973 × 421	*c* 845 × 754	*d* 417 × 278
e 536 × 768	*f* 784 × 230	*g* 903 × 518	*h* 623 × 319
i 473 × 926	*j* 245 × 397	*k* 724 × 839	*l* 956 × 680
m 638 × 817	*n* 413 × 947	*o* 365 × 365	*p* 429 × 398
q 219 × 609	*r* 937 × 346	*s* 158 × 415	*t* 538 × 709
u 498 × 794	*v* 734 × 457	*w* 673 × 295	*x* 462 × 897

ONE FRACTIONAL FACTOR

DRILL EXERCISES

1. Give answers:

a ¼ of 84	*b* 96 × ⅛	*c* ⅓ of 54	*d* 78 × ½
e ¾ " 48	*f* 64 × ⅜	*g* ⅔ " 36	*h* 84 × 7/12
i ⅙ " 84	*j* 48 × ⅝	*k* ⅚ " 72	*l* 60 × 11/12
m ⅝ " 48	*n* 56 × ⅞	*o* ⅝ " 81	*p* 96 × 1/16
q ⅓ " 84	*r* 96 × 1/12	*s* ⅞ " 45	*t* 80 × 3/16
u ⅔ " 96	*v* 48 × 5/12	*w* ⅝ " 63	*x* 64 × 5/16

2. Multiply:

a ¼ of 83	*b* 97 × ⅛	*c* 7/12 of 13	*d* 15 × 7/9
e ¾ " 21	*f* 33 × ⅜	*g* 11/12 " 11	*h* 11 × 5/9
i ⅙ " 95	*j* 17 × ⅝	*k* 1/9 " 82	*l* 85 × 1/16
m ⅚ " 11	*n* 13 × ⅞	*o* 2/9 " 43	*p* 27 × 3/16
q ⅓ " 88	*r* 95 × 1/12	*s* 4/9 " 16	*t* 15 × 5/16
u ⅔ " 35	*v* 13 × 5/12	*w* 5/9 " 17	*x* 13 × 7/16

SIGHT EXERCISES

1. What is the cost of (*a*) 12 pounds of sugar at 7¾¢? (*b*) Of 4½ pounds of meat at 32¢?

PROCESS

(*a*) Think 3 (¼ of 12), 9 (3 times 3). Think 84 (7 times 12), 93 (carrying 9). Write 93. 93¢ Ans.

(*b*) Think 128 (4 × 32), 144 (carrying 16, ½ of 32). Write 144. $1.44 Ans.

2. Give products:

a 66 × 1½	*b* 1½ × 52	*c* 63 × 1⅓
d 84 × 1⅓	*e* 1¼ × 84	*f* 66 × 1⅙
g 56 × 1¼	*h* 1⅕ × 60	*i* 84 × 1⅐
j 20 × 1¾	*k* 1⅛ × 88	*l* 81 × 1⅑
m 24 × 1⅜	*n* 1⅑ × 81	*o* 36 × 1⅑
p 48 × 1⅜	*q* 1⅑ × 45	*r* 54 × 1⅙
s 32 × 1⅑	*t* 1⅕ × 56	*u* 40 × 1⅖
v 16 × 1⅞	*w* 1⅐ × 27	*x* 60 × 1⅗

3. Multiply:

a 20 × 1⅑	*b* 14 × 2½	*c* 15 × 2½
d 17 × 1⅛	*e* 15 × 3⅓	*f* 16 × 3⅓
g 19 × 1⅙	*h* 16 × 2¼	*i* 17 × 2¼
j 22 × 1⅕	*k* 30 × 3⅕	*l* 21 × 3⅕

m $23 \times 1\frac{1}{4}$	*n* $18 \times 2\frac{1}{6}$	*o* $13 \times 2\frac{1}{6}$
p $37 \times 1\frac{1}{10}$	*q* $16 \times 3\frac{1}{8}$	*r* $17 \times 3\frac{1}{8}$
s $25 \times 1\frac{1}{12}$	*t* $27 \times 2\frac{1}{9}$	*u* $28 \times 2\frac{1}{9}$
v $33 \times 1\frac{1}{16}$	*w* $20 \times 4\frac{1}{10}$	*x* $21 \times 4\frac{1}{10}$

WRITTEN EXERCISES

1. How many pounds of flour are there in 18 bags containing 24½ pounds each?

PROCESS

Think 9 (18 halves).
Think 72 (18 times 4), 81 (carrying 9); write 1.
Think 36 (18 times 2), 44 (carrying 8); write 44.
441 lb. Ans.
Test by using 2 and 9 as factors.

NOTE: By taking 2 as the first factor, the first product is an integer.

2. Write answers from the book. Test:

a $12 \times 110\frac{1}{2}$	*b* $110\frac{1}{4} \times 24$	*c* $8 \times 112\frac{3}{4}$	*d* $231\frac{1}{3} \times 15$
e $10 \times 346\frac{3}{5}$	*f* $108\frac{1}{4} \times 20$	*g* $6 \times 109\frac{2}{3}$	*h* $322\frac{2}{3} \times 18$

3. Mr. Schlaefer raised an average of $112\frac{3}{4}$ bushels of potatoes to the acre. What was the yield of a 24-acre field at that rate?

PROCESS

$112\frac{3}{4}$ bu.
$\times 24$
2706 bu. Ans.

Think 18 (¾ of 24)
Think 48 (24 × 2), 66 (carrying 18). Write 6.
Think 24 (24 × 1), 30 (carrying 6); write 0.
Think 24 (24 × 1), 27 (carrying 3); write 27.

4. Write answers from the book:

a $8 \times 246\frac{1}{2}$	*b* $246\frac{1}{2} \times 12$	*c* $212\frac{1}{4} \times 24$
d $9 \times 246\frac{3}{8}$	*e* $325\frac{1}{3} \times 12$	*f* $111\frac{1}{2} \times 32$
g $6 \times 246\frac{5}{8}$	*h* $418\frac{5}{6} \times 12$	*i* $101\frac{1}{8} \times 48$
j $8 \times 275\frac{3}{4}$	*k* $522\frac{3}{4} \times 12$	*l* $321\frac{1}{6} \times 18$
m $8 \times 317\frac{5}{8}$	*n* $607\frac{3}{8} \times 12$	*o* $321\frac{1}{5} \times 15$

5. How many yards are there in 29 pieces of gingham averaging $35\frac{3}{4}$ yards to the piece?

PROCESS

$35\frac{3}{4}$ yd.
$\times 29$
$4)87$
$21\frac{3}{4}$
315
70
$1036\frac{3}{4}$ yd. Ans.

Multiply $\frac{3}{4}$ by 29, by multiplying 29 by 3 and dividing the product by 4 (do not write 4). To the result, $21\frac{3}{4}$, add 9 times 35, and 2 (tens) times 35.

TEST

Multiply 29 by 36, and from the product deduct $\frac{1}{4}$ of 29.

$29 \times 35\frac{3}{4}$

174	Product by	6
1044	" "	36
$-\ 7\frac{1}{4}$	" "	$\frac{1}{4}$
$1036\frac{3}{4}$	" "	$35\frac{3}{4}$

6. Multiply:

a $45\frac{3}{8} \times 79$	*b* $74 \times 73\frac{3}{4}$	*c* $16\frac{3}{8} \times 129$
d $32\frac{3}{8} \times 83$	*e* $53 \times 81\frac{5}{8}$	*f* $17\frac{3}{8} \times 135$
g $63\frac{3}{8} \times 67$	*h* $65 \times 64\frac{4}{5}$	*i* $18\frac{5}{8} \times 215$
j $24\frac{3}{4} \times 55$	*k* $81 \times 43\frac{5}{8}$	*l* $19\frac{7}{8} \times 223$
m $58\frac{3}{8} \times 91$	*n* $95 \times 52\frac{5}{8}$	*o* $15\frac{3}{8} \times 329$

7. What is the cost (*a*) of 123 shares of stock at $\$83\frac{3}{4}$ a share? (*b*) Of 137 shares at $\$85\frac{5}{8}$ a share?

PROCESS

(*a*) 123
$83¾
369 Product by 3
92¼ " " ¾
984 " " 8 (tens)
$10301¼ Ans.

Use the product by 3 as the first partial product. Divide this by 4 for the product by ¾

First multiply 137 by 5, then by ⅝ by dividing the first product by 8. Then multiply by 8 (tens).

(*b*) 137
$85⅝
695 Product by 5
86⅞ " " ⅝
1096 " " 8 (tens)
$11741⅛ Ans.

8. Multiply:

a 13¾ × 25 *b* 126 × 15⅝ *c* 237 × 23⅜
d 27⅞ × 37 *e* 217 × 37⅞ *f* 369 × 42⅝

9. Multiply 267 (*a*) by ⅞. (*b*) By 9⅘. (*c*) By 19⅜. (*d*) By 39¾. (*e*) By 49⅝.

PROCESS

(*a*) 267 × ⅞
Less ⅛ 33⅜
Ans. 233⅝

(*b*) 267 × 9⅘
2670 Product by 10
Less 53⅖ " " ⅕
2616⅖ Ans.

(*c*) 267 × 19⅜
5340 Product by 20
Less 89 " " ⅛
5251 Ans.

(*d*) 267 × 39¾
10680 Product by 40
66¾ " " ¼
10613¼ Ans.

10. Find products:

a $\frac{5}{6} \times 365$	*b* $\frac{3}{4} \times 291$	*c* $9\frac{7}{8} \times 58$	*d* $19\frac{4}{5} \times 98$
e $\frac{7}{8} \times 213$	*f* $\frac{5}{9} \times 577$	*g* $9\frac{4}{5} \times 63$	*h* $29\frac{5}{6} \times 83$
i $\frac{4}{7} \times 415$	*j* $\frac{4}{5} \times 364$	*k* $9\frac{3}{4} \times 95$	*l* $39\frac{3}{8} \times 71$
m $\frac{2}{3} \times 127$	*n* $\frac{11}{12} \times 427$	*o* $9\frac{5}{6} \times 89$	*p* $49\frac{7}{8} \times 65$
q $\frac{9}{10} \times 567$	*r* $\frac{15}{16} \times 336$	*s* $9\frac{1}{2} \times 77$	*t* $59\frac{3}{4} \times 77$
u $\frac{7}{8} \times 643$	*v* $\frac{14}{15} \times 616$	*w* $9\frac{2}{3} \times 97$	*x* $69\frac{1}{2} \times 83$

MULTIPLYING FRACTIONS

GENERAL METHOD

WRITTEN EXERCISES

1. How many square yards of oil cloth are there in a piece $57\frac{1}{3}$ yards long and (*a*) $2\frac{5}{8}$ yards wide? (*b*) $3\frac{9}{16}$ yards wide?

PROCESS

(*a*) $57\frac{1}{3} \times 2\frac{5}{8} = \frac{\overset{43}{\cancel{172}}}{\underset{3}{\cancel{3}}} \times \frac{7}{\underset{8}{\cancel{21}}} = \frac{301}{2} = 150\frac{1}{2}$ (sq. yd.) Ans.

Omit the denomination. Change each mixed number to an improper fraction. Cancel. Write the product of the new numerators over the new denominator. Reduce to a mixed number. After the result write *sq. yd.* in a parenthesis.

Use no figures beyond those shown above.

(*b*) $57\frac{1}{3} \times 3\frac{9}{16} = \frac{172}{3} \times \frac{57}{16} = ?$

2. Find products:

a $44\frac{2}{3} \times 9\frac{3}{4}$	*b* $75\frac{7}{8} \times 12\frac{2}{3}$	*c* $124\frac{1}{2} \times 9\frac{2}{3}$
d $18\frac{3}{4} \times 8\frac{1}{2}$	*e* $65\frac{1}{3} \times 10\frac{1}{4}$	*f* $105\frac{1}{8} \times 8\frac{3}{4}$
g $27\frac{1}{5} \times 7\frac{1}{3}$	*h* $59\frac{1}{2} \times 11\frac{1}{6}$	*i* $132\frac{1}{3} \times 7\frac{3}{8}$

SPECIAL METHOD

SIGHT EXERCISES

1. Find $\frac{1}{4}$ (*a*) of $1\frac{3}{5}$. (*b*) Of $2\frac{1}{2}$. (*c*) Of $3\frac{1}{3}$.

PROCESS

(*a*) Change $1\frac{3}{5}$ to $\frac{8}{5}$; $\frac{1}{4}$ of $\frac{8}{5} = \frac{2}{5}$. Ans.
(*b*) Change $2\frac{1}{2}$ to $\frac{5}{2}$; $\frac{1}{4}$ of $\frac{5}{2} = \frac{5}{8}$. Ans.
(*c*) Change $3\frac{1}{3}$ to $\frac{10}{3}$; $\frac{1}{4}$ of $\frac{10}{3} = \frac{10}{12} = \frac{5}{6}$. Ans.

2. Give answers:

a $\frac{1}{2}$ of $1\frac{1}{2}$	*b* $\frac{1}{3}$ of $2\frac{1}{2}$	*c* $\frac{1}{4}$ of $3\frac{1}{2}$	*d* $\frac{1}{3}$ of $4\frac{1}{2}$
e $\frac{1}{2}$ of $1\frac{1}{3}$	*f* $\frac{1}{3}$ of $2\frac{1}{4}$	*g* $\frac{1}{4}$ of $3\frac{1}{5}$	*h* $\frac{1}{8}$ of $5\frac{1}{3}$
i $\frac{1}{2}$ of $1\frac{2}{3}$	*j* $\frac{1}{3}$ of $2\frac{3}{4}$	*k* $\frac{1}{4}$ of $2\frac{1}{2}$	*l* $\frac{1}{8}$ of $6\frac{2}{3}$
m $\frac{1}{2}$ of $1\frac{3}{4}$	*n* $\frac{1}{3}$ of $1\frac{2}{3}$	*o* $\frac{1}{4}$ of $3\frac{1}{4}$	*p* $\frac{1}{8}$ of $7\frac{1}{2}$

3. Write answers from the book:

a $\frac{1}{2}$ of $1\frac{1}{2}$	*b* $\frac{1}{2}$ of $8\frac{1}{2}$	*c* $\frac{1}{2}$ of $9\frac{1}{2}$	*d* $\frac{1}{2}$ of $25\frac{1}{2}$	*e* $\frac{1}{2}$ of $125\frac{1}{3}$
f $\frac{1}{3}$ of $2\frac{1}{2}$	*g* $\frac{1}{3}$ of $8\frac{1}{2}$	*h* $\frac{1}{3}$ of $9\frac{1}{2}$	*i* $\frac{1}{3}$ of $25\frac{1}{2}$	*j* $\frac{1}{3}$ of $125\frac{1}{4}$
k $\frac{1}{4}$ of $3\frac{1}{2}$	*l* $\frac{1}{4}$ of $8\frac{1}{2}$	*m* $\frac{1}{4}$ of $9\frac{1}{2}$	*n* $\frac{1}{4}$ of $25\frac{1}{3}$	*o* $\frac{1}{4}$ of $125\frac{1}{3}$
p $\frac{1}{5}$ of $4\frac{1}{2}$	*q* $\frac{1}{5}$ of $6\frac{1}{2}$	*r* $\frac{1}{5}$ of $7\frac{1}{2}$	*s* $\frac{1}{5}$ of $25\frac{1}{3}$	*t* $\frac{1}{5}$ of $126\frac{1}{5}$

WRITTEN EXERCISES

1. A dealer had four pieces of cloth containing (*a*) $57\frac{1}{2}$ yd., (*b*) $57\frac{1}{3}$ yd., (*c*) $57\frac{1}{4}$ yd., and (*d*) $57\frac{1}{5}$ yd., re-

spectively. He sold ⅓ of (*a*), ¼ of (*b*), ⅛ of (*c*), and 1/16 of (*d*). How many yards of each did he sell?

PROCESS

(*a*) ⅓ of 57½ yd. = 19⅙ yd. Ans.

(*b*) ¼ of 57⅔ yd. = 14⁵⁄₁₂ yd. Ans.

Divide 57⅔ by 4. This gives a quotient of 14 (write 14), and a remainder of 1⅔, or ⅝. ¼ of ⅝ is ⁵⁄₁₂. Write ⁵⁄₁₂.

(*c*) ⅛ of 57¾ yd. = 11¹¹⁄₂₀ yd. Ans.

Write the quotient 11. Reduce 2¾, the remainder, to ¹¹⁄₄. ⅛ of ¹¹⁄₄ is ¹¹⁄₂₀. Write ¹¹⁄₂₀.

(*d*) 1/16 of 57⅜ yd. = 9⅜ yd. Ans.

Write 9, the quotient. Reduce 3⅜, the remainder, to ¹⁸⁄₅. 1/16 of ¹⁸⁄₅ is ⅜. Write ⅜.

TESTS

Check the results by covering the answers. In (*a*) multiply 19⅙ by 3. In (*b*) multiply 14⁵⁄₁₂ by 4. In (*c*) multiply 11¹¹⁄₂₀ by 5. In (*d*) multiply 9⅜ by 6.

2. Write answers from the book:

a ½ of 269½	*b* ⅓ of 374⅔	*c* ¼ of 475½
d ⅕ of 598¾	*e* 1/16 of 675½	*f* 1/17 of 750⅛
g ⅛ of 845½	*h* 1/19 of 932¾	*i* 1/11 of 803½

3. Multiply 365¾ (*a*) by 1¼. (*b*) by 1⅕. (*c*) by 1⅐.

PROCESS

(*a*) $365\frac{3}{4} \times 1\frac{1}{4}$ (*b*) $365\frac{3}{4} \times 1\frac{1}{5}$ (*c*) $365\frac{3}{4} \times 1\frac{1}{7}$

$\frac{1}{4}$ $91\frac{7}{16}$ $\frac{1}{5}$ $73\frac{3}{20}$ $\frac{1}{7}$ $52\frac{1}{4}$

$457\frac{3}{16}$ Ans. $438\frac{3}{10}$ Ans. 418 Ans.

TESTS

(*a*) $91\frac{7}{16} \times 5$ (*b*) $73\frac{3}{20}$ by 6 (*c*) $52\frac{1}{4}$ by 8.

4. Find products:

a $1\frac{1}{2} \times 157\frac{1}{3}$	*b* $1\frac{1}{3} \times 206\frac{1}{2}$	*c* $1\frac{1}{4} \times 314\frac{2}{5}$
d $1\frac{1}{5} \times 408\frac{3}{4}$	*e* $1\frac{1}{6} \times 512\frac{3}{4}$	*f* $1\frac{1}{7} \times 621\frac{1}{8}$
g $1\frac{1}{8} \times 732\frac{1}{2}$	*h* $1\frac{1}{9} \times 815\frac{1}{7}$	*i* $1\frac{1}{10} \times 993\frac{1}{3}$

5. Multiply (*a*) $365\frac{3}{4}$ by $4\frac{1}{2}$. (*b*) $257\frac{1}{5}$ by $5\frac{1}{3}$. (*c*) $189\frac{3}{8}$ by $8\frac{1}{5}$.

PROCESS

(*a*) $365\frac{3}{4}$

$\times \quad 4\frac{1}{2}$

1463 Product by 4

$+182\frac{7}{8}$ " " $\frac{1}{2}$

$1645\frac{7}{8}$ Ans.

(*b*) $257\frac{1}{5}$

$\times \quad 5\frac{1}{3}$

1289 Product by 5

$+ \ 85\frac{14}{15}$ " " $\frac{1}{3}$

$1374\frac{14}{15}$ Ans.

(*c*) $189\frac{3}{8}$

$\times \quad 8\frac{1}{5}$

1515 Product by 8

$+ \ 37\frac{7}{8}$ " " $\frac{1}{5}$

$1552\frac{7}{8}$ Ans.

TESTS

Multiply (*a*) $182\frac{7}{8}$ by 9, (*b*) $85\frac{14}{15} \times 16$, (*c*) $37\frac{7}{8}$ by 41. Why?

6. Multiply $126\frac{1}{2}$ (*a*) by $3\frac{3}{5}$, (*b*) by $4\frac{4}{7}$, (*c*) by $5\frac{5}{9}$.

PROCESS

(*a*) $126\frac{1}{2}$
$\times\ 3\frac{3}{5}$
$379\frac{1}{2}$ Product by 3
$(\frac{1}{5})\ 75\frac{9}{10}$ " " $\frac{3}{5}$
$455\frac{2}{5}$ Ans.

(*b*) $126\frac{1}{2}$
$\times\ 4\frac{4}{7}$
506 Product by 4
$(\frac{1}{7})\ 72\frac{2}{7}$ " " $\frac{4}{7}$
$578\frac{2}{7}$ Ans.

(*c*) $126\frac{1}{2}$
$5\frac{5}{9}$
$632\frac{1}{2}$ Product by 5
$(\frac{1}{9})\ 70\frac{5}{18}$ " " $\frac{5}{9}$
$702\frac{7}{9}$ Ans.

(*a*) $\frac{1}{5}$ of the product by 3 is the product by $\frac{3}{5}$
(*b*) $\frac{1}{7}$ " " " " 4 " " " " $\frac{4}{7}$
(*c*) $\frac{1}{9}$ " " " " 5 " " " " $\frac{5}{9}$

TESTS

(*a*) Multiply $75\frac{9}{10}$ ($\frac{3}{5}$ of $126\frac{1}{2}$) by 6. This gives $\frac{18}{5}$ of it
(*b*) " $72\frac{2}{7}$ ($\frac{4}{7}$ " ") " 8. " " $\frac{32}{7}$ " "
(*c*) " $70\frac{5}{18}$ ($\frac{5}{9}$ " ") " 10. " " $\frac{50}{9}$ " "

$\frac{18}{5} = 3\frac{3}{5}$ $\frac{32}{7} = 4\frac{4}{7}$ $\frac{50}{9} = 5\frac{5}{9}$

7. Find products:

a $3\frac{3}{5} \times 126\frac{1}{3}$	*b* $4\frac{4}{7} \times 126\frac{1}{4}$	*c* $5\frac{5}{8} \times 126\frac{1}{5}$
d $2\frac{2}{7} \times 126\frac{1}{2}$	*e* $6\frac{6}{11} \times 126\frac{1}{6}$	*f* $7\frac{7}{9} \times 126\frac{1}{7}$
g $8\frac{8}{11} \times 126\frac{1}{2}$	*h* $9\frac{9}{11} \times 126\frac{1}{3}$	*i* $2\frac{2}{5} \times 253\frac{1}{2}$
j $3\frac{3}{8} \times 315\frac{5}{7}$	*k* $4\frac{4}{9} \times 407\frac{1}{3}$	*l* $5\frac{5}{9} \times 512\frac{1}{4}$

8. Multiply $215\frac{2}{5}$ (*a*) by $2\frac{3}{4}$. (*b*) By $3\frac{1}{3}$. (*c*) By $5\frac{7}{8}$.

PROCESS

(*a*) $215\frac{2}{5} \times 2\frac{3}{4}$

$646\frac{1}{5}$ Product by 3

Less $53\frac{17}{20}$ " " $\frac{1}{4}$

$592\frac{7}{20}$ Ans.

(*b*) $215\frac{2}{5} \times 3\frac{1}{3}$

$861\frac{3}{5}$ Product by 4

Less $71\frac{1}{5}$ " " $\frac{1}{3}$

$789\frac{2}{5}$ Ans.

(*c*) $215\frac{2}{5} \times 5\frac{7}{8}$

$1292\frac{2}{5}$ Product by 6

Less $26\frac{37}{40}$ " " $\frac{1}{8}$

$1265\frac{19}{40}$ Ans.

9. Multiply:

a $8\frac{2}{3} \times 215\frac{1}{2}$	*b* $9\frac{3}{4} \times 136\frac{2}{3}$	*c* $7\frac{7}{10} \times 257\frac{1}{5}$
d $3\frac{1}{5} \times 427\frac{1}{8}$	*e* $4\frac{5}{6} \times 215\frac{2}{3}$	*f* $5\frac{10}{11} \times 123\frac{1}{2}$
g $5\frac{7}{8} \times 224\frac{3}{5}$	*h* $7\frac{7}{9} \times 152\frac{2}{5}$	*i* $3\frac{11}{12} \times 306\frac{1}{3}$

SOME SHORT METHODS

ALIQUOT PARTS

A number that is a factor of another number is said to be an *aliquot part* of the latter.

Thus 50¢ is an aliquot part of a dollar; 10 days is an aliquot part of a month of 30 days; 6 hours is an aliquot part of a day.

Seventy-five cents, which is $\$\frac{3}{4}$, is called an *aliquant part* of a dollar, although it is an aliquot part of $3. While 20 days is not an aliquot part of a month it is an aliquot part of a year.

In performing computations, any number may be decomposed into others that are aliquot parts.

MULTIPLYING BY ALIQUOT PARTS OF 100

$25 = \frac{100}{4}$ $12\frac{1}{2} = \frac{100}{8}$ $33\frac{1}{3} = \frac{100}{3}$
$16\frac{2}{3} = \frac{100}{6}$ $50 = \frac{100}{2}$ $6\frac{1}{4} = \frac{100}{16}$

SIGHT EXERCISES

1. What is the cost of 25 rugs (*a*) at \$48 each? (*b*) At \$35 each?

PROCESS

(*a*) At \$48 each, 100 rugs, would cost 48 hundred dollars; ¼ of 100 rugs would cost ¼ of 48 hundred dollars or 12 hundred dollars. \$1200 Ans.

(*b*) At \$35 each, 100 rugs would cost 35 hundred dollars; ¼ of 100 rugs would therefore cost ¼ of 35 hundred dollars, or 8¾ hundred dollars. \$875 Ans.

2. Multiply by 25:

a 27	*b* 33	*c* 46	*d* 85	*e* 124	*f* 165
g 38	*h* 49	*i* 83	*j* 96	*k* 169	*l* 205

3. Multiply by 33⅓:

a 27	*b* 15	*c* 17	*d* 29	*e* 31	*f* 154
g 39	*h* 62	*i* 66	*j* 69	*k* 97	*l* 128

4. (*a*) In multiplying 48 by 12½, what fraction of 48 hundred is the result? (*b*) What is ⅛ of 49 hundred? (*c*) What number is equal to ⅛ hundred? (*d*) To ⅜ hun-

dred? (*e*) To ⅜ hundred? (*f*) To ⅝ hundred? (*g*) To ⅝ hundred? (*h*) To ⅞ hundred? (*i*) To ⅞ hundred?

5. Multiply by 12½:

a 24	*b* 32	*c* 25	*d* 34	*e* 169	*f* 249
g 44	*h* 69	*i* 89	*j* 51	*k* 321	*l* 404

6. What is (*a*) ⅛ of 100? (*b*) ⅜? (*c*) ⅝ ? (*d*) ⅞? (*e*) ⅝?

7. Multiply by 16⅔:

a 24	*b* 25	*c* 32	*d* 37	*e* 48	*f* 185
g 19	*h* 29	*i* 50	*j* 66	*k* 69	*l* 241

WRITTEN EXERCISES

1. What is the cost (*a*) of building 25 miles of railroad at $8765 a mile? (*b*) Of building 125 miles?

PROCESS

(*a*) $8765 × 25 = $219125 Ans.

Divide 8765 hundred by 4, which gives 2191¼ hundred. Substitute 25 for ¼ hundred.

(*b*) Since 125 is ⅛ thousand divide 8765 thousand by 8, which gives 1095⅝ thousand. Substitute 625 for ⅝ thousand.

$8765 × 125 = $1095625 Ans.

Write answers directly from the book:

2. Multiply by 25:

a 1625	*b* 3463	*c* 2345	*d* 1296	*e* 3579	*f* 1143
g 2305	*h* 4425	*i* 3617	*j* 1234	*k* 4061	*l* 3582

3. Multiply by 125:

a 1627	*b* 3565	*c* 2347	*d* 1298	*e* 5064	*f* 2076
g 2316	*h* 3666	*i* 2468	*j* 5670	*k* 4492	*l* 3786

4. Multiply by 33⅓:

a 1628	*b* 3464	*c* 2349	*d* 1297	*e* 4565	*f* 1116
g 4538	*h* 4016	*i* 3527	*j* 1357	*k* 2348	*l* 3691

5. Multiply by 12½:

a 1626	*b* 3665	*c* 2248	*d* 1089	*e* 2033	*f* 1854
g 2316	*h* 1444	*i* 1246	*j* 3458	*k* 2244	*l* 5687

6. Multiply by 16⅔:

a 1629	*b* 3352	*c* 2234	*d* 1185	*e* 2468	*f* 1965
g 3427	*h* 2555	*i* 1357	*j* 4569	*k* 3251	*l* 5408

WRITTEN EXERCISES

1. Mr. Sterrett sold 1465 shares of stock in the Corrigan Paper Mill for $175.25 a share. What did he receive?

	PROCESS
1465 175.25 36625 (*a*) $256,741.25 Ans.	Multiply 1465 by 25 by taking ¼ of it. Bring down 25. Find the product of 1465 by 175 by multiplying (*a*) by 7. Add in 366, the remaining figures of the first partial product.

2. Multiply by 17,525:

a 1245 *b* 2467 *c* 3488 *d* 4179 *e* 5234 *f* 6432

3. Multiply by 25,175:

a 2346 *b* 3578 *c* 4569 *d* 5283 *e* 6345 *f* 7543

4. Multiply by 7525:

a 3457 *b* 4689 *c* 5671 *d* 6394 *e* 7456 *f* 8654

5. Multiply by 2575:

a 4568 *b* 5792 *c* 6783 *d* 7405 *e* 8567 *f* 9765

OTHER SHORT METHODS

SIGHT EXERCISES

1. Find the cost of 88 shares of stock (*a*) at \$99 a share. (*b*) At \$99½. (*c*) At \$99¼. (*d*) At \$99⅞.

PROCESS

At \$100 a share, 88 shares would cost 88 hundred dollars. Diminish 88 hundred: in (*a*) by 88; in (*b*) by 44, ½ of 88; in (*c*) by 22, ¼ of 88; in (*d*) by 11, ⅛ of 88.

2. Give products:

a 24×99	*b* $24 \times 99\frac{1}{2}$	*c* $24 \times 99\frac{1}{4}$	*d* $24 \times 99\frac{7}{8}$
e 99×99	*f* $24 \times 99\frac{3}{8}$	*g* $24 \times 99\frac{5}{8}$	*h* $24 \times 99\frac{11}{12}$
i 99×48	*j* $48 \times 99\frac{1}{2}$	*k* $48 \times 99\frac{11}{12}$	*l* $48 \times 99\frac{1}{2}$
m 57×99	*n* $84 \times 99\frac{3}{4}$	*o* $99 \times 99\frac{5}{8}$	*p* $66 \times 99\frac{3}{8}$
q 99×63	*r* $16 \times 99\frac{7}{8}$	*s* $84 \times 99\frac{1}{4}$	*t* $88 \times 99\frac{3}{4}$
u 98×99	*v* $36 \times 99\frac{5}{8}$	*w* $99 \times 99\frac{10}{11}$	*x* $72 \times 99\frac{5}{8}$

3. Find the cost of 48 yards of ribbon (*a*) at 51¢ a yard. (*b*) At 26¢. (*c*) At 13½¢. (*d*) At 34⅓¢. (*e*) At 17⅔¢.

PROCESS

The price a yard is 1 cent more than $\frac{1}{2}$¢ in (*a*), than $\frac{1}{4}$¢ in (*b*), than $\frac{1}{8}$¢ in (*c*), than $\frac{1}{3}$¢ in (*d*), and than $\frac{1}{6}$¢ in (*e*).

Add, therefore, 48¢ to $\frac{1}{2}$ of $48 in (*a*), to $\frac{1}{4}$ of $48 in (*b*), to $\frac{1}{8}$ of $48 in (*c*), to $\frac{1}{3}$ of $48 in (*d*), and to $\frac{1}{6}$ of $48 in (*e*).

4. Give products:

a 86×51	*b* $88 \times 13\frac{1}{2}$	*c* $17\frac{2}{3} \times 24$	*d* $36 \times 34\frac{1}{3}$
e 26×84	*f* $32 \times 13\frac{1}{2}$	*g* $17\frac{2}{3} \times 72$	*h* $99 \times 34\frac{1}{3}$
i 46×51	*j* $64 \times 13\frac{1}{2}$	*k* $17\frac{2}{3} \times 42$	*l* $69 \times 34\frac{1}{3}$
m 26×24	*n* $96 \times 13\frac{1}{2}$	*o* $17\frac{2}{3} \times 54$	*p* $39 \times 34\frac{1}{3}$
q 51×72	*r* $72 \times 13\frac{1}{2}$	*s* $17\frac{2}{3} \times 66$	*t* $66 \times 34\frac{1}{3}$
u 28×26	*v* $56 \times 13\frac{1}{2}$	*w* $17\frac{2}{3} \times 78$	*x* $96 \times 34\frac{1}{3}$

5. What is the cost of 48 yards of embroidery (*a*) at 49¢ a yard? (*b*) At 24¢? (*c*) At 11½¢? (*d*) At 32⅓¢? (*e*) At 15⅔¢?

PROCESS

The price a yard is 1 cent less than $\frac{1}{2}$¢ in (*a*), than $\frac{1}{4}$¢ in (*b*), than $\frac{1}{8}$¢ in (*c*), than $\frac{1}{3}$¢ in (*d*), and than $\frac{1}{6}$¢ in (*e*).

Deduct, therefore, 48¢ from $\frac{1}{2}$ of $48 in (*a*), from $\frac{1}{4}$ of $48 in (*b*), from $\frac{1}{8}$ of $48 in (*c*), from $\frac{1}{3}$ of $48 in (*d*) and from $\frac{1}{6}$ of $48 in (*e*).

6. Give products:

a 86×49	*b* $88 \times 11\frac{1}{2}$	*c* $15\frac{2}{3} \times 72$	*d* $36 \times 32\frac{1}{3}$
e 24×84	*f* $32 \times 11\frac{1}{2}$	*g* $15\frac{2}{3} \times 42$	*h* $99 \times 32\frac{1}{3}$
i 46×49	*j* $64 \times 11\frac{1}{2}$	*k* $15\frac{2}{3} \times 24$	*l* $69 \times 32\frac{1}{3}$
m 24×24	*n* $96 \times 11\frac{1}{2}$	*o* $15\frac{2}{3} \times 54$	*p* $39 \times 32\frac{1}{3}$
q 72×49	*r* $72 \times 11\frac{1}{2}$	*s* $15\frac{2}{3} \times 66$	*t* $66 \times 32\frac{1}{3}$
u 24×28	*v* $56 \times 11\frac{1}{2}$	*w* $15\frac{2}{3} \times 78$	*x* $96 \times 32\frac{1}{3}$

WRITTEN EXERCISES

1. Find the area of a rectangle 344 yards long (*a*) 99 yards wide, (*b*) 97 yards wide, (*c*) 95 yards wide.

PROCESS

(*a*) 344 (yd.) × 99 (yd.)
34056 (sq. yd.) Ans.

Deduct 344 from 100 times 344 without writing the latter product.

In (*b*) deduct 3 times 344 from 100 times 344.
Think 12 (3 × 4) and *8* (writing 8) are 20.

(*b*) 344 (yd.) × 97 (yd.)
33368 (sq. yd.) Ans.

Think 12 (3 × 4), 14 (carrying 2) and *6* (writing 6) are 20.
Think 9 (3 × 3), 11 (carrying 2) and *3* (writing 3) are 14.
Think 1 and *3* (writing 3) are 4. Bring down 3.

2. Multiply:

a 456×99	*b* 98×375	*c* 576×999
d 567×97	*e* 96×486	*f* 389×998
g 678×95	*h* 96×598	*i* 437×997
j 789×94	*k* 95×864	*l* 684×996
m 234×98	*n* 97×864	*o* 886×995

3. At \$248 an acre, find the cost (*a*) of 37½ acres, (*b*) Of 35 acres, (*c*) Of 27½ acres, (*d*) Of 26¼ acres, (*e*) Of 75 acres, (*f*) Of 87½ acres, (*g*) Of 97½ acres, (*h*) Of 62½ acres.

PROCESS

(*a*)	25 A.	\$6200		(*b*)	25 A.	?	
+	12½ "	?		+	10 "	?	
	37½ A.	?	Ans.		35 A.	?	Ans.
(*c*)	25 A.	?		(*d*)	25 A.	?	
+	2½ "	?		+	1¼ "	?	
	27½ A.	?	Ans.		26¼ A.	?	Ans.
(*e*)	100 A.	?		(*f*)	100 A.	?	
−	25 "	?		−	12½ "	?	
	75 A.	?	Ans.		87½ A.	?	Ans.
(*g*)	100 A.	?		(*h*)	50 A.	?	
−	2½ "	?		+	12½ "	?	
	97½ A.	?	Ans.		62½ A.	?	Ans.

In (*a*) find the cost of 12½ A. by taking ½ the cost of 25 A.
In (*b*) find the cost of 10 A. by multiplying \$248 by 10.
In (*c*) the cost of 2½ A. is $\frac{1}{10}$ the cost of 25 A.
etc.

4. Multiply:

a 136 × 37½	*b* 212 × 75	*c* 384 × 87½
d 444 × 43⅓	*e* 516 × 35	*f* 639 × 66⅔
g 712 × 62½	*h* 883 × 45	*i* 969 × 76⅔
j 842 × 47½	*k* 715 × 97	*l* 624 × 17⅞

5. Find the cost of 1347[1] yards of prints (*a*) at 11$\frac{9}{16}$ cents a yard. (*b*) At 10$\frac{3}{8}$ cents.

METHOD

Save time and figures by using aliquot parts of $\frac{9}{16}$ and of $\frac{3}{8}$.

(*a*)	1347½		(*b*)	1347½	
At 11¢	$161.22½		At 10¢	$134.75	
" ½¢	6.73¾	(I)	" ¼	3.36⅞	(III)
" $\frac{1}{16}$¢	84$\frac{7}{32}$	(II)	" ⅛	1.68$\frac{7}{16}$	(IV)
	$168.80 Ans.			$139.80 Ans.	

For (II) take ⅛ of (I) For (IV) take ½ of (III)

6. Find products:

a 2⅜ × 484	*b* 6$\frac{5}{16}$ × 184	*c* 10¾ × 272½
d 3⅝ × 576	*e* 7$\frac{9}{16}$ × 908	*f* 18⅜ × 104½
g 4$\frac{3}{16}$ × 328	*h* 8$\frac{11}{16}$ × 736	*i* 12⅝ × 364⅓
j 5$\frac{7}{16}$ × 254	*k* 9$\frac{13}{16}$ × 864	*l* 16$\frac{9}{16}$ × 406¼

DECIMALS

ONE DECIMAL FACTOR

WRITTEN EXERCISES

1. What is the area of a plot (*a*) 7 rods long 6.85 rods wide? (*b*) 15 rods long 8.4 rods wide?

METHOD

(*a*)	6.85 (rd.)	(*b*)	8.4 (rd.)
	× 7 "		× 15 "
Ans.	47.95 (sq. rd.)	Ans.	126.0 (sq. yd.)

When the product can be written at once, insert the decimal point when it is reached in performing the multiplication.

2. Write answers from the book:

a 8×13.52	*b* 12×2.345	*c* $32 \times .0204$
d 6×2.345	*e* 15×12.34	*f* 13×120.5
g $5 \times .1768$	*h* 21×3.421	*i* 16×24.31

SIGHT EXERCISES

1. Multiply by 10:

a 34.26	*b* 4.32	*c* .897	*d* .0345	*e* .0059	*f* 4.6
g 19.84	*h* 5.67	*i* .603	*j* .0567	*k* .0006	*l* 5.2

2. Multiply by 100:

a 62.43	*b* 9.84	*c* .789	*d* .0534	*e* .0095	*f* 6.4
g 35.18	*h* 5.23	*i* .264	*j* .0402	*k* .0007	*l* 8.3

3. Multiply by 1000:

a 26.43	*b* 8.49	*c* .978	*d* .0435	*e* .0062	*f* 5.7
g 17.09	*h* 3.46	*i* .104	*j* .0926	*k* .0004	*l* 9.1

WRITTEN EXERCISES

1. The coin value of a franc is \$.193. What is the value (*a*) of 10 francs? (*b*) Of 20 francs? (*c*) Of 60 francs?

PROCESS

(*a*) To multiply \$.193 by 10, shift the decimal point one place to the right. For (*b*) multiply by 2 the result obtained in (*a*).

2. Write answers from the book:

a 40×3.1416	*b* $\$4.8665 \times 100$	*c* 70×2.345
d $50 \times .1975$	*e* $.3937 \times 200$	*f* 80×13.81
g 60×123.4	*h* 2.2046×300	*i* 90×347.2

3. (*a*) How many pounds are there in 900 kilos of 2.2046 pounds each? (*b*) Find the coin value of £4000 at \$4.8665 to the £.

METHOD

(*a*)	$\cancel{2.}20.46$ lb.	(*b*)	$\$4\cancel{.}866.5$
	$\times\ 9\cancel{0}\cancel{0}$		$4\cancel{0}\cancel{0}\cancel{0}$
	1984.14 lb. Ans.		$\$19466.\cancel{0}$ Ans.

In (*a*), multiply 2.2046 by 100 by shifting the decimal point two places to the right. Multiply the changed multiplicand by 9.

In (*b*) multiply 4.8665 by 1000 by shifting the decimal point three places to the right. Multiply the changed multiplicand by 4. Cancel the decimal cipher.

4. Multiply. Test:

a 900×174.9862	*b* 1200×14.756	*c* 700×1374.64
d $2400 \times .02345$	*e* 600×24.385	*f* 3000×3.0098

MULTIPLYING DECIMALS

WRITTEN EXERCISES

1. Multiply (*a*) $.243 \times .37$. (*b*) 3.65×8.6.

Changing the decimals to common fractions the problem becomes

$$(a)\ \frac{243}{1000} \times \frac{37}{100} = \frac{8991}{100000} = .08991 \text{ Ans.}$$

$$(b)\ \frac{365}{100} \times \frac{86}{10} = \frac{31390}{1000} = 31.39 \text{ Ans.}$$

The numbers of ciphers in the denominator of the product is equal to the combined number in both factors.

PROCESS

(*a*) .243
× .37
729
.08991 Ans.

Ignoring the decimal points, multiply 243 by 37. Since there are three decimal places in the multiplicand and two in the multiplier, point off five (3 + 2) decimal places in the product. To point off this number, prefix a decimal cipher to the original product.

(*b*) Cut off 3 decimal places. Cancel the terminal cipher

(*b*) 3.65
× 8.6
2920
31.39ø Ans.

TEST

As a preliminary to the test of (*b*) note that the product must be more than 8 times 3 and less than 9 times 4.

2. Find products. Test:

a 7.29 × 4.8 *b* 7.0234 × 819 *c* 25.09 × 8.32
d .0385 × 7.2 *e* 6.1408 × .042 *f* 531.75 × .484

3. Multiply 513.583 (I) by 3.25, (II) by 23.25, (III) by 37.5

METHOD

(I) *a* 513.583 × 3.25
b 128.39575 Product by ¼
Ans. 1669.14475 " of (*a* by 3) + *b*

In doing the work substitute ¼ for .25

(II)
513.583
23.25
128.39575
1540.749
10271.66

When the multiplier is a mixed decimal containing such decimals as .25, .125, etc., some accountants prefer to place the latter beyond the last figure of the multiplicand, as they do in writing a mixed number. In this case they insert the decimal point in each partial product.

(III) *a* 513.583 × 37.5 (300/8)
b 154074.9 Product of *a* by 300
19259.3625 " " *b* " ⅛

Test by using aliquot parts, multiplying first by 25, then by 12½.

4. Multiply:

a .864 × .625	*b* 84.624 × 33⅓	*c* 78.6 × .135
d 97.25 × .23	*e* 14.9375 × 34⅓	*f* 437 × .995
g 27.055 × 3.4	*h* 8312.34 × .99	*i* .115 × 6.288
j .875 × 3.08	*k* 99.7 × 6.086	*l* 95.704 × 32⅓
m 88.5 × .6248	*n* 74.2 × .375	*o* 437.6 × .099
p 18.75 × 2.94	*q* 63.8 × .0495	*r* 28.4 × .076

MULTIPLYING DENOMINATE NUMBERS

WRITTEN EXERCISES

1. How many bushels of apples are there in 15 barrels averaging 2 bushels 3 pecks each?

METHOD

2 bu. 3 pk.
× 15
41 bu. 1 pk.

Think 45 pk. (15 × 3 pk.), 11 bu. 1 pk. (changing to bushels and pecks). Write 1 pk.
Think 30 bu. (15 × 2 bu.), 41 bu. (carrying 11 bu.) Write 41 bu. Ans. 41 bu. 1 pk.

2. Write answers from the book:

a 3 lb. 9 oz. × 4 | *b* 8 bu. 2 pk. × 3 | *c* 4 pk. 3 qt. × 6 | *d* 5 gal. 3 qt. × 5

e £4 8s. × 7 | *f* 5s. 6d. × 8 | *g* 6 qt. 1 pt. × 9 | *h* 3 ft. 4 in. × 10

3. A family uses 2 quarts 1 pint of milk a day. How much does it use (*a*) in a week? (*b*) During June?

4. Write products from the book:

a 3 lb. 10 oz. × 18 | *b* 3 qt. 1 pt. × 16 | *c* 3 yd. 1 ft. × 24 | *d* 46s. × 12

e 10s. 6d. × 18 | *f* 1 bu. 3 pk. × 20 | *g* 3 pk. 4 qt. × 15 | *h* 2 ft. 6 in. × 40

CHAPTER NINE

DIVISION

DIVIDING BY AN INTEGER

SIGHT DRILLS

	A	B	C	D	E	F	G	H
a	45	54	102	104	105	106	108	140
b	56	65	110	111	114	116	117	143
c	69	76	118	119	120	121	123	144
d	78	87	125	126	128	130	132	145
e	86	98	133	135	135	136	138	147

1. Name rapidly (I) by columns, (II) by lines, the multiples (*a*) of 2. (*b*) Of 4. (*c*) Of 8. (*d*) Of 3. (*e*) Of 9. (*f*) Of 6. (*g*) Of 5. (*h*) Of 7. (*i*) Of 11.

2. State (I) by columns, (II) by lines, the remainders when the following numbers are divided: (*a*) By 3. (*b*) By 5. (*c*) By 9. (*d*) By 11. (*e*) By 4. (*f*) By 8. (*g*) By 25.

	A	B	C	D	E	F	G	H
a	345	554	102	504	905	506	109	543
b	656	165	210	611	814	416	217	677
c	469	987	325	729	623	321	423	709
d	278	887	425	826	735	239	332	821

3. Give two factors of:

a	65	*b*	69	*c*	86	*d*	133	*e*	106	*f*	111
g	87	*h*	46	*i*	57	*j*	119	*k*	143	*l*	121
m	51	*n*	95	*o*	62	*p*	145	*q*	134	*r*	118
s	93	*t*	58	*u*	91	*v*	158	*w*	155	*x*	146

ORAL PROBLEMS

1. In how many weeks will a man earn \$1440 when his weekly earnings are (*a*) \$15? (*b*) \$16? (*c*) \$18? (*d*) \$24? (*e*) \$30? (*f*) \$36?

2. How many pounds are there in (*a*) 96 oz.? (*b*) 144 oz.? (*c*) 80 oz.? (*d*) 112 oz.? (*e*) 176 oz.? (*f*) 128 oz.?

3. How many feet are there in (*a*) 132 in.? (*b*) 168 in.? (*c*) 96 in.? (*d*) 192 in.? (*e*) 288 in.? (*f*) 960 in.?

4. (*a*) At 60 pounds to the bushel, how many bushels of potatoes weigh 9600 pounds? (*b*) How many bushels of oats, weighing 32 pounds to the bushel, will have the same weight?

SIGNS OF DIVISION

To indicate that 15 is to be divided by 3, write 3, the divisor, then a curved line, then the dividend. Write 5, the quotient, underneath. This form may be read "3 into 15 (goes) 5 times."

$$3\overline{)15}$$
$$5$$

Or, write the dividend above a line and the divisor underneath. Follow with a sign of equality and then the quotient. This form may be read "15 over 3 equals 5."

$$\frac{15}{3} = 5$$

Or, write the dividend, then the division sign (÷), then the sign of equality, concluding with the quotient. This form is read "15 divided by 3 equals 5." The other two forms may also be read in this way.

$$15 \div 3 = 5$$

The second form is generally employed when either term is compound or contains literal numbers.

$$\frac{3 \times 16}{2}, \quad \frac{ab}{c}, \quad \frac{y^2 - 16}{y + 4}$$

In many European countries the colon, which in this country is employed to denote ratio, is used as the division sign. In some of them our division sign (÷) indicates subtraction.

SHORT DIVISION

WRITTEN EXERCISES

1. (*a*) When 15 barrels of sugar weigh 4620 pounds, what is the average weight per barrel? (*b*) How many overcoats at \$15 each will cost \$5250?

PROCESS

(*a*) 15)4620 lb. Divide 4620 lb. by 15, the *number* of barrels.
Ans. 308 lb.

(*b*) \$15)\$5250 Divide \$5250 by \$15, the cost of a coat. The quotient, 350, is the *number* of coats. Write *coats* in a parenthesis.
Ans. 350 (coats)

Test each result, covering the dividend with a strip of paper on which you write the product of the quotient by the divisor. Remove the paper and compare this product with the dividend.

Be careful to place the first quotient figure under the right-hand figure of its partial dividend, and to place a quotient figure or cipher under each remaining figure of the dividend.

2. Write quotients from the book. Under each write its product by the divisor:

a 32)1024	*b* 18)5418	*c* 15)3600	*d* 19)4408
e 21)2331	*f* 14)2002	*g* 13)3952	*h* 17)8517
i 16)2128	*j* 24)2952	*k* 23)7590	*l* 25)5375

DIVIDING BY A MULTIPLE OF 10

PREPARATORY EXERCISES

1. At \$200 an acre how many acres of land will cost \$2800?

2. At 2000 pounds to the ton, how many tons are there in 86000 pounds of coal?

3. At 160 square rods to the acre, how many acres are there in 3200 square rods?

4. (*a*) Why is the quotient of $2800 \div 200$ the same as the quotient of $28 \div 2$? (*b*) By what number are both terms of $2800 \div 200$ divided to change them to $28 \div 2$?

> Dividing the divisor and the dividend by the same number makes no change in the quotient.

WRITTEN EXERCISES

1. (*a*) At \$60 each, how many cows will cost \$57,600? (*b*) What is the average cost of a rug when \$52,000 is paid for 800 rugs?

METHOD

(*a*) 6|0)6760|0 (cows) Ans.

(*b*) 8|00)\$520|00 \$ Ans.

In (*a*) divide both terms by 10 by cutting off the final cipher. Divide 5760 by 6.

In (*b*) divide both terms by 100 by cutting off the last two ciphers. Divide \$520 by 8.

2. Write quotients from the book:

a 120)67,800	*b* 800)36,800	*c* 1500)106,500
d 600)45,600	*e* 160)38,560	*f* 4000)292,000

REMAINDERS

PREPARATORY EXERCISES

1. At \$16 each, how many calves can be bought for \$500, and how much money will remain?

2. At \$16 a ton, how much iron ore can be bought for \$500? Give answer as a mixed decimal.

3. How many miles an hour does a train travel when it goes 500 miles in 16 hours? Give answer as a mixed number.

4. If a vessel goes 16 miles an hour, how many hours and minutes will it require to go 500 miles?

5. How many persons can be properly accommodated in a room having 500 square feet of floor space if each is to have at least 16 square feet?

6. How many auto cars are needed to transport 500 soldiers at one time if each can carry only 16 soldiers?

7. Why should the remainder be given in the answer to Ex. 1 and omitted in the answer to Ex. 5?

8. What is done with the remainder in the answer to Ex. 6?

WRITTEN EXERCISES

1. Write answers from the book, giving each quotient as an abstract number and the remainder as a concrete number:

a 17 lb.)8523 lb.	*b* \$14)\$4258	*c* 16 T.)8163 T.
d 15 yd.)4539 yd.	*e* 13 mi.)7973 mi.	*f* 24 A.)7227 A.

2. At 2000 pounds to the ton, give the number of tons and pounds (*a*) in 96,875 pounds of coal. (*b*) In 97,000 pounds. (*c*) In 97,450 pounds.

METHOD

(*a*) 2|000 lb.)96|875 lb.

48(T.)875 lb. Ans.

When 96875 lb. is divided by 1000, there is a remainder of 875 lb. Write this remainder after 48, the quotient of 96 (thousand) divided by 2 (thousand). After 48, the number of tons, write T. in a parenthesis.

(*b*) 2|000 lb.)97|000 lb.

48(T.)1000 lb. Ans.

2 (thousand) is contained in 97 (thousand) 48 times with a remainder of 1 (thousand)

(*c*) 2|000 lb.)97|450 lb.

48(T.)1450 lb. Ans.

Divide 97 by 2, writing 48, the quotient. Prefix 1, the remainder, to 450, the figures of the dividend that are cut off.

To divide an integer by a number ending in one or more ciphers cut off the terminal cipher or ciphers in the divisor and the same number of figures from the right of the dividend. Divide the remaining figures of the dividend by the remaining figures of the divisor. Write as the remainder the figures of the dividend that have been cut off, prefixing the partial remainder, if any, left after performing the division.

3. Find quotients and remainders:

a 120)57650	*b* 700)50200	*c* 1800)203500
d 210)25635	*e* 140)48320	*f* 3200)99805

DECIMAL QUOTIENTS

When the quotient of two integers is to be given as a decimal, continue the division by mentally annexing decimal ciphers to the dividend.

WRITTEN EXERCISES

1. (*a*) I paid \$18 for 8 bushels of potatoes; what was the cost a bushel? (*b*) At \$16 a ton, how many tons of straw can be bought for \$250?

	PROCESS
(*a*) 8)\$18. \$ 2.25 Ans.	Place a decimal point after \$18. Place a decimal point in the quotient under the one in the dividend.
(*b*) 16)250. 15.625(T) Ans.	Place a decimal point after 250. Do not write the decimal ciphers. Place a decimal point in the quotient under the one in the dividend.

2. Write quotients from the book, as mixed decimals:

a 12)246	*b* 8)243	*c* 16)340	*d* 4)3926
e 14)175	*f* 6)201	*g* 18)405	*h* 2)4321

DECIMAL DIVIDENDS

3. Divide (*a*) 24.3 by 8. (*b*) .246 by 12.

PROCESS

(*a*) 8)24.3 — 3.0375 (*b*) 12).246 — .0205

Place a decimal point in the quotient under the one in the dividend. In (*b*) place a cipher under the 2 in the dividend.

4. Write quotients from the book:

a 12)36.6	*b* 8)3.23	*c* 16).344	*d* 9)34.2
e 12)4.86	*f* 8).324	*g* 18)3.69	*h* 4)1.02

5. How many tons are there (*a*) in 96,875 pounds? (*b*) In 97,000 pounds? (*c*) In 97,450 pounds? Give results as mixed decimals.

METHOD

(*a*) 2000)96.875/ Ans. 48.4375 (T.)

(*b*) 2000)97.000/ 48.5 (T.)

(*c*) 2000)97.450/ Ans. 48.775 (T.)

Since the answer is to be given as a decimal, cancel the three ciphers in the divisor (dividing it by 1000) and divide each dividend by 1000 by setting off three decimal places.

To show that the original dividend is an integer, place after it a decimal point, and cancel it when the new one is written.

6. How many tons are there in 14,760 pounds?

7. At 160 square rods to the acre, how many acres are there in 2440 square rods?

8. Express quotients in decimal form:

a 120)426 *b* 800)1244 *c* 16000)4040 *d* 160)808

9. Divide (*a*) 62.4 by 120, (*b*) 14.84 by 16000.

METHOD

Divide the divisor by 10 by canceling the cipher. Divide the dividend by 10 by moving the decimal point one place to the left. Divide the changed dividend by the changed divisor.

(*a*) 12Ø)6.2,4
.52 Ans.

To move the decimal point in the dividend three places to the left, prefix a decimal cipher.

(*b*) 16ØØØ).014,84

10. Express quotients in decimal form:

a 300)4.26 *b* 6000)524.4 *c* 150).606 *d* 90)47.7

FRACTIONAL QUOTIENTS

When the result of a division is to be expressed as a mixed number, write the remainder over the divisor in the form of a fraction.

11. Express each quotient as a mixed number. Write answers from the book.

a 11)37561 *b* 13)29400 *c* 12)39863 *d* 15)46702

12. In the following give the fraction in the quotient in lowest terms. Write answers from the book.

a 14)30107 *b* 16)50100 *c* 21)84639 *d* 18)90372

COMPOUND NUMBER QUOTIENTS

The result of the division of a denominate number by an abstract number may be expressed as a compound number, as shown by the first example (*a*) below.

13. Divide (*a*) 54 bushels by 16. (*b*) 42 yd. 2 ft. 3 in. by 9.

PROCESS

54 bu. divided by 16 gives a quotient of 3 bu. with a remainder of 6 bu. Change the latter to 24 pk. This divided by 16 gives a quotient of 1 pk., with a remainder of 8 pk. Change the latter to 64 qt. This divided by 16 gives a quotient of 4 qt.

(*a*) 16)54 bu.
Ans. 3 bu. 1 pk. 4 qt.

After dividing 42 yd. by 9, and writing the quotient, 4 yd., change 6 yd., the remainder, to 18 ft. To this add 2 ft., making the next dividend 20 ft. Write 2 ft., the quotient, and change the remainder, 2 ft., to 24 in. To this add 3 in., making the next dividend 27 in. Write 3 in., the next quotient.

(*b*) 9)42 yd. 2 ft. 3 in.
Ans. 4 yd. 2 ft. 3 in.

14. Express each quotient as a compound denominate number. Write answers from the book.

a 8)34 lb. *b* 6)39 lb. 6 oz. *c* 12)51bu. 2 pk. 4 qt.

d 9)40 yd. *e* 7)33 gal. 1 qt. *f* 10)95 yd. 1 ft. 10 in.

g 8)37 bu. *h* 5)63 yd. 1 ft. *i* 11)86 gal. 2 qt. 1 pt.

15. Divide 65 yd. 10 in. by 10.

METHOD

10)65 yd. 0 ft. 10 in. Insert the missing denomination.

16. Express quotients as compound numbers.

a 10)62 yd. 8 in. *b* 12)76 bu. 4 qt. *c* 9)73 gal. 1 pt.

FRACTIONAL DIVIDENDS

SIGHT EXERCISES

1. Give quotients.

a $\frac{2}{3} \div 2$	*b* $\frac{4}{3} \div 2$	*c* $1\frac{1}{3} \div 2$	*d* $1\frac{1}{5} \div 2$
e $\frac{3}{4} \div 3$	*f* $\frac{9}{4} \div 3$	*g* $2\frac{1}{4} \div 3$	*h* $1\frac{7}{8} \div 3$
i $\frac{4}{5} \div 2$	*j* $\frac{8}{5} \div 2$	*k* $1\frac{3}{5} \div 2$	*l* $1\frac{3}{5} \div 2$
m $\frac{5}{6} \div 5$	*n* $\frac{10}{3} \div 5$	*o* $3\frac{1}{3} \div 5$	*p* $2\frac{1}{2} \div 5$
q $\frac{3}{8} \div 4$	*r* $\frac{16}{3} \div 4$	*s* $1\frac{7}{8} \div 4$	*t* $3\frac{1}{5} \div 4$

Dividing the numerator divides a fraction.

2. Give answers:

a $\frac{1}{2}$ of $\frac{1}{2}$ = ? *b* $\frac{1}{2}$ of $\frac{3}{2}$ = ? *c* $\frac{1}{2}$ of $1\frac{1}{2}$ = ?

d $\frac{1}{2} \div 2$ = ? *e* $\frac{3}{2} \div 2$ = ? *f* $1\frac{1}{2} \div 2$ = ?

Multiplying the denominator divides a fraction.

3. Give quotients:

a $\frac{1}{3} \div 2$	*b* $\frac{2}{3} \div 2$	*c* $1\frac{1}{3} \div 2$	*d* $1\frac{1}{4} \div 2$
e $\frac{1}{2} \div 3$	*f* $\frac{5}{2} \div 3$	*g* $2\frac{1}{2} \div 3$	*h* $2\frac{1}{3} \div 3$
i $\frac{3}{4} \div 5$	*j* $\frac{4}{3} \div 5$	*k* $1\frac{1}{3} \div 5$	*l* $3\frac{1}{2} \div 5$
m $\frac{5}{6} \div 4$	*n* $\frac{3}{2} \div 4$	*o* $1\frac{1}{2} \div 4$	*p* $3\frac{2}{3} \div 4$
q $\frac{5}{9} \div 6$	*r* $\frac{7}{5} \div 6$	*s* $1\frac{2}{5} \div 6$	*t* $4\frac{3}{5} \div 6$

WRITTEN EXERCISES

1. A farmer raised 1865½ bushels of potatoes on 7 acres. (*a*) What was the average yield an acre? (*b*) Divide 1765½ by 8.

METHOD

(*a*) 7)1865½ bu.
266½ bu. Ans.

Change the remainder, 3½, to the improper fraction, ½. ½ ÷ 7 = 1/14

(*b*) 8)1765½
220 11/16

Change the remainder 5½ to 11/2. 11/2 ÷ 8 = 11/16

2. Divide. Write answers from the book:

a 9)3087 9/10	*b* 8)4456½	*c* 7)5005⅞	*d* 6)2352⅚
e 5)6095½	*f* 4)7348⅗	*g* 3)9423⅔	*h* 2)8932½
i 3)7345⅕	*j* 4)6934⅔	*k* 5)4767½	*l* 6)3872⅖
m 7)2986½	*n* 8)3597⅜	*o* 9)4627⅓	*p* 8)5983¼

In dividing a mixed number by an integer divide the fractional remainder by the divisor when the quotient is to be expressed as a mixed number.

DIVIDING BY FACTORS

WRITTEN EXERCISES

1. A farmer delivered 55,104 pounds of corn to a dealer. (*a*) How many bushels were there at 56 pounds to the bushel? (*b*) Find its value at \$1.47½ a bushel.

METHOD

8)55,104
. 7) 6888
Ans. 984 (bu.)

Instead of dividing by 56, many accountants use the factors. Test the result by covering the dividend, and writing the product of 984 by 8, and this product by 7.

2. Find quotients:

a 59,544 ÷ 72	*b* 68,096 ÷ 64	*c* 91,584 ÷ 96
d 99,616 ÷ 88	*e* 88,816 ÷ 56	*f* 78,197 ÷ 49

3. Divide (*a*) 86,347 by 91 (*b*) 228,338 by 66.

METHOD

(*a*) 7)86,347
13)12,335⅞
948⁷⁄₉₁ Ans.

(*b*) 11)228,338
6)20,758
3459⅝ Ans.

4. Express quotients as mixed numbers:

a 68,551 ÷ 81	*b* 81,745 ÷ 66	*c* 95,239 ÷ 54
d 83,015 ÷ 42	*e* 90,583 ÷ 39	*f* 71,229 ÷ 28

Decimal quotients are frequently limited to two or to three places. In such a case carry out the result to an additional place. When this last quotient figure is less than 5, reject it in stating the result; when it is 5 or more use it to increase the preceding figure by 1.

5. Divide (*a*) 34,885, (*b*) 24,703 by 96, giving quotients to nearest thousandth, to nearest hundredth.

METHOD

(*a*) 8)34,885
12)4360.625
363.3854

(*b*) 8)24,703
12)3087.875
257.3229

(I) Quotient to nearest thousandth
(*a*) 363.385 Ans. (*b*) 257.323 Ans.

(II) Quotient to nearest hundredth
(*a*) 363.39 Ans. (*b*) 257.32 Ans.

6. Give quotients (I) to nearest thousandth, (II) to nearest hundredth.

a 58,843 ÷ 48	*b* 45,678 ÷ 72	*c* 62,943 ÷ 77
d 37,945 ÷ 56	*e* 86,745 ÷ 84	*f* 76,849 ÷ 63

7. Divide (*a*) 86,347 by 91, (*b*) 228,338 by 66; (I) expressing each result as a mixed number, (II) giving quotients and remainders.

METHOD

(*a*) 7)86,347	(*b*) 6)228,338	
13)12,335⅞	11)38,056⅓	
(I)	948$^{7}/_{91}$ Ans.	3459⅓ Ans.
(II)	948; rem. 79 Ans.	3459; rem. 44 Ans.

In (*a*), the denominator of $^{7}/_{91}$, the fraction in the mixed number quotient (I), being the same as the regular divisor, 91, write 79, its numerator, as the remainder in (II).

In (*b*) multiply ⅓, the fraction in the answer (I) by 66, the regular divisor, which gives 44 as the remainder (II).

By omitting to reduce the fractions ⅞ and $^{44}/_{66}$ to lowest terms, the denominator of the latter would correspond with the regular divisor, and its numerator would be the remainder.

(*b*) 6)228,338
11)38,056⅞
3459$^{44}/_{66}$

8. Divide; (I) expressing results as mixed numbers, (II) giving quotients and remainders.

a 68,551 ÷ 81	*b* 83,015 ÷ 42	*c* 94,724 ÷ 48
d 34,672 ÷ 77	*e* 53,219 ÷ 32	*f* 95,239 ÷ 54
g 11,572 ÷ 33	*h* 81,745 ÷ 66	*i* 96,583 ÷ 39
j 48,312 ÷ 42	*k* 42,094 ÷ 84	*l* 71,229 ÷ 28

DIVIDING BY MULTIPLES

PREPARATORY EXERCISES

1. At $1½ each what will be the cost of 12 baseballs?

2. How many baseballs costing 3 half dollars each an be bought for 36 half dollars?

4. At \$7½ a ton, how many tons of coal can be bought (*a*) for \$324? (*b*) For \$318¾?

METHOD

(*a*) 7½)324
15)648
43½ (T.) Ans.

(*b*) 7½)318¾
15)637½
42½ (T.) Ans.

5. Divide. Test:

a 1½)110¼ *b* 1¼)103⅛ *c* 1⅓)120⅝

d 1⅜)120⅝ *e* 1½)123⅜ *f* 2½)163¾

6. Divide 83647 by 65. Give quotient (*a*) correct to nearest hundredth. (*b*) As a mixed number.

METHOD

65)83,647
13∅)16,729.4
1286.88 Ans.

Multiply the divisor and the dividend by 2. Cancel the cipher in the divisor and cut off one decimal place in the dividend.

7. Divide by 35, giving the result correct to the nearest hundredth:

a 17,463 *b* 23,986 *c* 35,207 *d* 43,916 *e* 54,268

8. Divide by 45, giving the quotient as a mixed number:

a 63,482 *b* 74,006 *c* 82,954 *d* 96,875 *e* 83,108

9. Divide by 55, giving the quotient to the nearest thousandth:

a 70,034 *b* 62,158 *c* 51,329 *d* 47,676 *e* 32,983

10. Divide (*a*) 43,816 by 25. (*b*) 3619.4 by 33⅓ (*c*) 260.78 by 12½. (*d*) 16.547 by 16⅔. Give quotients as mixed decimals.

METHOD

(*a*) 25) 43,816
1ØØ)1752.64 Ans.

(*b*) 33⅓)3619.4
1ØØ)144.77⅙ Ans.

(*c*) 12½)260.78
100)16.86¼ Ans.

(*d*) 16⅔)16.547
100).99⅓282 Ans.

Multiply divisor and dividend by 4 in (*a*), by 3 in (*b*), by 8 in (*c*), and by 6 in (*d*). Divide by 100 by shifting the decimal point in each new dividend two places to the left.

11. Divide by 25. Write answers from the book:

a 164.5 *b* 321.25 *c* 2408 *d* 12.34 *e* 3750 *f* 477.5

12. Divide by 16⅔. Write answers from the book:

a 123.5 *b* 506.75 *c* 4802 *d* 40.25 *e* 7250 *f* 307.5

13. Divide by 33⅓. Write answers from the book:

a 106.4 *b* 312.25 *c* 3504 *d* 20.44 *e* 4360 *f* 206.5

14. Divide by 12½. Write answers from the book:

a 125.5 *b* 273.25 *c* 1216 *d* 30.75 *e* 2430 *f* 512.5

15. How many tons of armor plate can be bought for \$55,440 when the price is (*a*) \$175 a ton? (*b*) \$225 a ton? (*c*) \$275 a ton?

METHOD

(*a*) 175) 55,440
700)221,760
Ans.

Multiply the divisor and the dividend by 4. (T.) Give the result as a mixed decimal.

Test the result by multiplying it by 175, using the aliquot part method.

16. Divide by 175:

a 78,995 *b* 86,387 *c* 97,244 *d* 67,011 *e* 59,822

17. Divide by 225:

a 39,798 *b* 28,368 *c* 48,438 *d* 79,326 *e* 65,286

18. Divide by 275:

a 93,071 *b* 85,679 *c* 54,670 *d* 43,692 *e* 74,283

19. How many acres of land can be bought for \$21,000 when the rate is (*a*) \$37½ an acre? (*b*) \$62½ an acre? (*c*) \$87½ an acre?

METHOD

a 37½) 21,000
300)168,000
Ans.

Multiply the divisor and the dividend by 8. (A.)

20. Divide by 37½:

a 24,000 *b* 31,500 *c* 42,360 *d* 54,813 *e* 62,172

21. Divide by 66⅔ (multiply both terms by 3):

a 84,000 *b* 73,200 *c* 96,480 *d* 86,214 *e* 71,203

22. Divide by 62½:

a 63,000 *b* 50,500 *c* 43,280 *d* 32,105 *e* 23,456

23. Divide by 75 (multiply both terms by 4):

a 18,000 *b* 23,400 *c* 36,070 *d* 42,315 *e* 50,306

24. Divide by 87½:

a 63,000 *b* 75,600 *c* 89,530 *d* 97,293 *e* 86,436

25. Divide by 112½:

a 72,000 *b* 60,300 *c* 53,820 *d* 53,065 *e* 32,148

26. Divide by 125. Give the quotient as a mixed decimal. Write answers directly from the book. (Multiply both terms by 8.)

a 14,723 *b* 2345.6 *c* 325.19 *d* 40.876 *e* .51,234

27. Divide by 375:

a 63,273 *b* 76,543 *c* 81,951 *d* 93,252 *e* 87,345

28. Divide by 625. Try to write answers from the book by multiplying both terms by 16:

a 43,210 *b* 32,104 *c* 21,043 *d* 10,234 *e* 54,321

29. Divide by 875:

a 30,387 *b* 26,614 *c* 43,638 *d* 60,501 *e* 10,605

30. Divide by 133⅓:

a 12,345 *b* 23,456 *c* 34,567 *d* 45,678 *e* 56,789

31. Divide by 166⅔. Write answers from the book. (Multiply both terms by 6.)

a 63,421 *b* 70,416 *c* 86,235 *d* 91,314 *e* 82,031

DECIMAL DIVISORS

WRITTEN EXERCISES

1. (*a*) A man pays annually as interest .06 of the sum he borrowed to help to pay for a house. If the interest is $135 a year, how much did he borrow?

(*b*) Divide 8.36 by 1.6

METHOD

(*a*) ,06)$135,00
Ans. $2250

Multiply the divisor by 100 by canceling the decimal point. Multiply the dividend by 100 by annexing two ciphers.

Divide the new dividend by the new divisor.

(*b*) Multiply the divisor by 10 by canceling the decimal point. Multiply the dividend by 10 by shifting the decimal point one place to the left, canceling the original one.

(*b*) 1,6)8,3.6
Ans. 5.225

(*a*) To show the change in the dividend, place a decimal point after 5. Cancel it when annexing the ciphers.

2. Find quotients:

a 1.5)2.97 *b* .16)18 *c* .08)322 *d* .007).1001

e .4)2.762 *f* .09)538.2 *g* 1.1)16.016 *h* .12)836.1

SIGHT EXERCISES

When the decimal divisor is an aliquot part of 1, that is, when it equals ½, ¼, ⅛, 1/16, change it to the equivalent fraction.

1. Give answers:

a 3.1 ÷ .5 *b* 62 ÷ .25 *c* .81 ÷ .125 *d* 2.2 ÷ .0625

2. Divide:

a 3.2 ÷ .4 *b* 21 ÷ .375 *c* 3.5 ÷ .625 *d* .21 ÷ .875

WRITTEN EXERCISES

Change each of the following divisors to a whole number by multiplying it by 2, 4, or 8.

1. Find quotients. Write answers from the book:

a 63.4 ÷ .25 *b* 216 ÷ 2.5 *c* 9.03 ÷ 1.25 *d* 876 ÷ .125

2. Divide:

a 63.5 ÷ .375 *b* 495 ÷ .625 *c* 8.03 ÷ .875 *d* 8.03 ÷ 6.25

e 4.96 ÷ 12.5 *f* 333 ÷ 3.75 *g* 4.05 ÷ 62.5 *h* 119 ÷ 875

DIVIDING BY 99

A computer would not use long division in dividing by 99 or 999.

PREPARATORY EXERCISES

1. Give quotients and remainders:

a 99)100 *b* 99)200 *c* 99)300 *d* 99)500 *e* 99)800

How does each quotient compare with each remainder?

2. In dividing (*a*) 103, (*b*) 305, (*c*) 509, (*d*) 713, (*e*) 937, by 99 how much more than 3 is the remainder in (*a*)? Than 5 is the remainder in (*b*)? Than 9 is the remainder in (*c*)? Than 13 is the remainder in (*d*)? Than 37 is the remainder in (*e*)?

3. In dividing 12 hundred by 99, what is (*a*) the quotient? (*b*) The remainder?

4. In dividing 24 hundred 72 (2472) by 99, (*a*) What is the quotient? (*b*) How much more than 72 is the remainder? (*c*) What is the remainder?

To divide a three- or a four-place number by 99, cut off the two right-hand figures (tens and ones). The remaining figures give the quotient; to obtain the remainder add the quotient figures to those cut off.

SIGHT EXERCISES

1. (*a*) At 99 cents each, now many baseballs can be bought for $12.75, and how much will remain? (*b*) Divide 2445 by 99.

METHOD

(*a*) 1275¢ ÷ 99¢ = 12 (baseballs); remainder 87¢ (12¢ + 75¢)

(*b*) 2445 ÷ 99 = 24; remainder 69 (24 + 45)

2. Divide by 99. Give quotients and remainders:

a 1841 *b* 2306 *c* 3560 *d* 4324 *e* 5009 *f* 6213

3. Divide by 99 (*a*) 8514; (*b*) 7230.

In (*a*) the quotient, according to the foregoing rule, is 85 and the remainder is 99; the correct quotient is, therefore, 86.

In (*b*) the remainder 102 indicates that the number 72 should be increased by 1, making the quotient 73; the remainder is 3 (102–99).

4. Divide by 99. Give quotients and remainders:

a 2080 *b* 3090 *c* 4060 *d* 2182 *e* 3180 *f* 4260

These examples are given to show pupils that such a divisor as 99, which seems a difficult one to them, is really a very simple one to computers; 999 is a still easier divisor.

A similar method is used in dividing by 98 and 998; also by 89, 79, 69, etc.

LONG DIVISION

SIGHT DRILLS

The guesses made by some pupils to obtain the successive quotient figures in a long division example show their need of sight drills similar to those given below:

1. Find quotients:

a 20)140	*b* 30)270	*c* 40)160	*d* 50)250
e 60)420	*f* 70)210	*g* 80)480	*h* 90)360
i 80)560	*j* 70)490	*k* 60)360	*l* 50)450

Obtain the quotient of each of the foregoing by dividing 14 by 2, 27 by 3, etc., rejecting the cipher in each term.

In the next set, think of 19, 29, etc., as less than 20, 30, etc. Since 20 is contained 9 times in 180, 19 must be contained 9 times with a remainder.

2. Give quotients, omitting remainders:

a 19)140	*b* 29)270	*c* 39)160	*d* 49)250
e 59)420	*f* 69)210	*g* 79)480	*h* 89)360
i 79)560	*j* 69)490	*k* 59)360	*l* 49)450

Since 7 times 20 is 140, 7 times 21 is more than 140; the latter, therefore, will contain 21 only 6 times with a remainder. In giving the quotient, say 6, omitting the remainder.

3. Give quotients. Omit remainders:

a 21)140	*b* 31)270	*c* 41)160	*d* 51)250
e 61)420	*f* 71)210	*g* 81)480	*h* 91)360
i 81)560	*j* 71)490	*k* 61)360	*l* 51)450

4. Omit a remainder when there is one:

a 210)840	*b* 430)860	*c* 310)930	*d* 510)3570
e 209)840	*f* 429)860	*g* 309)930	*h* 509)3570
i 211)840	*j* 431)860	*k* 311)930	*l* 511)3570

m 150)750	*n* 199)800	*o* 209)680	*p* 499)2510
q 149)900	*r* 202)800	*s* 129)390	*t* 391)2420
u 151)600	*v* 296)900	*w* 131)390	*x* 411)3690

Before a set of long division examples is worked, these should be used as drills, in which successive pupils are asked to announce rapidly the first quotient figure of each.

MULTIPLYING AND SUBTRACTING

PREPARATORY EXERCISES

1. Express $\frac{345}{47}$ as a mixed number.

METHOD

$\frac{345}{47} = 7\frac{}{47}$

First write 7 as the integral part of the quotient and 47 as the denominator of the fraction. Obtain the figures of the numerator thus:

Think 49 (7 times 7) and *6* (writing 6) are 55.
Think 28 (7 times 4), 33 (carrying 5) and *1* (writing 1) are 34. Ans. $7\frac{16}{47}$

2. Give quotient as mixed number. Write answers from the book.

a $\frac{400}{41}$	*b* $\frac{587}{61}$	*c* $\frac{632}{71}$	*d* $\frac{358}{53}$	*e* $\frac{615}{73}$	*f* $\frac{700}{83}$
g $\frac{294}{43}$	*h* $\frac{486}{67}$	*i* $\frac{200}{37}$	*j* $\frac{614}{79}$	*k* $\frac{876}{97}$	*l* $\frac{423}{59}$

WRITTEN EXERCISES

1. At 59 tons to the car, how many car loads are there in 43,567 tons, and how many tons are there over?

ONE WAY

Ans. 738 (C. L.); remainder 25 T.

59 T.)43567 T.

413

226

177

497

472

25

In long division write the quotient above the dividend. The first partial dividend is 435. Obtain 7, the first quotient figure, by dividing 43 by 6 (the divisor being nearly 60). Write 7 above 5, the last figure of the partial dividend.

After 738, the quotient, write C. L. (car load) in a parenthesis, followed by the remainder, 25 T.

Thousands of European children are taught only the following method in long division. Surely American boys and girls should be willing to use it if time can thus be saved. The greater concentration required tends to secure accuracy.

2. Divide 417,739 by 59.

ABBREVIATED FORM

Ans. 7080; remainder 19.

59)417739
473
19

This method consists in multiplying and subtracting in one operation, omitting the partial products.

Having written 7, the first quotient figure, think 63 (7 times 9) and *4* (writing 4) are 67; think 35 (7 times 5), 41 carrying 6. This being the same as the remaining figures of the partial dividend, the remainder is 4.

To 4, the remainder, annex 7, the next figure of the dividend, making 47 the next partial dividend.

Since 47 does not contain 59, write a cipher in the quotient and bring down 3, the next figure of the dividend, making 473, the next partial dividend.

Write 8, the next quotient figure, and think 72 (8 times 9), and *1* (writing 1) are 73; think 40 (8 times 5), 47 (carrying 7).

To 1, the remainder, annex 9, making 19 the next partial dividend.

Since 19 does not contain 59, write a cipher in the quotient.

Be careful to locate properly the first quotient figure, and to write a figure (which may be a cipher) over each of the remaining figures of the dividend. Before making one of the tests specified below, note that the quotient has the required number of figures.

Test the foregoing result by multiplying 7080 (the quotient) by 59 (the divisor) and adding to the product 19 (the remainder).

Or by casting out 11's, multiply 7 (the quotient excess) by 4 (the divisor excess). To this product, add 8 (the remainder excess), which gives 3 as the excess. The dividend excess is 3, which agrees with the other result.

SIGHT DRILLS

Before taking up the following examples there should be a rapid drill in giving the first quotient figure of each example and the number of figures in the quotient.

WRITTEN EXERCISES

1. Divide, using either the abbreviated form or the longer one. Test each result:

a 4000 ÷ 41	*b* 13,582 ÷ 501	*c* 908,671 ÷ 1023
d 5871 ÷ 61	*e* 26,154 ÷ 702	*f* 777,349 ÷ 2056
g 6325 ÷ 71	*h* 70,000 ÷ 803	*i* 470,493 ÷ 3142
j 7320 ÷ 81	*k* 52,610 ÷ 423	*l* 839,264 ÷ 4026
m 9065 ÷ 51	*n* 46,792 ÷ 315	*o* 678,579 ÷ 3205
p 5216 ÷ 91	*q* 63,130 ÷ 269	*r* 708,000 ÷ 1684
s 9653 ÷ 31	*t* 83,746 ÷ 137	*u* 853,568 ÷ 2245
v 6885 ÷ 21	*w* 94,217 ÷ 621	*x* 956,308 ÷ 3189

2. At a railroad terminal 794 car loads of freight were received, weighing 63,018 tons. What was the average load per car? Give answer (*a*) to the nearest hundredth. (*b*) To the nearest tenth. (*c*) To the nearest integer.

Test each result.

PROCESS

```
      79.367        (a) 79.37 T. Ans.
794)63018.
    7338            (b) 79.4 T.  "
     2920
      5380          (c) 79 T.    "
       6160
```

Test (*a*) by adding 616 to the product of 7936 and 794. Test (*b*) by adding 538 to the product of 793 and 794. Test (*c*) by adding 292 to the product of 79 and 794.

In giving the answer to (*a*) write 7 as the fourth figure of the quotient, since 7, the next figure, is greater than 5. In giving the answer to (*b*) write 4 as the third quotient figure, since 6, the fourth figure, is greater than 5. In giving the answer to (*c*) write 79, the next figure being less than 5.

3. Divide. Give each answer to the nearest integer:

a 4000 ÷ 43 *b* 23,582 ÷ 431 *c* 129,456 ÷ 531

4. Divide. Give answers to the nearest tenth:

a 5871 ÷ 51 *b* 36154 ÷ 732 *c* 248,673 ÷ 727

5. Divide. Give answers to the nearest hundredth:

a 6325 ÷ 81 *b* 47,000 ÷ 723 *c* 320,000 ÷ 337
d 7974 ÷ 91 *e* 52,318 ÷ 864 *f* 562,873 ÷ 739

6. (*a*) How many kilos of 2.2046 pounds each are equivalent to 54.75 pounds? (*b*) What decimal of a meter, 39.37 inches, is a yard?

PROCESS

(*a*) Ans. 24.834 (kilos)

2,2046.)54,7500.
10 6580
1 83960
75920
97820
9636

Multiply the divisor by 10,000 by shifting the decimal point 4 places to the right. Do the same with the dividend, annexing two ciphers. Carry the result to three decimal places.

(*b*) Write the dividend as 36 inches, to give it the same denomination as the divisor. Give the result to the nearest thousandth.

.9 etc.
39,37.)36,00.0

7. Give quotients to nearest thousandth:

a 38.765 ÷ 3.9	*b* 18,612 ÷ .095	*c* 844.73 ÷ .77
d 6.5772 ÷ .85	*e* 511.347 ÷ .67	*f* 105.28 ÷ 5.9
g 58.6 ÷ .049	*h* 2406 ÷ .43	*i* 53.95 ÷ .083

8. Divide 292.2331903 by 46.72352, giving the result correct to thousandths.

46.724)292.233

9. Give quotients correct to thousandths:

a 18.54875 ÷ 3.23852	*b* 76.824 ÷ .74325
c 8.32346 ÷ .72149	*d* 1634.7853 ÷ 22.37425
e .093285 ÷ .005873	*f* 398.647 ÷ 93.0286
g .0065437 ÷ .85436	*h* 73.0248 ÷ 1.63387

MULTIPLYING AND DIVIDING

CANCELLATION

PREPARATORY EXERCISES

1. (*a*) At \$16 a week of 44 hours, what is the pay of a girl who works 33 hours? (*b*) What wages does a boy receive for 39 hours' work at the rate of \$18 a week of 54 hours?

METHOD

In (*a*) multiply the weekly rate, \$16, by ¾, the fraction of the week she works.

In (*b*) multiply the hourly rate, \$⅓ by 39, the number of hours she works.

2. (*a*) What is the value of 321 eggs when they are worth 36 cents a dozen? (*b*) Of 360 eggs at the rate of 43 cents a dozen?

3. Give the cost (*a*) of 131 pounds of potatoes at \$1.80 a bushel of 60 pounds. (*b*) Of 120 pounds at \$1.70 a bushel.

4. Give answers:

a $16 \times \frac{33}{44}$	*b* $\frac{18}{54} \times 39$	*c* $321 \times \frac{36}{12}$	*d* $\frac{360}{12} \times 43$
e $\frac{180}{60} \times 131$	*f* $170 \times \frac{120}{60}$	*g* $\frac{17}{85} \times 60$	*h* $84 \times \frac{37}{74}$
i $\frac{246 \times 72}{82}$	*j* $\frac{123 \times 93}{279}$	*k* $\frac{130 \times 42}{65}$	*l* $\frac{24 \times 125}{120}$
m $\frac{82 \times 72}{246}$	*n* $\frac{123 \times 279}{93}$	*o* $\frac{65 \times 42}{130}$	*p* $\frac{120 \times 125}{24}$

WRITTEN EXERCISES

1. Find the cost of a rectangular plot 154 yards long 68 yards wide at the rate of $275 an acre (4840 sq. yd.).

ONE WAY

First find the number of square yards by multiplying 154 by 68, which gives 10472. Divide this by 4840 to ascertain the number of acres ($2\frac{632}{1210}$ A.). Multiply $275 by $2\frac{632}{1210}$.

A BETTER WAY

$$\frac{154 \times 68 \times \$275}{4840}$$

Indicate the area in square yards by writing 154×68. Draw a line and indicate the number of acres by writing 4840 underneath (as a divisor). Indicate that this result is to be used to multiply $275, by writing after it the latter preceded by a multiplication sign. Cancel.

In computations involving only multiplication and division, indicate the operations, then shorten the work by cancellation.

2. Find answers:

a $\dfrac{12 \times 7 \times 153}{19 \times 49}$ $\quad b$ $\dfrac{85 \times 126 \times 13}{17 \times 91}$ $\quad c$ $\dfrac{454 \times 198 \times 72}{81 \times 63 \times 35}$

3. Find the value of:

a $\dfrac{484 \times .06 \times 210}{360}$ $\quad b$ $\dfrac{24.5 \times 18.7}{.238}$

PROCESS

a $\dfrac{484 \times .\not{0}\not{6} \times 21\not{0}}{\overset{.01}{\not{3}\not{6}\not{0}}}$ $\quad 6$

In canceling .06 and 36 by 6, be careful to write .01 above the former.

b Since the divisor .238 contains three decimal places, annex a terminal decimal cipher to one of the numbers above the line, giving a total of three places in the dividend. Cancel all the decimal points.

$$\frac{24\not{.}5 \times 18\not{.}70}{\not{.}238}$$

This multiplies the divisor by 1000, and the dividend by 10 and again by 100.

4. Find the value of each of the following:

a $\dfrac{18.9 \times 12 \times 49}{6.3 \times 21}$ $\quad b$ $\dfrac{1.89 \times 12 \times 4.9}{2.1 \times .63}$ $\quad c$ $\dfrac{18.9 \times .12 \times .49}{.63 \times .21}$

DIVISION OF FRACTIONS

PREPARATORY EXERCISES

1. How many times is 2 thirds contained in 3 thirds? Give answer (a) as a mixed number, (b) as an improper fraction.

2. Divide 1 by $\frac{2}{3}$, giving quotient as an improper fraction.

3. If $\frac{2}{3}$ goes $\frac{1}{2}$ times into 1, how many times does it go (a) into 5 times 1? (b) Into 7 times 1? (c) Into 9 times 1?

4. Give quotients:

a 3 halves)2 halves *b* ½)⅝ *c* ½)1 *d* 1½)1

e 1 ÷ ⅝ *f* 1 ÷ ¾ *g* 1 ÷ ⅝ *h* 1 ÷ ⅓

The quotient of 1 divided by a number is called the *reciprocal* of a number.

5. Give the reciprocal of the following:

a 1⅓ *b* 2 *c* ⅓ *d* ¾ *e* 5⅕ *f* 2⅔

To get the reciprocal of a common fraction invert its terms.

WRITTEN EXERCISES

1. Find the average yield per acre (*a*) when 27 acres yield 496⅛ bushels. (*b*) When 28⅞ acres yield 693 bushels. (*c*) When 36¾ acres yield 790⅛ bushels.

PROCESS

(I)	(II)	(III)	
(*a*) $496\frac{1}{8} \div 27$	$= \frac{3969}{8} \div \frac{27}{1}$	$= \frac{3969}{8} \times \frac{1}{27}$	Cancel
(*b*) $693 \div 28\frac{7}{8}$	$= \frac{693}{1} \div \frac{231}{8}$	$= \frac{693}{1} \times \frac{8}{231}$	Cancel
(*c*) $790\frac{1}{8} \div 36\frac{3}{4}$	$= \frac{6321}{8} \div \frac{147}{4}$	$= \frac{6321}{8} \times \frac{4}{147}$	Cancel

(I) shows each example in the original form; (II) shows the divisor and the dividends written as fractions; (III) substitutes for each divisor its reciprocal and changes the sign of division to that of multiplication.

TEST

Multiply the result by the divisor.

2. Find quotients. Estimate the integral part of each result before beginning work:

a	$436 \div 18\frac{2}{3}$	*b*	$330\frac{5}{8} \div 23$	*c*	$907\frac{1}{2} \div 24\frac{1}{5}$
d	$973 \div 34\frac{3}{4}$	*e*	$924\frac{3}{4} \div 19$	*f*	$787\frac{1}{16} \div 42\frac{5}{6}$
g	$678 \div 42\frac{3}{8}$	*h*	$788\frac{1}{2} \div 57$	*i*	$870\frac{5}{6} \div 36\frac{3}{8}$
j	$279 \div 13\frac{1}{2}$	*k*	$425\frac{1}{3} \div 16$	*l*	$665\frac{1}{2} \div 16\frac{1}{2}$
m	$777 \div 29\frac{1}{3}$	*n*	$806\frac{1}{4} \div 30$	*o*	$231\frac{2}{5} \div 10\frac{5}{8}$
p	$357 \div 14\frac{1}{6}$	*q*	$528\frac{1}{2} \div 21$	*r*	$404\frac{1}{8} \div 13\frac{1}{3}$
s	$825 \div 31\frac{1}{4}$	*t*	$291\frac{3}{5} \div 18$	*u*	$545\frac{5}{12} \div 21\frac{1}{4}$
v	$564 \div 15\frac{2}{3}$	*w*	$607\frac{3}{4} \div 22$	*x*	$374\frac{5}{8} \div 18\frac{1}{2}$

3. Find the value of $\dfrac{8\frac{1}{3} \times 6\frac{2}{5} \times 4\frac{1}{2} \times 1\frac{1}{6}}{5\frac{1}{3} \times 3\frac{3}{4} \times 5\frac{5}{8} \times 2\frac{1}{4}}$

METHOD

$2\frac{5}{8} \times 3\frac{2}{5} \times \frac{9}{2} \times \frac{7}{6} \times \frac{3}{16} \times \frac{1}{15} \times \frac{5}{28} \times \frac{1}{9}$

Change the mixed numbers in the compound dividend to improper fractions. Then write as multipliers the reciprocals of the fractions in the divisor. Cancel.

4. Find the value of:

a $\dfrac{6\frac{3}{4} \times 5\frac{1}{2}}{27 \times 3\frac{1}{5}}$ *b* $\dfrac{3\frac{1}{2} \times 16\frac{1}{3}}{2\frac{1}{3} \times \frac{3}{4}}$ *c* $\dfrac{60\frac{1}{2} \times 3\frac{3}{4}}{14\frac{2}{3} \times \frac{3}{8}}$

5. At $\$3\frac{3}{4}$ a yard, how many yards of dress goods can be bought for $\$127\frac{1}{2}$?

NOTE: In dividing by $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{8}$, etc., business men frequently use the reciprocals in their mixed number forms, viz., $1\frac{1}{2}$, $1\frac{1}{3}$, $1\frac{1}{4}$, $1\frac{1}{5}$, etc., instead of the respective improper fractions: $\frac{2}{1}$, $\frac{3}{1}$, $\frac{4}{1}$, $\frac{5}{1}$, etc.

6. Divide $127\frac{1}{2}$ (*a*) by $\frac{3}{4}$, (*b*) by $\frac{4}{5}$, (*c*) by $\frac{5}{6}$.

METHOD

(*a*) $127\frac{1}{2} (\div \frac{3}{4}) \times 1\frac{1}{3}$

Add $\frac{1}{3}$ $42\frac{1}{2}$

170 Ans.

(*b*) $127\frac{1}{2} (\div \frac{4}{5}) \times 1\frac{1}{4}$

Add $\frac{1}{4}$ $31\frac{7}{8}$

$159\frac{3}{8}$ Ans.

(*c*) $127\frac{1}{2} (\div \frac{5}{6}) \times 1\frac{1}{5}$

Add $\frac{1}{5}$ $25\frac{1}{2}$

153

Inclose each divisor in a parenthesis and write its reciprocal as a multiplier. Multiply $127\frac{1}{2}$ by $1\frac{1}{3}$ in (*a*), by $1\frac{1}{4}$ in (*b*), by $1\frac{1}{5}$ in (*c*), by adding to it $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, respectively, of itself. Test the result in (*a*) by multiplying $42\frac{1}{2}$ by 4; in (*b*) by multiplying $31\frac{7}{8}$ by 5; in (*c*) by multiplying $25\frac{1}{2}$ by 6.

7. Find quotients:

a $1356\frac{3}{8} \div \frac{3}{4}$	*b* $6876\frac{3}{4} \div \frac{2}{3}$	*c* $7594\frac{2}{3} \div \frac{5}{8}$
d $2796\frac{5}{8} \div \frac{1}{5}$	*e* $7641\frac{1}{2} \div \frac{3}{4}$	*f* $6463\frac{1}{6} \div \frac{2}{3}$
g $3475\frac{3}{8} \div \frac{5}{6}$	*h* $8438\frac{2}{3} \div \frac{1}{5}$	*i* $5929\frac{1}{2} \div \frac{3}{4}$
j $4004\frac{7}{16} \div \frac{7}{8}$	*k* $9567\frac{1}{2} \div \frac{5}{6}$	*l* $4613\frac{1}{2} \div \frac{1}{5}$
m $5256\frac{3}{15} \div \frac{3}{5}$	*n* $8360\frac{2}{3} \div \frac{7}{8}$	*o* $3722\frac{2}{3} \div \frac{5}{6}$
p $6234\frac{2}{5} \div \frac{2}{3}$	*q* $7642\frac{1}{2} \div \frac{5}{6}$	*r* $2363\frac{1}{3} \div \frac{3}{8}$
s $7042\frac{1}{3} \div \frac{3}{4}$	*t* $6907\frac{1}{5} \div \frac{6}{7}$	*u* $1276\frac{1}{2} \div \frac{9}{10}$
v $8629\frac{1}{2} \div \frac{4}{5}$	*w* $5005\frac{1}{2} \div \frac{7}{8}$	*x* $2563\frac{1}{3} \div \frac{10}{11}$

8. At $\$1\frac{1}{4}$ a bushel, how many bushels of potatoes can be bought for $\$127\frac{1}{2}$?

9. Divide $127\frac{1}{2}$ (*a*) by $1\frac{1}{2}$, (*b*) by $1\frac{3}{8}$, (*c*) by $1\frac{1}{5}$. Test each result.

METHOD

(*a*) $127\frac{1}{2} (\div 1\frac{1}{2}) \times \frac{2}{3}$

Deduct $\frac{1}{3}$ $42\frac{1}{2}$

85 Ans.

(*b*) $127\frac{1}{2} (\div 1\frac{1}{3}) \times \frac{3}{4}$

Deduct $\frac{1}{4}$ $31\frac{7}{8}$

$95\frac{5}{8}$ Ans.

(*c*) $127\frac{1}{2} (\div 1\frac{1}{5}) \times \frac{5}{6}$

Deduct $\frac{1}{6}$ $21\frac{1}{4}$

$106\frac{1}{4}$ Ans.

Inclose each divisor in a parenthesis and write its reciprocal as a multiplier.

Multiply $127\frac{1}{2}$ by $\frac{2}{3}$ in (*a*), by $\frac{3}{4}$ in (*b*) and by $\frac{5}{6}$ in (*c*), by deducting from it $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{6}$, respectively, of itself.

Test the result in (*a*) by multiplying $42\frac{1}{2}$ by 2; in (*b*) by multiplying $31\frac{7}{8}$ by 3; in (*c*) by multiplying $21\frac{1}{4}$ by 5.

10. Find quotients:

a $1353\frac{3}{8} \div 1\frac{1}{2}$	*b* $5005\frac{1}{5} \div 1\frac{1}{5}$	*c* $7164\frac{9}{10} \div 1\frac{1}{8}$
d $2496\frac{2}{3} \div 1\frac{1}{3}$	*e* $5005\frac{1}{4} \div 1\frac{1}{6}$	*f* $8363\frac{1}{3} \div 1\frac{1}{9}$
g $3075\frac{5}{8} \div 1\frac{1}{4}$	*h* $6328\frac{1}{2} \div 1\frac{1}{7}$	*i* $9009\frac{11}{12} \div 1\frac{1}{10}$
j $4238\frac{1}{4} \div 1\frac{1}{2}$	*k* $7593\frac{3}{5} \div 1\frac{1}{5}$	*l* $8657\frac{1}{2} \div 1\frac{1}{9}$

SECTION IV

PRODUCTION AND CONSUMPTION

CHAPTER ONE

PROBLEMS OF THE CONSUMER

HOUSEHOLD EXPENSES

The calling followed by the largest number of persons is that of home keeping. Success in this line, as in any other, requires the employment of business methods. Efficient management is just as important in spending the income as it is in earning it.

FAMILY BUDGETS

The following table shows the average outlay of a large number of families in various sections of the United States for food, for shelter, and for clothing, arranged by classes having incomes as specified.

Yearly Income	Expenditures for			Remainder for operating expenses, savings, etc.
	Food	Shelter	Clothing	
$600	$258	$114	$78	(*a*)
900	378	162	126	(*b*)
1200	444	204	180	(*c*)
1500	510	255	270	(*d*)

WRITTEN EXERCISES

1. Write from the book the sum represented by (*a*), by (*b*), by (*c*) and by (*d*), respectively, in the foregoing table.

2. Make out a table similar in form to the foregoing, but changing the money to per cents in the last four columns.

3. Before the war the following was Mrs. Kirby's estimate of the minimum food requirements of Mr. Kirby and herself, and their three children.

Meats, etc.		*Cereals, etc.*	
5 lb. beef	@ 16¢	21 loaves bread	@ 5¢
½ " " (stew)	" 12¢	1 doz. rolls	" 10¢
½ " pork	" 28¢	2 lb. cake	" 10¢
½ " ham	" 36¢	¼ " rice	" 8¢
1 " chicken	" 18¢	2 " flour	" 4½¢
1½ " fish	" 12¢	2½ " oatmeal	" 4¢

Milk, Eggs, etc.		*Sugar, Tea, etc.*	
1 lb. butter	@ 32¢	1 lb. coffee	@ 20¢
½ " cheese	" 20¢	2 " sugar	" 7½¢
2 doz. eggs	" 32¢	½ pt. sirup	" 6¢
16 qt. milk	" 6¢	¼ lb. tea	" 40¢

Vegetables, etc.			
¾ pk. potatoes	@ 64¢	1 can tomatoes	@ 20¢
Turnips or carrots	" 5¢	¼ " corn	" 14¢
2 lb. onions	" 3¢	Fruit	" 25¢
Other vegetables	" 66¢	¼ lb. prunes	" 14¢
Beans and peas	" 5¢	Pickles, spices, etc.	" 6¢

4. Find the weekly cost (*a*) of the meats, etc., (*b*) of the cereals, etc., (*c*) of the milk, eggs, etc.,

(*d*) of the sugar, tea, etc., (*e*) of the vegetables, etc. (*f*) Find the total weekly cost. (*g*) Find the cost for 52½ weeks.

5. How much did the yearly cost exceed 42% of Mr. Kirby's pay for 300 working days at $3 a day?

6. The following was the estimated cost of clothing. Mrs. Kirby's hats and coats, and Mr. Kirby's overcoat were supposed to last two years, one half the given price being allowed for one year. The prices were taken from a schedule issued before the war.

Mother's Clothing			*Father's Clothing*		
2 hats (½)	@	$3.—	1 cap	@	$0.25
1 coat (½)	"	8.—	1 hat	"	.75
1 suit	"	8.—	1 suit	"	10.—
3 waists	"	.66⅔	1 overcoat (½)	"	10.—
2 dresses	"	1.25	1 pr. trousers	"	2.—
2 petticoats	"	.50	3 shirts	"	1.50
3 aprons	"	.15	2 shirts	"	1.—
6 handkerchiefs	"	.07½	6 collars	"	.10
6 prs. stockings	"	.10	2 pr. overalls	"	.75
2 pr. shoes	"	2.—	4 ties	"	.12½
Repairing shoes	"	1.—	4 handkerchiefs	"	.05
3 suits underwear	"	.20	6 pr. socks	"	.10
2 " "	"	.70	Gloves and mittens	"	.50
Linen	"	6.—	2 pr. shoes	"	2.—
Rubbers	"	.50	Repairing	"	1.50
Sundries	"	3.—	2 suits underwear	"	.50
			2 " "	"	.75

7. Find the annual cost of (*a*) the mother's clothing. (*b*) That of the father.

(*c*) Find the total cost of the family clothing, including $23.35 for that of the girl and $16.30 for each of the two boys.

(*d*) How much more did the clothing cost than 14% of $900? (*e*) What should have been the annual income, in order that 14% of it would purchase the specified clothing at the prices given?

8. The following are estimates of the other expenses for a year:

Rent	$156.—	Recreation	$35.—
Car fare	31.20	Church dues	10.—
Fuel and light	52.—	Utensils	15.—
Furniture	52.—	Spending money (father)	5.—
Insurance	52.—	Sundries	5.—
Reading matter	5.—		

a What is the total of the foregoing items?

b What was the total amount of the year's expenditures including those for food and clothing?

9. If Mr. Kirby's earnings of $900 were supplemented by a year's interest at 4% on $1000 he has in the savings bank, how much of his income should remain at the end of the year?

Finding that a house in the suburbs with a piece of ground could be bought for a cash payment of $600 and monthly instalments of $15 each until the remainder, $1200, was paid, with interest at 6%, Mr. Kirby bought it, taking possession February 1.

10. Make out a statement of the first year's payments, showing (*a*) the principal at the beginning of each month; (*b*) the interest for the month; (*c*) the amount due, including interest, etc.; (*d*) the payment made; (*e*) the balance remaining. (*f*) How much did his payments for the year exceed the total of the interest?

Date	Principal	Interest to date	Amount at date	Payment	Bal.
Mar. 1	$1200.—	$6.—	$1206.—	$15	$1191.—
Apr. 1	1191.—	5.96	1196.96	15	1181.96
May 1	1181.96	5.91	1187.87	etc.	etc.
etc.	etc.	etc.	etc.	etc.	etc.

NOTE: Use no side calculations. The monthly interest is ½% of the principal. Do not give fractions of a cent in stating the interest; write $5.96 for $5.95½, $5.91 for $5.9098, etc.

11. (*a*) How much did his interest payments amount to for the first year?

For purposes of taxation, his property was assessed at $1400, on which he paid a tax of ⅝%. (*b*) What were his taxes for the year? He insured his house for $1500 at 44 cents per $100 for three years. (*c*) What was the cost of the three years' insurance? He made most of his repairs himself with the help of his boys, paying $15 for material, and only $7 for outside help. (*d*) How much did he pay for interest, taxes, one year's insurance, and repairs? (*e*) How much less did these amount to than the yearly rent of his former residence? (*f*) What per cent of $1800, the cost of the house, was its assessed value of $1400? (*g*) For what per cent of its cost was it insured?

12. How much did he reduce his mortgage of $1200 by the end of the year?

As soon as they were settled in the new house, Mr. Kirby with the help of the boys fenced off a portion of the land for chickens, and bought materials for a henhouse as follows:

a 226 bd. ft. scantling
b 850 " " lumber
c 622 " " boards
d 2 pr. hinges
e 150 sq. ft. roofing paper
f 5 lb. nails
g 56 sq. ft. poultry wire

13. Find the cost of each of the foregoing items at \$32 per M (1000 board feet) for the scantling, \$30 per M for the lumber, \$36 per M for the boards, 50¢ per pair for the hinges, 2½¢ per square foot for the roofing paper, 6¢ per pound for the nails, and 1½¢ per square foot for the wire.

14. He bought 23 hens from a neighbor at 75 cents each. (*a*) Find the cost of the hens. The following is the egg-production of a year, with the average price prevailing during the month:

Month	Eggs Laid	Value per doz.	Month	Eggs laid	Value per doz.
Feb.	330	40¢	Aug.	334	34¢
Mar.	461	36	Sep.	129	36
Apr.	393	32	Oct.	99	40
May	358	30	Nov.	104	42
Jun.	357	32	Dec.	153	44
Jul.	344	33	Jan.	254	42

(*b*) Find the number of eggs laid during the year, (*c*) their value, (*d*) the average value per dozen.

15. During the year the garden patch yielded the following:

15	quarts	of string beans	2	bushels	of turnips
2½	"	" lima "	12	"	" tomatoes
2½	"	" navy "	100	heads	" cabbage
3	bushels	of beets	20	bunches	" carrots
2	"	" onions	150	"	" radishes
3	"	" peas	10	dozen	cucumbers
1½	"	" spinach			

Find the value of the foregoing at the prices prevailing in the vicinity of the school.

16. Mr. Kirby's payments for food were reduced $2.80 a week, owing to the home production of eggs and vegetables, the daily surplus being preserved for winter consumption. Find the saving of 52½ weeks.

"Balanced" Meals

A day's meals should supply, in proper quantities, protein, carbohydrates, fats, mineral matter, water.

Protein supplies the tissue building materials, together with some of the heat and energy. It is chiefly obtained from the whites of eggs, lean meat, skimmed milk, gluten of wheat, etc.

The best known of the carbohydrates are the sugar and the starch of foods. These yield energy in the form of heat and the power to do work.

Fat is obtained from cereals, eggs, nuts, cream, etc. It yields a larger amount of energy according to its weight than does either of the other two groups.

Mineral matter comes from green vegetables, fruits, cereals, and milk.

The day's meals of a mechanic should provide about 4 ounces of protein and 3500 calories of energy.

The following are the meals of a man doing clerical work:

Kind of Food	Weight	Cost	Protein	Energy
Breakfast				
Cereal Hominy	2 oz.	$\frac{5}{16}$¢	.166 oz.	206 calories
Meat Sausage cakes	3 "	$3\frac{3}{4}$	.390 "	398 "
Bread Toast,	3 "	1	.273 "	210 "
Butter ½ cu. in.	½ "	$1\frac{1}{8}$	—	112 "
Beverage Coffee	½ "	$1\frac{1}{8}$	—	—
Beverage Sugar	½ "	$\frac{3}{16}$	—	57 "
Fruit Prunes	2 "	1	.042 "	175 "
Total for meal	(*a*)	(*b*)	(*c*)	(*d*)
Dinner				
Meat Beef stew	8 oz.	$3\frac{3}{16}$¢	.449 oz.	330 calories
Vegetable Rice	2 "	$1\frac{1}{4}$	.160 "	324 "
" *Green* Spinach	2 "	$1\frac{1}{8}$	.042 "	32 "
Dessert Cherry roll	$5\frac{1}{2}$ "	$2\frac{3}{8}$	.217 "	353 "
Sauce	$1\frac{1}{2}$ "	$1\frac{3}{4}$	—	225 "
Bread 2 slices	2 "	$\frac{3}{8}$	.182 "	140 "
Butter ½ cu. in.	½ "	$1\frac{1}{8}$	—	112 "
Total for meal	(*e*)	(*f*)	(*g*)	(*h*)
Supper				
Animal food Cottage cheese	5 oz.	$1\frac{3}{16}$¢	1.145 oz.	160 calories
Vegetable Potato cakes	4 "	1	.088 "	96 "
Bread 3 slices	3 "	1	.273 "	210 "
Butter ¾ cu. in.	¾ "	$1\frac{11}{16}$	—	140 "
Beverage Cocoa	$5\frac{1}{4}$ "	$1\frac{5}{8}$	.216 "	193 "
Total for meal	(*i*)	(*j*)	(*k*)	(*l*)
Total for day	(*m*)	(*n*)	(*o*)	(*p*)

WRITTEN EXERCISES

1. Find the weight of the food for each meal, (*a*), (*e*), and (*i*); the total for the day (*m*); the cost for

each meal (*b*), (*f*), and (*j*); the total cost for the day (*n*); the quantity of protein for each meal, (*c*), (*g*), and (*k*); the total for the day (*o*); the amount of energy supplied by each meal, (*d*), (*h*), and (*l*); the total for the day (*p*).

The following table gives the most important items of one type of a United States soldier's service ration for a day:

		Protein	Calories per lb.	Protein	Calories per item
Bacon	16 oz.	15%	2080	(*a*)	(*j*)
Flour	18 "	11.5%	1680	(*b*)	(*k*)
Beans	2.4 "	22.5%	1600	(*c*)	(*l*)
Potatoes	20 "	1.8%	320	(*d*)	(*m*)
Prunes	1.28 "	2.1%	1440	(*e*)	(*n*)
Sugar	3.2 "	—	1868	(*f*)	(*o*)
Milk	.5 "	9½%	784	(*g*)	(*p*)
Lard	.64 "	—	4320	(*h*)	(*q*)
Butter	.5 "	1%	3608	(*i*)	(*r*)
			Totals	(*s*)	(*t*)

2. From the foregoing, find the quantity of protein in each item, (*a*) to (*i*); the number of calories in each, (*j*) to (*r*); the total amount of protein, (*s*); and the total number of calories, (*t*).

3. How many times 4 ounces is (*s*)?

4. How many times 3500 calories is (*t*)?

5. Find the number of pounds of each of the following required for a detachment of 6000 men:

(*a*) Bacon
(*b*) Flour
(*c*) Beans
(*d*) Potatoes
(*e*) Prunes
(*f*) Sugar
(*g*) Milk
(*h*) Lard
(*i*) Butter

6. Find the total quantity of food required a day for the same detachment, including the following:

Coffee	420 lb.	Pepper	60 lb.
Vinegar	120 "	Cinnamon	21 "
Sirup	120 "	Lemon Extract	21 "

7. Find the daily quantity allowed to each man.

A Sample United States Menu

The cost to the Government of the standard ration is now 38¼¢ a day. For the various specified items, the use of other articles is authorized when the latter supply the proper nourishment, and the cost of the ration does not exceed the Government allowance.

The following table gives the cost of a day's meal to 1700 men at a training camp:

	Breakfast	
Cereal	Milk Toast	$19.60
Meat	Ham Omelet	55.60
Vegetable	Fried Potatoes	20.00
Bread	Rolls and Butter	25.00
Beverage	Coffee and Milk	36.00
	Total	(*a*)

	Dinner	
Soup	Beans	$24.60
Meat	Fried Liver	80.00
Vegetable	Baked Potatoes	36.00
"	Fried Onions	22.80
Bread	Bread	16.00
Dessert	Coffee Cake	30.00
Beverage	Lemonade	33.00
	Total	(*b*)

SUPPER

Meat	Beef Roll	$32.00
Vegetable	Sweet Potatoes	21.00
Bread	Bread and Butter	17.00
Dessert	Cake	30.00
	Lemon Sauce	12.65
Beverage	Coffee with Milk	20.00
	Total	(*c*)

SOLDIERS AT MESS

WRITTEN EXERCISES

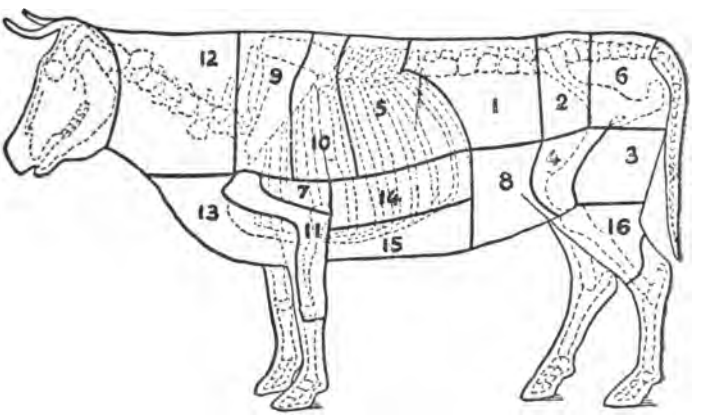
Find the cost of each meal, (*a*), (*b*), and (*c*). Find (*d*) the total cost for the day. Find (*e*) the daily cost per soldier. Find (*f*) the difference for 1700 soldiers between (*d*) and the Government allowance of 38¼¢ a day.

EFFICIENCY IN HOME KEEPING

No matter how small the sum available for food, the efficient manager supplies the necessary nourishment. She purchases cheaper cuts of meat, and makes them just as palatable as the higher-priced ones. By carefully watching the market she is able to give her table the needed variety.

She is careful to require the butcher to give her all

the fat and the bone that have been weighed and charged for. The water in which vegetables have been boiled, and which contains important mineral constituents, she uses in making soup. No stale bread is wasted, being made into tasty desserts.



THE NUMBERS ON THIS PICTURE LOCATE VARIOUS CUTS OF BEEF

High Cost of Meat

The most expensive item in food is frequently the meat, even when care is taken in its purchase.

The following shows the weight of the various cuts, and the retail price per pound:

1. Porterhouse	54 lb.	@ 32¢
2. Sirloin	45 "	" 30"
3. Round	37½ "	" 28"
4. Top Sirloin	24 "	" 26"
5. Rib Roast	40 "	" 24"

6. Rump	21	"	" 24"
7. Cross Rib	12½	"	" 24"
8. Flank	4½	"	" 20"
9. Chuck	52½	"	" 20"
10. Blade	15	"	" 20"
11. Shoulder	12	"	" 19"
12. Neck	12	"	" 18"
13. Brisket	20	"	" 15"
14. Plate	72	"	" 15"
15. Navel	48	"	" 15"
16. Shin	30	"	" 12"

WRITTEN EXERCISES

1. Find (*a*) the total weight of the foregoing cuts; (*b*) the amount paid; (*c*) the average price a pound. (*d*) How many pounds are sold above the average price? (*e*) How many below?

2. What is the total amount obtained by the butcher if he receives 6 cents a pound for 25 pounds of suet, 3 cents a pound for 25 pounds of scraps, and ⅝ cent a pound for 40 pounds of bones?

3. What is the average price a pound received for the entire carcass, including suet, scraps, and bone?

4. What per cent of the live weight of 1000 pounds is the weight of the carcass?

5. (*a*) What did the butcher pay for the meat at $7.50 per 100 pounds? (*b*) How much more did he receive for it?

6. How many live cattle weighing 1000 pounds each will be required to supply a day's rations for

6000 men at 1¾ pounds per man if the dressed weight of the cattle is 60 % of the live weight?

7. Complete the following table, which shows the daily food of 528 students in a Government school, at contract prices before the war:

Article	*Total Food*	*Oz. per pupil*	*Price per lb.*	*Total Cost*
Bread	891 lb.		5¢	
Beef	379½ "		8¢	
Oatmeal	33 "		4¢	
Potatoes	247½ "		2¢	
Sugar	10½ "		6¢	
Sirup	5½ "		8¢	
Cornstarch	11 "		8¢	
Corn Bread	121 "		3¢	
Butter	16½ "		30¢	
Flour	88 "		4½¢	
Milk	181½ "		3¢	
Coffee	5½ "		16¢	
Tea	1$\frac{1}{32}$ "		32¢	
Onions	11 "		4¢	
Raisins	11 "		8¢	
Tomatoes	5½ "		4¢	
Total	(*a*)	(*b*)		(*c*)

8. Find (*a*) the total weight of the daily rations for 528 pupils; (*b*) the number of ounces to a pupil's daily ration; (*c*) the total cost of 528 rations; (*d*) the cost for each pupil each day.

ORAL PROBLEMS

1. How many cents a day is the cost of a wife's food if it is .8 of the cost of her husband's food, which is 30 cents?

2. How many times the cost of the father's food is that of the family consisting of the parents and their three children, if the mother's food is .8 of that of the father, and the food of the children is .7, .6, and .5, respectively, that of the father?

3. Multiply 30 cents by 3.6.

4. When three ounces of sausage cake cost 3¾ cents, what was the cost (*a*) of an ounce? (*b*) Of a pound?

5. How much starch is lost by peeling a 4-ounce potato before boiling, if it loses 2.7% when peeled and .2% when boiled with skin?

6. (*a*) When round roast costs 30 cents a pound, and 4 ounces are lost in the bones and fat not eaten what is the cost an ounce of the cooked meat that is eaten? (*b*) What per cent is waste?

7. If rib roast beef costs 40 cents a pound uncooked, and the waste is 50% what is the cost (*a*) of a pound of cooked meat? (*b*) Of an ounce?

8. How much is saved on a pound of cooked meat if round roast costing 30 cents a pound uncooked is used instead of rib roast?

9. To a detachment of 6000 men the following items are issued:

Tomatoes, 7500 lb.	Dried peaches, 48 lb.
Onions, 1500 lb.	Jam, 75 lb.

Express the weight of each for a man (*a*) in pounds or decimal of a pound. (*b*) In ounces or decimal.

10. Give the per cent of waste in a pound of steak that contains ten ounces of lean meat.

Red Cross Articles

WRITTEN EXERCISES

1. Find the value of the materials used by the children of a small school in making the following articles for soldiers' use:

a 24 pajamas, each requiring 6 yards outing flannel at 12 cents a yard;
7 buttons at 10 cents a dozen;
1¼ yards of tape at 4 cents per 4-yard piece.

The following were used in making all of the pajamas:
1½ doz. spools of cotton at 45 cents a dozen;
50 needles at 5 cents a paper of 25;
2000 pins at 10 cents a M;

b 24 operating gowns, each requiring 5 yards twilled muslin at 26 cents a yard;
1¾ yd. tape at 4 cents a 4-yard piece.

The following were used in making all of the gowns:
1 doz. spools of cotton at 45 cents a dozen;
50 needles at 5 cents a paper of 25;
1000 pins at 10 cents a M.

c 20 bed shirts, each requiring 4½ yards of twilled muslin at 26 cents a yard;
1¾ yards of tape at 4 cents a 4-yard piece.

The following were used in making all of the shirts:
10 spools of cotton at 45 cents a dozen;
50 needles at 5 cents a paper of 25;
800 pins at 10 cents a M.

d 120 operating caps, each requiring ¾ yard of twilled muslin at 26 cents a yard.

The following were used in making all of the caps:

10 spools of cotton at 45 cents a dozen;
50 needles at 5 cents a paper of 25;
500 pins at 10 cents a M.

e 80 operating helmets, each requiring ¾ yard of cheese-cloth at 22 cents a yard;
1 yard of tape at 7 cents a 4-yard piece;
Cotton, needles, and pins as in (*d*).

f 80 ice-bag covers, each requiring ¾ yard canton flannel at 20 cents a yard;
2 yards of tape at 4 cents a 4-yard piece;
Cotton, needles, etc. as in (*d*).

g 120 comfort bags, each requiring ½ yard of cretonne at 25 cents a yard;
1¾ yards of tape at 7 cents a 4-yard piece;
Cotton, needles, etc. as in (*d*).

2. Find the value of the contents (*a*) of a bag, (*b*) of 120 bags.

1 spool of cotton, white	$0.05
1 " " " khaki	.05
1 " " darning cotton	.05
1 paper of needles	.05
5 darning needles at 5 cents per paper of 25	
1 doz. white buttons	.05
1 " khaki "	.05
1 thimble	.02
1 pr. scissors	.10
1 cake of soap	.05
1 paper of pins	.05
1 paper of safety pins	.05

1 comb	.15
1 tooth brush	.25
1 small mirror	.25
6 handkerchiefs @ 25¢ a package of 3	.
1 lead pencil	.02
1 writing pad	.10
24 envelopes @ 3¢ a dozen	
10 postal cards @ 2¢	
1 collapsible drinking cup	.10
1 pen knife	.50
2 pr. tan shoe laces @ 10¢	

3. How many articles were made?

4. Find the total value of the materials, and the contents of the bags, all of which were contributed by friends of the school.

The Millinery Class

1. Find the cost of the materials used in making a silk-covered hat as follows:

¾ yd. buckram at $3.08 a roll of 16 yd.
3 " brace wire at 12 cents a roll of 30 yd.
⅛ " crinoline at 7 cents a yd.
1¾ " satin at $1.48 a yd. (trimming)

For 20 hats there were needed:

4 spools Kerr's thread at 14 cents
40 milliners' needles at $1.125 a M.

2. Find the cost of a straw hat, as follows:

10 yd. brace wire at 12 cents a 30-yd. roll
¾ " cape net at 25 cents
1½ pc. straw braid at $1.25

3. Find the cost of the materials used in making a spray of three poppies, each flower requiring:

¾ yd. ribbon at 52 cents
3 centers at 72 cents a gross
1½ yd. tie wire at 11 cents a 25-yd. spool
1 spray of leaves at 65 cents a dozen.
3 stems at 36 cents a gross (144)

One Form of a Household Account

Mrs. Goldstone keeps her accounts in an ordinary blankbook. She gives a double page to each month, and groups the monthly summaries on the thirteenth page, from which she ascertains the receipts and the expenditures for the year.

The receipts are chiefly Mr. Goldstone's regular weekly salary of $25, which is supplemented by pay for extra work, and by interest on his savings.

Mrs. Goldstone makes her entries at the close of each day. On June 1, she first writes 37.34, the balance remaining at the close of May 31. She then enters her two expenditures, from the total of which she finds that the balance then on hand should be 35.84. Finding that this agrees with the cash, she knows that she has omitted no expenditure. Then she enters 1.50 in the "Total" column, and 35.84 in the last one.

In making her entries at the close of June 7, she writes 28.00 in the second column, as the day's cash receipts, placing 9.38, the previous day's balance, in the first column.

In the monthly "Summary," she inserts the total of each column except the first, the third, and the last. In the first, she writes the balance on hand at the close of May. In the third she writes the sum of this balance and the total receipts of the month. In the last column she writes the difference between the summary of the third column and that of the next column to the last, which gives the total expenditures of the month.

On succeeding pages is shown the June page of Mrs. Goldstone's account.

WRITTEN EXERCISES

1. (*a*) Find the total of each day's expenditures, and the balance at the close of each day. (*b*) Find the monthly summary of each item of expenditure, and of the total expenditure. (*c*) Write the summary for June; underneath it, write the May summary, as given below; on the following lines insert the increase or the decrease with respect to each item. (*d*) Determine the balance on hand at the close of April 30, from the data given in the May summary.

2. Mr. Goldstone's income for the year was $1440. (*a*) What per cent of this sum was spent during the year for rent at the monthly rate of $18? (*b*) What per cent of the June income was spent for food during that month? (*c*) What per cent was spent for food during the year if the monthly average was $43.20?

3. The cost of the family's clothing for the year was $352. What per cent of Mr. Goldstone's income was expended for this purpose?

JUNE, 1919 C–21

Day	On Hand			Expenditures				
	Balance	Receipts	Total	Food	Rent	Clothing	Fuel	Light
1			37.34					
2			35.84	3.14	18.—			
3				.20				
4						4.30		
5								
6				.15				
7	9.38	28.—	37.38	9.80				
8								
9				2.60			7.25	
10						.65		
11		5.70		.63				
12								
13								
14		25.—		8.74				
15								1.10
16								
17				1.15				
18						8.—		
19								
20								
21		29.40		9.20				
22								
23				.12				
24								
25								
26				.27				
27								
28		25.—		7.65				
29								
30				.59				
Summary		113.10	150.44					
Last month	37.34	136.10		48.15	18.—	16.30	2.10	1.30
Increase								
Decrease								

Insurance	Ch'rch	Ice	Reading	Utensils	Recre'tion	Health	Sundries	Bank	Dues	Total	Balance
	.35				1.15					1.50	35.84
		.35	.15							21.64	14.20
										.20	
							.17				
					.35						9.38
										9.80	27.58
	.40					2.—					
		.35	.15		.25						
									1.—		
				1.63							
					.40			5.—			
	.30										
		.35	.15								
$8.40					40						
	.30						.42				
		.35	.15		.25						
				.24				5.—			
			.10								
	.40										
							.23				
2.10	1.50	.95	1.01	3.15	3.80	3.10	—	15.—	1.—		37.84

The Household Inventory

The following excerpt is taken from the Standard Fire Insurance Policy, used in some states:

A Fire Scene

> *If fire occur, the insured shall forthwith separate the damaged and undamaged personal property . . . make a complete inventory of the same, stating the quantity and the cost of each article and the amount claimed thereon. . . .*

In order to be able to comply with this requirement in the event of a fire in his home, Mr. Helm has listed in a blankbook an inventory of the personal property contained in his residence. This book Mr. Helm keeps with his policy in his office downtown in order

that they escape destruction in a fire that may destroy his property.

A page is given to each room and one to the recapitulation. Other pages contain itemized lists of the "Books," "Pictures," "Cut Glass," etc. A page of "Pictures," for instance, would give the title and value of each of the eleven (11) totaled in the parlor page as worth $174, together with the number and the value of all of the others in the house, specifying the room in which each is hung.

The inventory gives the cost of each item with the date of purchase. From the latter may be determined the sum that should be deducted for depreciation.

The following is a list of articles contained in the parlor when the inventory was made. As articles are added or removed, changes are noted.

PARLOR

Number	Article	Date of Purchase	Cost	Remarks
1	Carpet	XI–16–1908	$40.—	
7	Chairs	"	82.—	
1	Clock	XII–23–1910	20.—	Gift
4	Curtains	X–15–1909	12.—	
3	Electric Fixtures	XI–16–1908	25.—	
1	Jardiniere	"	5.—	Gift
1	Lamp	"	12.—	Electric
1	Mirror	"	25.—	
1	Music Cabinet	"	15.—	
1	Piano and Cover	"	375.—	
1	Piano Stool	"	5.—	
84	Piano Music	Various	37.50	See List
11	Pictures	"	175.—	" "
1	Portiere	IX–15–1911	10.—	
1	Rug	"	12.—	
4	Shades	XI–16–1908	4.—	
2	Sofas	"	40.—	
1	Statue	XII–23–1915	12.—	Gift
2	Tables	XI–16–1908	10.—	

WRITTEN EXERCISES

1. Find the original value of the contents of the parlor.

2. The following shows the last page. Include the value of the contents of the parlor, and ascertain the original value of all the furniture.

RECAPITULATION

Halls	$127.—	Servants' Room	$48.—
Parlor	———	Bath Room	25.—
Dining Room	647.50	Laundry	39.50
Living Room	765.—	Attic	168.—
Kitchen and Pantry	129.—	Linen Closet	94.50
Bedroom 1	347.50	Chests, etc.	47.80
" 2	263.25	Miscellaneous	123.—
" 3	189.50	Total	$

3. Mr. Helm insures the foregoing articles for $3000, at the rate of 55 cents a $100, for three years. What is the cost of the insurance?

4. In the event of the total destruction of all of the articles by fire, what sum should he receive from the insurance company if it deducts for depreciation 25 % of the original value of the articles?

CHAPTER TWO

PROBLEMS OF THE PRODUCER

FARMING AS A BUSINESS

Everybody, whatever his calling, is interested in the success of the 6½ millions of farmers, upon whom devolves the part of feeding and clothing, not only themselves and the remaining 70% of our population, but also many millions in other parts of the globe.

A FARM HOUSE

The world's welfare depends upon a maximum production, at reasonable cost, with prices that yield the farmer interest on his capital and a fair compensation for his time, muscle, and brains.

FARM ACCOUNTS

While the farmer's accounts must necessarily be few in number and easily kept, they should show him

the expense of production and the profit or the loss made by the sale of his products, giving as much detail as possible.

THE INVENTORY

Many farmers limit their accounts to the making of an annual inventory, each of which they compare with the preceding one to ascertain what may be the profit or loss for a year.

The following are two annual inventories.

INVENTORY OF BURGUNDY FARM

Items	Jan. 1, 1919		Jan. 1, 1920	
	Quantity	Value	Quantity	Value
Real Estate	400 A.	$38,000.—	400 A.	$40,000.—
Cows, etc.	59	3,750.—	6	3,573.—
Hogs	27	346.—	37	396.—
Horses	7	1,200.—	9	1,850.—
Sheep	87	624.—	100	875.—
Hens	167	107.—	132	83.50
Machinery, etc.	—	1,500.—	—	1,350.—
Corn	80 bu.	120.—	125 bu.	187.50
Oats	200 "	180.—	90 "	81.—
Potatoes	40 "	36.—	80 "	80.—
Hay	15 T.	390.—	24 T.	576.—
Silage	90 "	360.—	110 T.	440.—
Feed	1½ "	45.—	6½ T.	195.—
Bills Receivable		46.—		—
Cash		670.—		2,148.—
Total		(*a*)		(*c*)
Less Mortgage		8,500.—		6,500.—
		(*b*)		(*d*)
			Last year	(*b*)
		Increase for year		(*e*)

WRITTEN PROBLEMS

1. (*a*) Find the value of the land and equipment on Jan. 1, 1919. (*b*) The net value after the deduction of the mortgage. (*c*) The gross value Jan. 1, 1920. (*d*) The net value at this date. (*e*) The increase in a year.

2. Find the total value of the following items, which have not been included in the farm equipment: Furniture, $2475; automobile, $1800; 2 buggies at $75 each; harness, etc., $95.

3. Find the value of the machinery, as follows:

1 thresher	$340	1 manure spreader	$80
1 binder	135	1 wheat drill, hoe	75
1 mower, 5-ft.	35	1 " " , disk	60
1 " 4-ft.	30	2 harrows, disk	50
1 roller	12	2 " , spring tooth	27
1 dray	2	2 " , spike tooth	24
1 hay loader	60	2 plows, 2-horse	12
2 hay racks	20	2 plows, 3-horse	18
1 weeder	12	2 double cultivators	50
2 wagons, 4-horse	120	2 single "	14
1 wagon, 2-horse	50	4 double shovel plows	10
1 corn planter, dbl. r.	40	1 smoothing harrow	12
1 " " , single	12	1 weeder	12
1 horse rake, side del.	40	1 horse rake, spring tooth	20

4. If 10% should be charged off for a year's depreciation, (*a*) what is the value on Jan. 1, 1920 of machinery worth $1500 on Jan. 1, 1919? (*b*) What will be its value on Jan. 1, 1921?

5. (*a*) How much is 5% of the inventory value of Jan. 1, 1919? (*b*) How much does the year's increase exceed this sum?

6. Besides the profits shown by the inventory, there should be added $32.50 a month for the use of the house, $450 for the vegetables, eggs, etc., supplied by the farm, and $35 for wood, etc., used as fuel. How much do these items amount to in a year?

RECEIPTS AND EXPENDITURES

The following table gives Mr. Appich's receipts and expenditures for five years:

Items	1st year	2d year	3d year	4th year	5th year
Receipts					
Cattle	$2084.30	$200.—	$500.10	$1937.60	$720.80
Sheep, lambs, wool	460.—	590.40	550.—	550.—	750.—
Wheat	1230.—	751.—	1557.—	1157.—	1682.40
Corn	969.50	620.—	1000.—	750.—	906.—
Oats	—	—	—	60.—	75.40
Hay	—	709.—	860.—	905.—	980.—
Live hogs	135.60	309.80	200.—	475.—	756.20
Poultry & dairy products	375.—	403.—	462.—	690.10	698.40
Wood	180.25	—	—	—	—
Apples	506.—	—	1427.—	512.40	1680.80
Total Receipts	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
Expenditures					
Labor	$400.—	$450.—	$500.—	$700.—	$700.—
Taxes	150.—	138.—	145.—	140.—	168.—
Farm Supplies	300.—	300.—	300.—	200.—	400.—
Interest	360.—	360.—	344.—	260.—	240.—
Fertilizer	135.—	116.50	147.86	185.39	185.39
Seed	550.—	400.—	600.—	750.—	600.—
Feed	212.—	150.—	150.—	197.50	188.—
Cattle (for Feeding)	1763.—	—	—	781.24	—
Hogs (for Feeding)	36.—	—	—	—	—
Extra Labor	—	—	300.—	—	418.—
Total Expenditures	(*f*)	(*g*)	(*h*)	(*i*)	(*j*)
Net Income	(*k*)	(*l*)	(*m*)	(*n*)	(*o*)

7. Find the receipts for each year, (*a*) to (*e*).
8. Find the expenditures, (*f*) to (*j*).
9. Find the net income, (*k*) to (*o*).

Milk Production

The following table shows the quantity of milk yielded by a herd of 27 cows on the farm of Mr. Popkins, who arranges to have the winter production as great as possible:

Month	Quarts	Rate	Receipts	Month	Quarts	Rate	Receipts
Sep.	4,606	3½¢		April	11,226	3¢	
Oct.	5,708	3½		May	9,342	3	
Nov.	5,983	4		Jun.	6,253	3	
Dec.	10,510	4		Jul.	4,142	2½	
Jan.	13,008	4		Aug.	4,280	3	
Feb.	11,858	4		Sold	1,460	4	
Mar.	12,467	4		Used	2,190	3½	
				Total	(*a*)		(*b*)

10. Find (*a*) the number of quarts produced during the year; (*b*) the receipts, including the value of the milk used in the families of the owner and two hired men. (*c*) Find the average value a quart. (*d*) Find the total weight of the milk at 2⅛ pounds a quart. (*e*) Estimate the average number of quarts a cow for the year.

SIGHT PROBLEMS

1. A farm worth $40,000 is assessed for purposes of taxation at $24,000. (*a*) What per cent of the actual value is the assessed value? (*b*) The taxes for a year are $168; what is the tax rate on each $1000 of the assessed value?

2. Give the value of each of the following a bushel or a ton, from these data.

80	bu.	worth	$120	15	tons	worth	$390
200	"	"	180	90	"	"	360
40	"	"	96	1½	"	"	45
90	"	"	81	24	"	"	576
80	"	"	100	110	"	"	440
150	"	"	180	6½	"	"	195

3. In a year $150 is expended for food for a family of five, and the farm supplies food worth $450. (*a*) What is the total value of the food? (*b*) What per cent of the total is supplied by the farm?

A BARN AND SILO

4. The fuel consumed in a year is worth $60. Of this there is spent $25 for coal. The remainder is supplied by the farm. What per cent of the annual expense does the latter constitute?

5. If an apple tree occupies a space 2 rods by 2 rods, (*a*) how many square rods does it occupy? (*b*) How many trees would there be to an acre (160 square rods)? (*c*) To 30 acres?

6. Give the value of 2400 bushels (*a*) at $1 a bushel. (*b*) At 2½ cents a bushel. (*c*) At 97½ cents a bushel.

7. What would be the cost of 5 bushels of seed at \$5¾ a bushel?

8. What is the value of 40 acres of land at \$120 an acre? If it is assessed at 60% of its value, what is the assessed value?

9. Give the interest for a year on \$400 at 6%.

Cost of a Crop

Some farmers desire to know more about their business than is disclosed by a comparison of inventories, or even by a statement of receipts and expenditures. They wish to ascertain (*a*) the cost of producing certain crops, and (*b*) the profit, if any, of selling them at market prices.

WRITTEN EXERCISES

The following is a memorandum of expenditures before the war for labor in the production of corn on 40 acres:

Items	Dates	Labor		Cost		
		Man Hours	Horse Hours	Man	Horse	Total
Plowing	Mar. 25–Apr. 2	80	320	(*a*)	(*b*)	(*c*)
Disking	Apr. 7–Apr. 29	90	360	(*a*)	(*b*)	(*c*)
Harrowing	" 29–May 4	25	50	(*a*)	(*b*)	(*c*)
Planting	" 30– " 5	30	60	(*a*)	(*b*)	(*c*)
Harrowing	May 10– " 14	35	70	(*a*)	(*b*)	(*c*)
Cultivating	" 27– " 30	60	120	(*a*)	(*b*)	(*c*)
"	Jun. 3–Jun. 6	55	110	(*a*)	(*b*)	(*c*)
"	" 14– " 18	50	100	(*a*)	(*b*)	(*c*)
"	" 23–Jul. 5	60	120	(*a*)	(*b*)	(*c*)
Picking seed	Sep. 27–Oct. 7	60	—	(*a*)	(*b*)	(*c*)
Husking	Nov. 2–Nov. 22	300	600	(*a*)	(*b*)	(*c*)
		(*d*)	(*e*)	(*f*)	(*g*)	(*h*)

1. Find the cost (*a*) of the man labor on each item at 20 cents an hour; (*b*) of the horse labor at 12½ cents an hour; (*c*) of the total expense for labor on each item. (*d*) Find the total number of man hours; (*e*) of horse hours. (*f*) Find the cost of the man labor; (*g*) of the horse labor; (*h*) the total labor cost.

NOTE: Find (*f*) by adding the (*a*) column. Check the result by multiplying 20 cents by (*d*). Find (*g*) by adding the (*b*) column. Check the result by multiplying 12½ cents by (*e*). Find (*h*) by adding the (*c*) column. Check the result by comparing this total with the sum of (*f*) and (*g*).

2. Find (*a*) the interest at 6% on the value of the land, 40 acres at $120 an acre; (*b*) the taxes at $7 a $1000 on the assessed value of $2880; (*c*) the cost of 5¾ bushels of seed at $5 a bushel; (*d*) of 8 tons of fertilizer at $16 a ton; (*e*) the interest at 6% on $400, the value of the machinery used.

3. Make out a statement in the following form showing the receipts, expenditures, and profits from the foregoing crop.

FIELD B — 40 ACRES — CORN

Sold 2400 bu. @ 97½¢			
Value of Stalks	(*a*) 29 25		
Total Receipts			(*b*)
Labor costs	(*c*)		
Fertilizer	(*d*)		
Seed	(*e*)		
Interest on Land (Rent)	(*f*)		
Taxes	(*g*)		
Interest on Equipment	(*h*)		
Other Expenses	19 25		
			(*i*)
Total Expenses			
	Profit		(*j*)

(*b*) Write the total value of the crop. Insert at (*c*), (*d*), etc., the several expenditures specified in

previous examples, and the separate item, $19.25. At (*i*) write the total expenditures. At (*j*) write the profit.

4. From the data given in the previous examples, find the cost of production (*a*) an acre; (*b*) a bushel.

5. Fill in the missing items in the following account, with an acre of grapes during 3 years:

One Year		*Two Years*		*Three Years*	
Cleaning land	$25.—	Forward	(*c*)	Forward	(*h*)
Plants	10.—	Stakes	7.50	Cultivating	16.—
Setting plants	1.—	Setting stakes	3.50	Fertilizers	15.50
Fertilizers	12.—	Fertilizers	15.50	New stakes	2.—
Cultivating	5.—	Cultivating	16.—	Crates and picking	42.75
Total	(*a*)	Total	(*d*)	Total	(*i*)
Interest on (*a*) for 1 yr. at 6%	(*b*)	Interest on (*d*) for 1 yr. at 6%	(*e*)	Interest on (*i*) for 1 yr. at 6%	(*j*)
Carried forward	(*c*)	Total	(*f*)	Total	(*k*)
		Deduct sales of 52 crates @ $2.50	(*g*)	Deduct sales of 75 crates @ $2.50	(*l*)
		Balance forward	(*h*)	Balance	(*m*)

All of the permanent laborers on Mrs. Bruen's farm are men with families. Each is supplied with a house, an acre of ground for a garden, a cow with pasture, and firewood.

SIGHT PROBLEMS

1. Frank Kerr receives $200 a year in cash; the use of a house, which he considers as the equivalent of $10 additional monthly wages; with land and cow, from which he derives products worth $70 during the year. What is the value of the foregoing for a year?

2. His wages are increased $25, the following year, he is given corn to the value of $25, and he sells $25 worth of vegetables. What is his income the second year, including the use of the house and the value of the other products obtained from the land, cow, etc., which this year were worth $80?

ONE AND ONE-HALF ACRES IN TEN HOURS

3. During the next year he receives an additional $25 in cash, and 50 bushels of corn worth $1 a bushel. What is his income for the third year, including the use of the house, and $120 as the value of vegetables, milk, etc., sold and used?

4. During the fourth year he receives, in addition to other items given in the third year, 30 bushels of wheat worth $1.50 a bushel. What is his income for the fourth year?

5. Find the cost of plowing an acre of land when one man at \$2 a day, and 4 horses at \$1.25 each a day, plow (*a*) 2 acres in a day; (*b*) 3 acres.

6. If it requires 80 hours of a man's work, and 320 hours of a horse's work to plow 40 acres, (*a*) how many horses are used to the plow? At 10 hours a

TWENTY ACRES IN SIXTEEN HOURS

day, how many days are required (*b*) to plow 40 acres? (*c*) To plow 5 acres?

7. Assuming that a man and a team can do work as follows, find (I) the number of days of labor required to do each of the following for 40 acres; and (II) the cost of each at the rate of \$2 a day for a man, and \$2.50 for a team:

a Plowing, 1⅗ acres a day
b Disking, 6⅔ " " "

c Harrowing, twice, 10 acres a day each time
d Rolling, 13⅓ acres a day
e Seeding, 10 " " "
f Spreading manure, ¼ acre a day
g Cultivating three times, 6⅔ acres a day each time
h Harvesting, 6⅔ acres a day
i Tying and shocking, no horses, 3⅓ acres per man
j Husking, ⅝ acre a day, per man, no horses

8. (*a*) How many men and teams would be required to do the plowing in 5 days? (*b*) How many men would be required to do the husking in 8 days?

WRITTEN PROBLEMS

1. What are the average yearly receipts of James Reed for wages, when he is paid during four years $200, $275, $350 and $385 respectively?

2. (*a*) Find his average yearly receipts from the sale of vegetables, milk, etc., which, for the four years, are $25, $45, $70, and $95 respectively.

3. Find his average yearly income.

4. His cash expenditures for food average $70 per year, the remainder being supplied by the farm. Mrs. Reed boards the temporary help, the profits on which supply the family clothing. How much is left during the four years for expenses other than rent, food, and clothing?

5. Each of Mrs. Bruen's permanent hands works 2¾ hours on Sundays caring for the stock, 3½ hours on each of six holidays, 5½ hours on Saturdays, and 10

hours on the other days. How many hours a year does each work?

6. In calculating the labor cost per man-hour, Mrs. Bruen determines the annual expenditures for a man's yearly work from the following data: Wages, supplies, etc., \$385; 6% on \$50 for use of cow; 6% on \$250 for land for vegetables and pasture; 6% on \$1000, the cost of the house, as rent; and \$2 for insurance, etc. Find (*a*) the labor cost of a man a year. (*b*) Find the cost per man-hour based on the number of hours of work in example 5.

7. Find the total expense of a horse for a year, covering 6% interest on \$275, its cost; feed, 365 days, at 30 cents a day; shoeing, etc., \$9.

8. (*a*) If the horse works 1080 hours in a year, what is the expense an hour? (*b*) What is the expense an hour if the horse works only 1000 hours a year?

9. Arthur Gravely bought a tractor for \$2080. What is the yearly interest at 6% on its cost?

10. If the life of the tractor is 6½ years, what is the average yearly loss by depreciation?

11. Find the total of the yearly interest, depreciation, and repairs amounting to \$75.20.

12. The tractor is used for 104 days at an average of 12½ hours a day. How many hours of work does it furnish a year?

13. What is the expense an hour of work for interest, depreciation, and repairs?

SECTION V

FROM THE PRODUCER TO THE CONSUMER

CHAPTER ONE

BUYING AND SELLING AGENCIES

SIGHT EXERCISES

1. From the following data (*a*) give the price paid for a barrel of apples by the consumer:

Farmer receives	$2.50	Cartage to jobber	.15
Local buyer's profit	.25	Jobber's profit	.25
Freight and refrigeration	.35	Cartage to retailer	.25
Receiver's profit	.15	Retailer's profit	1.10

(*b*) What per cent of the price paid by the consumer is received by the farmer? (*c*) What is the total of the gross profits of the four dealers? (*d*) What per cent does each receive of the price paid by the consumer? (*e*) What per cent of this price goes for freight and refrigeration? (*f*) What per cent for the two cartage items?

2. When a farmer receives $1.10 per 100 pounds for onions that cost the consumer $2.50, what per cent of this selling price does the farmer receive?

3. A lot of cabbage is subject to the following charges from the farm to the retailer.

Farmer receives	$5 per ton	Jobber's profit	$3 per ton
Freight charges	10 " "	Commission	1 " "
Refrigerating	5 " "	Cartage	2 " "
Barrels and handling	2 " "	Wholesaler's profit	2 " "

(*a*) Give the total of the foregoing. (*b*) How much a ton does the retailer receive, at the rate of 3¢ a pound? What per cent of the cost to the customer does the farmer receive?

Commission

Compensation received by one person for buying or selling goods for another, for collecting money, for selling real estate, etc., is called a *commission*. The person doing this work is called the agent; the person for whom it is done is called the *principal*.

An agent receiving eggs, butter, vegetables, berries, etc., to be sold for the account of a distant principal is called a *commission merchant*. A shipment of this kind is called a *consignment*, the principal being the *consignor* and the agent the *consignee*.

A Consignment of Produce

A. T. Weekes, of Marquette, Kansas, ships to Sullivan and Conroy, commission merchants, Kansas City, 60 cases of eggs and 150 barrels of potatoes, to be sold for his account.

The shipper (consigner) delivers the goods at Marquette to the Missouri Pacific R.R. agent, from whom he receives a *bill of lading* (receipt), which sets forth that the railroad company has received from A. T.

Weekes the above-mentioned items, to be delivered to Sullivan and Conroy upon surrender of the bill of lading and payment of the amount due for freight.

The *consignees* (Sullivan and Conroy) present the bill of lading, pay the freight bill, and transfer the goods to their store. When all the articles are sold, they send an account of sales to Mr. Weekes with a check for the sum due him.

ACCOUNT OF SALES

KANSAS CITY MO., Aug. 25, 1919

SULLIVAN & CONROY

Sold for account of A. T. WEEKES,
Marquette, Kansas.

60 cases Eggs. 150 bbl. Potatoes.

1919					
Aug.	18	40 cases Eggs, 1200 doz. .32	(*a*)		
	20	80 bbl. Potatoes 3.20	(*b*)		
	23	20 cases Eggs, 600 doz. .31	(*c*)		
	24	70 bbl. Potatoes 3.15	(*d*)		(*e*)
		Charges			
Aug.	17	Freight and drayage	123	75	
	25	Commission, 4%	(*f*)		(*g*)
		Net proceeds by check inclosed			(*h*)

WRITTEN EXERCISES

1. Copy and complete the foregoing account. Insert the extensions (*a*) to (*d*) and the footing at (*e*). Find (*f*), which is 4% of (*e*). Insert at (*g*) the total charges. Find (*h*) by deducting (*g*) from (*e*).

2. Make out a check on the First National Bank for the sum due Mr. Weekes.

3. Find the weight (*a*) of a case of 30 dozen eggs at 22 ounces per dozen eggs and adding 8¾ pounds for the weight of the package. (*b*) Of the shipment of 30 cases.

4. Find the weight of a barrel containing 2¾ bushels of potatoes at 60 pounds to the bushel, adding 21 pounds as the weight of the barrel.

5. How much less than a minimum car load of 36,000 pounds is there in the total shipment?

6. (*a*) What is the commission at 2½% for collecting a debt of $240.75? (*b*) How much does the agent remit to his principal?

7. Find the commission on the sale of a house for $9750 at 5% on $1000, 2½% on $4000, and 1% on the remainder.

8. How much should a salesman sell in a year to yield him a commission of $5000 at 3¾% on his sales?

9. An agent bought for his principal 60,000 feet of lumber at $42 per 1000 ft. How much did the lumber cost the latter after he had paid freight amounting to $275 and the agent's commission of 2½%?

10. A commission merchant received a consignment of 60 crates of blackberries. He sold 20 crates at $2.40 each, 15 crates at $2.60, and the remainder at $2.50. Find the net proceeds after the deduction of charges amounting to $12.75 and commission at 5%.

The Local Buyer

The individual producer generally disposes of his goods in the neighborhood. His surplus eggs he exchanges for groceries at the nearest store. His

milk he sells to the creamery, his grain to the owner of the elevator at the railroad station, his cotton to the warehouse man, his cattle to a traveling buyer, etc.

WRITTEN EXERCISES

1. A farmer delivered 12 loads of wheat to an elevator. The gross weights and the tares were as follows:

Gross	Tare	Gross	Tare	Gross	Tare
3150	1061	3216	1070	3168	1069
3210	1062	3420	1072	3056	1073
3095	1064	3175	1073	3384	1067

Find (*a*) the total gross weight, (*b*) the total tare, (*c*) the total net weight, (*d*) the sum received for the wheat at \$2.10 a bushel (60 lb.).

2. How much does a planter receive for 12 bales of cotton, weighing, respectively, 523 lb., 519 lb., 532 lb., 527 lb., 518 lb., 516 lb., 517 lb., 523 lb., 518 lb., 525 lb., 516 lb., 525 lb., when the deduction for tare is 22 pounds a bale, and the rate is 23¾ cents a pound?

3. A local buyer pays pickers of wild huckleberries 12 cents a quart for picking, and he delivers the berries to a local shipper at an advance of 2 cents a quart. The latter supplies crates holding 32 quart-cups and consigns the berries to a commission merchant. Find the shipper's profit on a crate when the berries bring 20 cents a quart less 10% commission; 50 cents is deducted for expressage, and 30 cents for the cups and the deterioration of the crate.

4. Copy and complete the following statement rendered by the commission merchant:

WILMINGTON, N. C., Aug. 1, 1919

E. K. WILSON
Fruit and Produce Commission Merchants
Sold for account of Mr. F. T. O'ROURKE,
Hamlet, N. C.

	12/32 Hbs.	.20	(a)		
	6/32 " (soft)	.18	(b)		(c)
	Express		(d)		
	Commisson		(e)		(f)
	Check herewith				(g)

NOTE: (a) 12/32 means 12 crates of 32 quarts each, which bring 20 cents per quart. (b) Six crates of 32 quarts each bring 18 cents per quart, being overripe. Insert at c the total receipts.

At (d) insert expressage on 18 crates at 45 cents per crate; at (e) the commission at 10% on the receipts (c); at (f), the total of (d) and (e); at (g) the difference between (c) and (f).

Observe the abbreviations used in expressing 12 crates of huckleberries, each containing 32 quarts.

5. Find the weight of a loaded crate, when the crate weighs 8 pounds, each quart-cup 1 ounce, and the berries 1½ pounds a quart.

SELLING THROUGH A BROKER

Tormey and Ryan are local grain buyers, at Fairview, near Marquette, Kansas.

Wishing to sell three car loads of No. 2 wheat in Kansas City, they load it on cars numbered 18790, 24360, and 29411 of the Missouri Pacific Railroad, consigned to their broker, George Smith, notifying the latter by the following telegram:

> Have shipped 1800 bu. 2 wheat, M. P., 18790, 24360, 29411. Have drawn for $3000.

The last sentence notifies the broker that they have made a sight draft on him for $3000, which he is to

pay to their credit at a bank in Kansas City, and receive the bill of lading.

The draft is in this form:

$3000 $^{00}/_{100}$ Marquette, Kan., July 30, 1919

At sight, pay to the order of Ourselves
Three Thousand 00/100——————Dollars
value received, and charge to account of
To George Smith
Kansas City, Mo.
Tormey & Ryan

They deposit this draft in The Farmers Bank of Marquette, for collection after indorsing it as follows:

Pay to the order of The Farmers Bank
for collection only.
Tormey & Ryan.

This draft they attach to the bill of lading. The Farmers Bank indorses it over to its Kansas City correspondent as follows:

Pay to the order of
The First National Bank, Kansas City, Mo.
for collection only
The Farmers Bank, Marquette, Kansas
A. T. Sullivan, *Cashier.*

sending it and the attached bill of lading with instructions to deliver the latter to George Smith when he "takes up" the draft.

When the draft with the bill of lading reaches The First National Bank, it immediately notifies George

Smith. The latter pays $3000 to the bank, receiving the draft and the bill of lading. The First National Bank transmits the $3000 less collection charges to The Marquette Bank, which places the proceeds to the credit of Tormey & Ryan.

As soon as Mr. Smith had learned by telegraph of the consignment, he offered the wheat to Chas. Scott & Co., who agreed to take it, on arrival, at $2.15, provided it proved, upon inspection, to be of No. 2 grade.

When the wheat reached Kansas City on Aug. 5, Chas. Scott & Co. accepted it, giving a check for its value at its arrival weight of 107,160 pounds.

George Smith rendered Tormey & Ryan the following account of sale:

KANSAS CITY, No., Aug. 6, 1919

GEORGE SMITH
Grain Receiver

Sold for account of
MESSRS. TORMEY & RYAN
Marquette, Kan.

1919 Aug.	5	107,160 # Wheat #2 2.15 Cars M. P. 18790, 24360, 29411	(a)			
		Charges				
		Freight 2¢ 36.—				
		Interest 4 da. @ 6% (b)				
		Weighing .40				
		Inspection .35				
		Commission (c)	(d)			
		Net proceeds to your credit	(e)			
		Draft	3000			
		Balance due you			(f)	
		George Smith per K.				

WRITTEN EXERCISES

1. (*a*) How many bushels are there in 107,160 pounds of wheat? (*b*) Find its value at $2.15 a bushel.

2. George Smith charges a commission of 1¢ a bushel, also interest at 6% for 4 days on $3000. What is (*a*) his commission? (*b*) The interest?

3. At 2 cents a bushel, for freight, what is the rate per 100 pounds?

4. Copy and complete the foregoing statement, from data given above: Observe that (*d*) is the total of the charges, that (*e*) is (*a*) less (*d*), and that (*f*) is (*e*) less $3000.

5. Write George Smith's check on The First National Bank in settlement of his account with Tormey & Ryan.

Buying through a Broker

A broker acts as purchasing agent as well as selling agent. George Smith having received an order from Burton & Billings, Milwaukee millers, to buy 6000 bushels of wheat for their account, made an agreement with Robert Black for the delivery of the wheat at $2.17¾, in Kansas City on cars "routed" for Milwaukee.

When the cars were loaded, Robert Black received a bill of lading consigning the shipment to Burton & Billings, Milwaukee. This bill he attached to a sight draft for the cost of the wheat, which he deposited for collection in the Commercial National Bank.

George Smith notified his principals by telegraph of the purchase, and mailed his bill of $60 for commission.

In this transaction the broker did not handle the money paid for the wheat. He gave the millers' order to Robert Black, and received his commission from them. Mr. Black collected the value of the wheat from the consignees by means of a draft.

WRITTEN EXERCISES

1. Find (*a*) the cost of 6000 bushels of wheat at $2.17¾. (*b*) The freight at 6 cents per 100 pounds. (*c*) The total cost in Milwaukee including freight and commission.

2. (*a*) Make out the sight draft drawn by Robert Black on Burton & Billings. (*b*) Write the indorsements transferring it to The Commercial Bank for collection, and the latter's indorsement, for collection, to the Wisconsin Trust Co., of Milwaukee.

3. Ryan & Co., brokers of Savannah, bought for a Lowell mill 800 bales of cotton weighing 394,000 pounds at 23⅞¢ delivered to a steamer. Find the cost of the cotton and the commission of $5 per 100 bales.

Storage

Railroad companies require the removal of goods from their cars within, say, 48 hours after arrival at terminus. If such merchandise is not intended for immediate sale, it is stored in a warehouse. Butter, eggs, poultry, etc., are bought by dealers at low rates and placed in cold storage until higher prices enable the dealers to withdraw them for sale at a profit.

WRITTEN EXERCISES

1. A dealer stored 175 cases of eggs for 5 months at the following rates:

For the first 25 cases, 20 cents a month
" " next 25 " 18 " " "
" " third 25 " 16 " " "

For each additional 25 cases 2 cents less a month than the rate for the previous 25. What was the cost of storage?

2. Find the cost of storing 1875 pounds of poultry for 3 months, at ¼¢ a pound for the first month and ⅛¢ a pound thereafter.

3. (*a*) Find the cost of storing 40,000 bushels of wheat for 40 days at ⅝¢ a bushel for receiving, weighing, and storing for 10 days; and ¼¢ a bushel for each succeeding 5 days. (*b*) Find the cost of screening and blowing it at ⅛¢ a bushel, and delivering it to an ocean vessel at ½¢ a bushel.

4. What is the difference in the expense of the storage in the last example, and the rate in another warehouse of 1¢ a bushel for 20 days and ⅒ a bushel for each subsequent day?

CHAPTER TWO

TRANSPORTATION PROBLEMS

SIGHT EXERCISES

1. Assuming 1000 pounds, including wagon, as the reasonable load for a mule to draw for 10 hours on a level dirt road, (*a*) how many pounds can 2 mules draw on a wagon weighing 1000 pounds? (*b*) 4 mules, on a wagon weighing 1500 pounds? (*c*) 6 mules, on a wagon weighing 2000 pounds?

2. Find the rate per ton-mile (1 ton for 1 mile) when it costs \$1 to transport 1 ton (*a*) for 4 miles by horse and wagon; (*b*) for 125 miles by rail; (*c*) 333⅓ miles by canal; (*d*) 1500 miles on the Great Lakes.

3. A team that can haul a load of 3000 pounds, including a 1000-pound wagon, on a level dirt road, can haul 5000 pounds on the same road after it is macadamized. What part of the expense of transportation is saved by the improvement of the road, assuming that the same wagon is used?

4. Find the cost (*a*) of 9 pounds of oats at 80 cents a bushel of 32 pounds. (*b*) Of 12 pounds of oats. (*c*) Of 14 pounds of hay at \$1.25 per 100 pounds.

5. Find the cost of hauling wheat when the time required to make the round trip was 10 hours, at the cost of 30 cents per man-hour for the driver and 6 cents per mule-hour for each of a pair of mules.

6. When the load consisted of 20 bushels and the distance to the station is 10 miles, give (*a*) the number of ton-miles; (*b*) the cost per ton-mile; (*c*) the cost per bushel.

NOTE: In estimating the number of ton-miles ignore the weight of the wagon; also the length of the return trip with the empty wagon.

7. How many pounds does a mule draw on rails when his load consists of 3 cars each weighing 4000 pounds and containing 3000 pounds of coal?

8. How many tons are there in a canal boat load of 8000 bushels of wheat?

9. How many ton-miles are represented by a canal boat containing 240 tons of freight and going 2 miles an hour for 24 hours? If 4 mules are used, how many ton-miles are obtained a mule?

10. At 2 miles an hour (*a*) how many hours would it require a mule-drawn boat to travel 288 miles? (*b*) How many days?

ANIMAL TRANSPORTATION

WRITTEN EXERCISES

1. An army mule's daily ration is 9 pounds of oats and 14 pounds of hay. (*a*) Find the daily cost at 80

cents a bushel of 32 pounds for oats, and $1.25 per hundred pounds for hay. (*b*) Find the cost for 365 days, including 100 pounds of straw monthly for bedding at 80 cents per hundred pounds.

2. Find the yearly cost of keeping a horse whose daily ration is 12 pounds of oats and 14 pounds of hay. Use the foregoing prices, and include the cost of bedding as above.

3. A planter's figures show that it cost 44 cents a day for a mule's feed for 210 days, and 12 cents a day for pasture, etc., for 155 days. (*a*) Find the cost for the mule's keep a year after adding $1 a month for interest, depreciation, etc. (*b*) Find the cost a mule-hour when it works 2000 hours a year.

4. (*a*) Find the cost of transporting 20 bushels of wheat to the railroad station when it requires 10 hours to make the round trip at 30 cents an hour for the driver and 12 cents an hour for the team. (*b*) What is the cost a bushel? (*c*) If the farm is 10 miles from the station, how many ton-miles are represented by a load of 1200 pounds hauled 10 miles? (*d*) What is the cost per ton-mile?

5. A mule travels 3 miles an hour for 8 hours a day, drawing 3 car loads of coal from the vein to the shaft and returning with the empty cars. If each loaded car contains 1½ tons of coal (*a*) how many ton-miles are represented by the loads drawn in 4 hours? (*b*) Give the cost a ton-mile, if the boy driver receives $2 a day and the mule's work is considered to be worth 70 cents a day.

Improved Roads

6. Mr. Wilson hauls on an average 64 tons a year a distance of 5½ miles. (*a*) How many ton-miles does this represent? (*b*) What is his annual saving in the cost of hauling, by the reduction of 18¾ cents a ton-mile by the improvement of his road? His share of the cost of the improvement was $375. (*c*) What per cent of this amount is the annual saving?

7. The year following the improvement of the roads of Bagnell County 100,000 tons of freight were hauled an average distance of 7½ miles at a reduction of 17½ cents a ton-mile in the cost of transportation. (*a*) What was the saving? The improvement of the 120 miles of roads cost $1750 a mile. The value of the property in the county was $18,000,000. (*b*) What per cent of the value of the property did the improvement cost? (*c*) What was Mr. Bradford's share of the cost, if his farm of 160 acres was valued at $112.50 an acre?

8. (*a*) How many square yards are there in the surface of a road a mile (1760 yards) long and 15 feet wide? (*b*) If a ton of broken stone will cover 3⅛ square yards to the proper depth, how many tons will be required for a mile of road?

Railroad Transportation

The freight rates between Missoula and Portland, Oregon, are as follows:

1st class	$1.60	a	100	lb.	Furniture, Dry Goods, etc.
2d "	1.36	"	"	"	Hardware, Copper, etc.
3d "	1.12	"	"	"	Paint, Plow Points, etc.
4th "	.96	"	"	"	Canned Vegetables, etc.
5th "	.80	"	"	"	Wrapping Paper, etc.

There are four other classes, A, B, C, and D, the rates being 64¢, 48¢, 40¢, and 32¢, respectively.

9. Find the cost a ton-mile at each of the foregoing rates from Missoula to Portland, 634 miles.

10. Find the freight from Missoula to Portland, for an automobile weighing 2750 pounds, when the rate is double the 1st class one.

11. The freight rates from Denver to Salt Lake City, 745.5 miles, are as follows a 100 pounds:

1st class, $1.54; 2d class, $1.31; 3d class, $1.15; 4th class, $ .96; 5th class, $ .79½. Find the rate a ton-mile for each class.

12. The rate on fruit for less than car loads (L.C.L.) is $1.54 a 100 pounds; for car-load lots (C.L.), it is $1 a 100 pounds. What per cent of the former rate is the latter?

13. How much will a shipper of 18,690 pounds of fruit (and packages) save by paying $1 a 100 pounds for a car load of 24,000 pounds, instead of the L.C.L. rate of $1.54 a 100 pounds on the actual weight of the shipment?

Commodity Rates

In addition to the 100-lb. rate for ordinary freight, there is frequently a "commodity" rate for such articles

as grain, which pays by the bushel; coal, by the ton; milk, by the 40-quart can, etc.

Water Transportation

1. (*a*) How many tons are there in a Lake steamer load of 400,000 bushels of wheat? (*b*) How many freight cars containing 36,000 pounds each are needed to transport this quantity of wheat? (*c*) How many such steamer loads will be required to fill a tow of 48 barges, each having a capacity of 1000 short tons?

2. Find the cost of transferring 400,000 bushels of wheat from canal boats to an ocean steamer as follows:

Harbor towing, \$4 a boat of 8,000 bushels
Transportation of floating elevator, $\frac{1}{8}$¢ a bushel
Weighing and transferring, $\frac{5}{8}$¢ a bushel
Trimming on canal boat, \$1.50 a 1000 bushels
Trimming on steamer, \$2.— a 1000 bushels

3. (*a*) When the rail freight rate was 16 cents a 100 pounds from Chicago to New York, what was the rate a bushel? (*b*) How much less a bushel would it cost at 1.2¢ a bushel by lake from Chicago to Buffalo, ½¢ a bushel for transferring from steamer to canal boat; and 5¢ a bushel by canal boat from Buffalo to New York?

4. At 9.6 cents for the freight on a bushel of wheat for 960 miles, what is the rate a ton-mile?

5. Find (*a*) the canal (and river) rate a ton-mile when the cost for transporting a bushel of wheat was .44¢ for 440 miles. (*b*) The ocean rate a ton-mile when the cost was 6¢ for a bushel 1800 miles.

6. When the steamboat rate on eggs is 47¢ a 100 pounds between Cincinnati and Memphis, find the ton-mile cost, taking (*a*) the distance by water, 750 miles; (*b*) the land distance of 500 miles by road. (*c*) Find the ton-mile rate by rail at 60 cents a 100 pounds for 494 miles.

7. How much more would it cost to ship 5400 dozen eggs in cases containing 30 dozen each than in cases containing 36 dozen each, when the freight is 23 cents a case regardless of its weight?

Shipments by Express

The minimum freight on a package is the rate per 100 pounds, while express rates cover all weights from a pound up.

The Union Express Company, which covers nearly every section of the United States, has nearly 300 "scales." The "scale" for any office gives the rate to every other office in the country.

Express packages are divided into three classes, according to their bulk, value, etc. Most of the packages are of the first class.

They are carried on a car forming part of a passenger train, which insures rapid transit. They are delivered to the stores or residences of city consignees.

Specimen Rate Scales

The cost of the expressage on a first-class package to New York from certain specified cities is as follows for certain weights up to 100 pounds.

RATES TO NEW YORK FROM

Weights	Chicago	St. Louis	Dallas	Denver	Butte	San Francisco
1 lb.	$0.30	$.30	$.33	$.33	$.36	$.38
2 "	.32	.33	.38	.40	.44	.49
3 "	.34	.35	.44	.45	.53	.60
4 "	.37	.37	.49	.52	.60	.73
5 "	.40	.41	.55	.57	.69	.84
6 "	.42	.43	.60	.64	.77	.95
7 "	.44	.45	.65	.69	.86	1.06
8 "	.47	.48	.71	.76	.93	1.17
9 "	.49	.51	.77	.81	1.02	1.28
10 "	.51	.53	.81	.87	1.10	1.39
—	—	—	—	—	—	—
20 lb.	.75	.79	1.36	1.47	1.94	2.51
—	—	—	—	—	—	—
30 lb.	.98	1.04	1.90	2.07	2.76	3.62
—	—	—	—	—	—	—
50 lb.	1.45	1.56	2.99	3.27	4.42	5.85
—	—	—	—	—	—	—
100 lb.	2.64	2.86	5.72	6.27	8.58	11.44

A fraction of a pound is taken as a pound.

WRITTEN EXERCISES

1. Find the expressage to New York on packages from Chicago as follows:

a 24 weighing 4½ pounds each.

b 47 weighing 6¼ pounds each.

c 36 weighing 8¾ pounds each.

2. Find the expressage to New York on packages from St. Louis as follows:

a 137 weighing 3 lb. 5 oz. each.

b 294 weighing 19 lb. 7 oz. each.

c 178 weighing 7 lb. 5 oz. each.

3. Find the expressage to New York on the following packages:

a 168 from Dallas, each weighing 3½ pounds.
b 209 from Denver, each weighing 9 lb. 14 oz.
c 329 from Butte, each weighing 29¾ pounds.
d 415 from San Francisco, each weighing 49 lb. 9 oz.

4. A merchant shipped from New York packages as follows, each weighing 99 lb. 1 oz.: 153 to Chicago, 217 to St. Louis, 98 to Dallas, 369 to Denver, 54 to Butte, and 147 to San Francisco. Find the total charge for expressage.

Mail Matter

Domestic

United States and Possessions

Domestic mail matter is divided into four classes:

First — Letters, postal cards, sealed packages.

Second — Periodical publications.

Third — Miscellaneous printed matter weighing four pounds or less

Fourth (Parcel Post) — All mailable matter not included in previous classes.

Rates of Postage

First-class Matter

Postal cards, 1 cent each. Letters and sealed packages 2 cents an ounce or fraction thereof. Mail carried by aeroplane, 6 cents for the first ounce or fraction thereof, and 6 cents for each additional ounce.

SIGHT EXERCISES

1. Give the cost of postage (*a*) on 246 letters each weighing less than 1 ounce. (*b*) On 122 similar letters weighing over 1 ounce, but less than 2 ounces.

2. What postage should you pay on a sealed package weighing 10 ounces?

3. Give the cost of mailing 25 letters by aeroplane post, each weighing three quarters of an ounce.

Second-class Matter

The rate of postage on newspapers and periodicals bearing notice of entry as second-class matter and sent unsealed by the public, is 1 cent for each 4 ounces or fraction thereof.

4. Give the postage on a newspaper weighing

a 1 oz. *b* 2½ oz. *c* 3¼ oz. *d* 8½ oz. *e* 11 oz.

5. Give the postage on a package of magazines weighing

a 18 oz. *b* 2 lb. 3 oz. *c* 3 lb. 1 oz. *d* 5 lb. 10 oz.

Third-class Matter

The rate of postage on circulars, newspapers, and periodicals not entered as second-class, and other printed matter (not books), is 1 cent for 2 ounces or fraction thereof. Limit of weight is 4 pounds.

6. Give the postage on a map weighing

a 5 oz. *b* 3 oz. *c* 7 oz. *d* 4½ oz. *e* 6½ oz. *f* 8 oz.

7. Give the postage on a package of pictures weighing

a 12 oz. *b* 1 lb. 10 oz. *c* 3 lb. 4 oz. *d* 2 lb. 9 oz. *e* 21 oz.

Fourth-class Matter

Parcels weighing 4 ounces or less, except books, seeds, plants, etc., pay 1 cent for each ounce or fraction thereof. Parcels weighing 8 ounces or less, con-

taining books, catalogues, seeds, plants, etc., pay 1 cent for each 2 ounces or fraction thereof.

Larger parcels of books, seeds, plants, and other mailable articles pay the parcel post rates given on another page. These are based on the weight of a parcel and the distance to which it is to be carried.

8. Give the postage on a package of seeds weighing *a* 1 oz. *b* 7 oz. *c* 3 oz. *d* 8 oz. *e* 5 oz. *f* 4 oz.

PARCEL POST RATES

Weight	Local Rates	1st and 2nd zones 50 to 150 mi.	3d zone 150 to 300 mi.	4th zone 300 to 600 mi.	5th zone 600 to 1000 mi.	6th zone 1000 to 1400 mi.	7th zone 1400 to 1800 mi.	8th zone over 1800 mi.
1 lb.	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2 "	.06	.06	.08	.11	.14	.17	.21	.24
3 "	.06	.07	.10	.15	.20	.25	.31	.36
4 "	.07	.08	.12	.19	.26	.33	.41	.48
5 "	.07	.09	.14	.23	.32	.41	.51	.60
6 "	.08	.10	.16	.27	.38	.49	.61	.72
7 "	.08	.11	.18	.31	.44	.57	.71	.84
8 "	.09	.12	.20	.35	.50	.65	.81	.96
9 "	.09	.13	.22	.39	.56	.73	.91	1.08
10 "	.10	.14	.24	.43	.62	.81	1.01	1.20
etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.	etc.
20 lb.	.15	.24	.44	.83	1.22	1.61		
etc.	etc.	etc.	etc.	etc.	etc.	etc.		
50 lb.	.30	.54	1.04	2.03	3.02	4.01		
etc.	etc.	etc.						
70 lb.	.40	.74						

Maximum weight 70 lb. | Maximum weight 50 lb.

9. Give the postage on each of the following packages mailed in Kansas City:

Place	*Zone*	*Weight*	*Any P. O.*	*Zone*	*Weight*
a Pierre, S. D.	4	9 lb. 13 oz.	*b* In Alaska	8	22 lb. 3 oz.
c Topeka, Kans.	1	69 lb.	*d* " Nevada	6	9 oz.
e Peoria, Ill.	3	17 lb. 4 oz.	*f* " S. C.	5	16 lb. 7 oz.
g San Francisco	7	33 lb. 8 oz.	*h* " Conn.	6	9 lb. 7 oz
i Lincoln, Neb.	2	48 lb.	*j* " Hawaii	8	14 oz.
k Kansas City	Local	64 lb.	*l* " D. C.	5	27 lb.

CHAPTER THREE

PROBLEMS OF THE MANUFACTURER

Making and Selling Bread

PREPARATORY EXERCISES

1. Mr. Taylor purchased an established bakery, the books of which showed the following expenditures for the preceding year: materials, $4500; help, $2920; sundry expenses, $820. Give the total expenditures.

2. What was the profit for the year if the sales amounted to $9000?

3. Find the cost (*a*) of 300 barrels of flour at $11 a bbl.; (*b*) of 60 barrels; (*c*) of 5 barrels; (*d*) of 365 barrels.

4. If the total cost of materials was $4500, find the amount spent for the other articles: sugar, yeast, salt, butter, eggs, etc.

5. If 16 tons of coal were used in 300 working days, how many working days would a ton last?

6. The rent of the building is $40 a month, ⅛ of which should be charged to the bakeshop and ½ to the store. (*a*) What fraction of the rent remains to be charged to the living expenses of the family? (*b*) How many dollars a year?

7. The expense for help included a baker's wages at $1200, the proprietor's services at $1200, and the pay of a girl in the store. How much did she receive (*a*) a year? (*b*) A week?

8. A barrel of flour makes 315 pounds of dough. If $\frac{1}{9}$ of this is lost in the process of baking, (*a*) how many pound-loaves of bread does a barrel of flour make? (*b*) How many loaves weighing 14 ounces each?

9. If $4000 was received from the yearly sales of bread and rolls, how much was realized from the sales of pies and cakes?

10. If $\frac{2}{3}$ of the baker's time was spent in making pies and cakes, how much of his yearly pay should be charged to the labor on these?

Mr. Taylor's Memoranda

From the accounts of the preceding year, Mr. Taylor made the following memorandum of

Manufacturing Costs

Materials	$4500	Flour, Yeast, Salt, Sugar, Fruit, etc.
Direct labor	1200	Wages of the baker
Prime cost	?	Sum of the foregoing
Indirect labor	600	One-half of the proprietor's services
Rent	80	One-sixth of rent of building
Fuel, light, etc.	210	Coal, gas bills, etc.
Depreciation, etc.	110	Repairs, insurance, etc.
Manufacturing costs	?	Total of the preceding five items.

11. What was the *prime cost*, which is the total spent for materials and the pay of the baker?

12. What were the *overhead factory expenses*, the sum of all other items of expenditures incurred in preparing the articles for sale?

13. What fraction of the prime cost was the overhead factory expense?

This fraction is called the *factory rate of burden.*

One-half of the proprietor's allowance is charged to the bakeshop.

Mr. Taylor made another memorandum showing

SELLING COSTS

Proprietor's pay	$600	One-half of allowance
Salesgirl's pay	520	$10 per week
Rent	240	One-half of $480
Heat, Light, etc.	90	Coal used to heat store, gas bills, etc.
Depreciation, etc.	90	Repairs, insurance, etc.
Selling costs	?	Total of the foregoing

Mr. Taylor combined the foregoing memoranda into the following:

Statement

Labor and Materials	$5700
Factory Expenses	1000
Selling Expenses	1540
Profit	?
Received from sales	$9000

14. What was the profit for the year?

15. Give the rate per cent of the profit on the gross receipts from sales.

Mr. Taylor made a detailed examination of the manufacturing and the selling costs to determine the percentage of profit on each of the separate items of (*a*) bread, (*b*) cakes, and (*c*) pies.

Two barrels of flour were used on each Saturday, and on the eve of a holiday, and one on each other working day.

One-half of each barrel was used for bread, one-third for cakes, and one-sixth for pies, and the cost of the flour for each was apportioned on this basis. The cost of the other materials, as shown by the books,

determined the rates as given for the cost of materials in the next table. The extra work required in preparing cake and pie for the oven, made one-third of the baker's pay the proper charge for each, notwithstanding the difference in the respective quantities made of the three varieties.

The extra space required in preparing several kinds of cakes and of pies over that needed in mixing dough, called for the given apportionment of rent. The apportionment for depreciation was made on the same basis.

WRITTEN EXERCISES

1. Complete the following table of Mr. Taylor's analysis of manufacturing costs:

ANALYSIS OF MANUFACTURING COSTS

	Total	Bread	Cake	Pie
Materials	$4500	7/15	16/45	8/45
Direct labor	1200	1/3	1/3	1/3
Prime cost	?	?	?	?
Indirect labor	600	1/3	1/3	1/3
Rent	80	1/6	2/6	2/6
Fuel, etc.	210	1/3	1/3	1/3
Depreciation	110	1/6	2/6	2/6
Manufacturing costs	?	?	?	?

The fractions indicate each item's part of the total in the same line. Change these fractions to dollars. Give totals in dollars in the spaces indicated by a question mark (?)

2. Complete the following table of Mr. Taylor's statement of selling costs:

ANALYSIS OF SELLING COSTS

	Total	Bread	Cake	Pie
Proprietor's allowance	$600	½	3/10	⅕
Salesgirl's pay	520	½	3/10	⅕
Rent	240	⅓	⅓	⅓
Fuel, light, etc.	90	⅓	⅓	⅓
Depreciation	90	⅓	⅓	⅓
Selling costs	?	?	?	?
Manufacturing costs	Insert from preceding example			
Total costs	?	?	?	?
Receipts from sales	?	4000	3000	2000
Profit	?	?	?	?

3. (*a*) Find the cost of 365 barrels of flour at the average rate of $10.50 a barrel. (*b*) Find the cost of the other bread materials if the total cost was $2100 including the cost of one-half of the flour.

4. One-third of the flour was used for cake. Find the cost of the sugar, shortening, eggs, currants, etc., if the total cost of the cake ingredients amounted to $1600.

5. The total cost of the pie ingredients was $800, including that of one-sixth of the flour. How much was paid for the remaining materials?

6. From a barrel of flour dough was made weighing 315 pounds, which lost ⅛ of its weight in baking. Find the number of one-pound loaves made during the year.

7. Of the foregoing, 220 stale loaves were sold at half price, and the others at 8 cents each. How much was received for the bread?

8. (*a*) What per cent of the sum received from the sales of bread was the profit? (*b*) Of the sum received

from the sales of cake? (*c*) Of the sum received from the sales of pie?

9. (*a*) In order to have made a profit of 12½% on his cake sales of $3000, how much less should have been spent for the materials, the other expenses remaining the same? (*b*) What per cent of the materials used could be obtained for this sum?

10. Mr. Taylor's predecessor sold his pies for 20 cents each. (*a*) How many pies did he make? (*b*) Find the average cost of a pie. (*c*) If he had charged ⅛ more than this cost, what would have been the selling price of a pie? (*d*) What per cent of this selling price would the profit have been?

11. The following is the cost sheet of a shirt factory for a year:

FACTORY COSTS

Raw materials	$250,000	
Direct labor	150,000	
Prime cost		(*a*)
Salaries of factory officials	3,500	
Wages	19,500	
Total indirect labor		(*b*)
Rent of factory	8,000	
Power, light, heat	3,000	
Repairs to equipment	2,000	
Fire insurance	1,500	
Other insurance	500	
Employer's liability, etc.	500	
Welfare work	1,500	
Taxes, state and local	500	
Other factory expense	6,000	
Total factory expense		(*c*)
Total factory overhead		(*d*)

Find (*a*), (*b*), and (*c*). Find (*d*) the total *factory overhead* which is the sum of the cost of indirect labor

(*b*) and of factory expense (*c*). Find (*e*) the *rate of burden;* viz., the percentage the total factory overhead (*d*) is of the prime cost (*a*).

Find (*f*) the administrative expense and (*g*) the selling expense from the next two tables.

The following sheet shows the expense of administration for a year.

ADMINISTRATIVE EXPENSE

Salaries of officials	$6,500
Salaries of office force	5,500
Rent of general office	1,500
Office expense	500
Collecting expense	2,000
Bad debts	2,500
Corporation tax	250
Other administrative expense	2,250
Total administrative expense	(*f*)

The following sheet shows for a year the cost of selling articles made in the factory.

SELLING EXPENSE

Salaries of officials	$6,000
Salaries, commissions, etc., of salesman	28,000
Wages	1,500
Rent of show and shipping rooms	2,500
Packing materials	2,250
Cartage and freight outward	3,250
Advertising	5,500
Other selling expense	10,500
Total selling expense	(*g*)

Find (*h*) the total cost of manufacturing, administering, and selling, including payments of $4500 interest on loans and $3500 deterioration on equipment.

(*i*) The net profit for the year on the sales, which amounted to $660,000. (*j*) The per cent of $660,000.

Make out a sheet in the following form, taking the amounts from (*k*) to (*t*) from the foregoing tables:

STATEMENT FOR 1920

Gross receipts from sales				(*k*)
Raw materials	(*l*)			
Direct labor	(*m*)			
Prime cost		(*n*)		
Indirect labor	(*o*)			
Factory expense	(*p*)			
Factory overhead		(*q*)		
Total manufacturing expense			(*r*)	
Administrative expense	(*s*)			
Selling expense	(*t*)			
Interest and deterioration	(*u*)			
Total expense of administration, selling, etc.			(*v*)	
Total gross cost of goods				(*w*)
Net profit for the year				(*x*)

12. From the last statement find the per cent (two decimal places).

a The cost of raw materials is of the prime cost.
b The cost of direct labor is of the prime cost.
c The prime cost is of the gross cost of goods.
d The factory overhead is of the prime cost.
e The total manufacturing expense is of the total gross cost of goods.
f The total gross cost of goods is of the selling price.

13. In determining the factory overhead some manufacturers include a share of the administrative expense. (*a*) How much would the total administrative and selling expenses, etc., in the last statement have been decreased if it had been reduced by ⅛ of the administrative expenses? (*b*) How much should be the factory overhead in this case?

14. What would be the net profit for the year if 5% interest on the capital of $50,000 were deducted from the net profit shown in the statement?

15. If the proprietor had allowed himself $3000 per year for his services what would then have been his net profit after deducting also the interest on his capital?

Profit and Loss

Mr. Paulsen in 1921 added to his wares a new implement which he sold at $8 each. The preceding year his sales had amounted to $50,000 with a profit of $5000. At the end of 1921 his receipts were $60,000, but his profits had fallen to $3000. Upon examining his accounts he found that his increase in receipts was largely due to sales of the new implement, which he then ascertained was sold by other manufacturers for $10.50. Careful calculations based upon the balance sheets for 1920 and 1921 showed that the implement cost him $9.50 to make and sell. He then added to his books a series of cost accounts that enabled him to avoid future mistakes of this kind.

16. Complete the following tables giving the materials required for a dozen shirt waists in each of two factories. (Give results in mills.)

Factory	Yd.	Price	Material	Trimmings	Boxes	Total	Discount	Net total
I	39	.66½	(*a*)	$1.130	$.270	(*b*)	$.563	(*c*)
II	38½	.66½	(*d*)	.870	.280	(*e*)	.915	(*f*)

17. Find the cost of a dozen of each of the foregoing, using the net total for materials obtained in the preceding example:

Factory	Cost of material	Direct labor	Factory expense	Selling expense	Total cost
I	(*c*)	$.3980	$2.882	$5.195	(*g*)
II	(*f*)	.5960	3.536	7.721	(*h*)

18. Each of the above lines is sold to retailers at $42 a dozen, less 6%. Find the gain or the loss a dozen by each of the two manufacturers.

Overhead Expenses

A manufacturer who finds that his factory overhead in 1920 is $45,000, which is 30% of the prime cost of $150,000, does not assume in fixing 1921 prices that he should take the same burden rate of 30% without carefully examining into all the conditions. This examination may determine his selection of 25%, or possibly 35%. How nearly right has been his choice can be determined definitely only when a new balance sheet is made up, which may not be until the end of 1921.

19. The factory expenses of the Brown Manufacturing Company comprise:

Rent of factory	$7,500
Power, light, and heat	3,800
Repairs and depreciation	600
Fire insurance	2,600
Workmen's compensation	100
Taxes	1,100
Superintendent, timekeepers, etc.	19,800
Office expenses	4,000

Find (*a*) the total of the foregoing factory expenses. (*b*) Find the total of the following selling expenses:

Rent of show and shipping rooms	$3,400
Salaries of office force	6,100
Salaries and commissions of salesmen	78,400
Packing materials	2,000
Cartage and outward freight	2,700
Advertising	18,700
Expense of collection	12,900
Other expenses	5,000

20. The balance sheet of a small factory shows net sales amounting to $104,000, and expenses as follows:

Raw materials	$51,454
Labor	32,665
Factory expenses	2,366
Administration	3,430
Selling expenses	8,300
Interest on loan	783
Depreciation, etc.	173

(*a*) Find the profit. (*b*) What per cent is realized on the capital invested, $50,000? (*c*) What per cent of the sales is the profit?

CHAPTER FOUR

THE MERCHANTS' PROBLEMS

A Retail Business

WRITTEN EXERCISES

1. From the following data find a butcher's profit during the year:

Sales for the year		$30,000
3 salesmen, $10, $12.50, $15 a week.	$(*a*)	
Delivery expense	900	
Rent $390 Miscellaneous $210	(*b*)	
Wrapping $70 Advertising $80	(*c*)	
Telephone and Postage $90 Bad debts $210	(*d*)	
Office boy and expenses	480	
Depreciation, etc.	600	
Cost of goods	23,700	(*e*)
Net profit		(*f*)

2. The net profit is what per cent (*a*) of the yearly sales. (*b*) Of the cost of the goods? What per cent of the sales are (*c*) the salaries of the salesmen; (*d*) the delivery expenses; (*e*) the rent; (*f*) the miscellaneous expenses; (*g*) postage and bad debts; (*h*) office expenses; (*i*) depreciation, etc.?

3. If the average number of packages delivered was 40 a day for 300 business days, (*a*) what was the cost of delivery for each package? (*b*) What would be

the expense of delivery at 3¾ cents a package, the rate charged by a newly-organized delivery company? (*c*) What would have been the saving last year by the employment of this company?

4. Find the total yearly expense of the delivery company, as follows:

Pay roll	$16,548.72	Wagon account	$210.—
Feed	3,734.20	Shoeing horses	708.35
Light and fuel	190.80	Harness	96.75
Repairs	724.98	Damage claims	119.80
Depreciation	325.—	Miscellaneous	930.40

5. (*a*) From the foregoing total find the average cost of a delivery, 786,300 deliveries having been made in the year. (*b*) What was the average number of deliveries a day for the 300 business days?

6. Find (*a*) to (*f*) the total daily sales of shoe salesmen as follows, (*h*) to (*m*) the weekly sales of each, and (*g*) the total weekly sales.

Salesmen	Z	Y	X	W	V	U	Daily Total
Monday	$42	$41	$40	$60	$55	$70	(*a*)
Tuesday	38	36	42	57	53	69	(*b*)
Wednesday	37	38	33	56	56	68	(*c*)
Thursday	43	40	41	58	52	72	(*d*)
Friday	39	40	42	49	53	69	(*e*)
Saturday	51	50	43	65	61	83	(*f*)
Total for week	(*h*)	(*i*)	(*j*)	(*k*)	(*l*)	(*m*)	(*g*)

7. Assuming that a salesman's weekly pay should be 8⅓% of his net sales, find the value of the weekly services of each of the foregoing to the nearest dollar.

First express each in dollars and twelfths, add them, and compare their sum with 8⅓% of (*g*). If the results agree, the rates are presumably correct. Express them in dollars, as called for by the problem.

8. Find (*a*) to (*l*) the total sales a month; (*n*) to (*s*) the annual sales of each, and (*m*) the total annual sales:

Salesmen	Z	Y	X	W	V	U	Monthly total
January	$887	$775	$840	$1216	$1095	$1464	(*a*)
February	863	816	785	1235	1208	1593	(*b*)
March	876	839	766	1256	1163	1509	(*c*)
April	904	850	808	1265	1084	1477	(*d*)
May	798	905	815	1298	1059	1448	(*e*)
June	875	879	825	1289	1158	1527	(*f*)
July	894	864	858	1137	1047	1496	(*g*)
August	856	837	798	1206	1053	1544	(*h*)
September	893	823	789	1234	1106	1409	(*i*)
October	841	787	768	1199	1095	1437	(*j*)
November	888	748	740	1286	1123	1518	(*k*)
December	863	806	829	1275	989	1406	(*l*)
Total for year	(*n*)	(*o*)	(*p*)	(*q*)	(*r*)	(*s*)	(*m*)

9. (*a*) Find the value of the annual services of each of the foregoing to the nearest dollar, taking 8⅓% of his annual sales as the rate. (*b*) What should be his weekly pay to the nearest dollar, taking 52 weeks to the year?

Check by expressing each in dollars and a fraction, then finding their sum, etc.

10. A butcher slow at figures and not having a computing scale, makes out a table showing the charge to be made for fractions of a pound at current prices. He expresses results in cents, considering each fraction of a cent a full cent.

Rates	37¢	29¢	27¢	23	21¢
3 oz.	7¢		6¢		
5 "	12¢				
7 "	16¢				
11 "					
13 "					

Fill out the foregoing portion of the table.

NOTE: In sales of over 3 or 4 pounds a dealer is likely to reject fractions of a cent below ½.

11. The balance sheet of a retail shoe store shows the receipts and the expenditures for a year as follows:

Sales during the year			$80,000
Purchases of stock		$59,200	
Buying expenses	$912		
Pay of salespeople	6800		
Advertising	1000		
Miscellaneous selling expenses	136		
Delivery	240		
Office salaries	1400		
Office supplies	160		
Rent	2640		
Heat and Light	480		
Insurance	304		
Taxes	296		
Repairs	96		
Depreciation	320		
Miscellaneous expenses	136		
Bad debts	80		
Total expenses		(*a*)	
	Total outlay		(*b*)
	Net profit		(*c*)

Find (*a*) the total expenses of buying, selling, etc.; (*b*) the total outlay, including cost of goods; (*c*) the net profit.

12. A furniture dealer who found that on sales of \$30,000 his selling expense was \$7500, fixed the price of a bedroom set by adding 35% to its cost of \$60, believing that this price would include an expense of 25% and a profit of 10% on it. Find (*a*) the selling price. (*b*) The selling expense at 25% of the selling price. (*c*) The profit.

13. (*a*) What should a dealer charge for a range that cost him \$32.50 so that he would include an expense of 25% on the selling price and a profit of 10% on the same price? (*b*) By what expense should he increase \$32.50?

14. The following shows the operating expenses of a retail shoe store for a year:

Buying expenses		
Freight and cartage to store		\$600
Selling expense		(*a*)
Pay of sales force	\$4,425	
Advertising	750	
Wrapping, etc.	85	
Delivery	140	
Management and office expense		1,100
Fixed charges and upkeep		(*b*)
Rent	1,650	
Heat and light	300	
Insurance	190	
Taxes	200	
Repairs and renewals	60	
Depreciation	200	
Miscellaneous expenses and bad debts		800
Total expense		(*c*)

Find (*a*), (*b*), and (*c*).

15. Find the net profit for the year if the sales are \$50,000, the invoice costs are \$36,200, and the total expense is that which is given in the previous example.

16. What did the proprietor pay for a pair of shoes that he sold for \$5, if he made the rate of profit shown by the figures in the last two examples?

17. A dry-goods store had on hand at the beginning of the year goods that cost \$4365.29. There were purchased during the year from four wholesalers goods invoiced at \$4234.56, \$5465.84, \$5798.77, and \$4867.58, respectively. There were on hand at the end of the year goods that had cost \$3757.83. Find the value of the goods sold.

Depreciation

Gartland & Sons, when fixing the value of their goods at a semi-annual inventory, make an arbitrary deduction from the invoice cost according to the length of time the goods had been in stock, a 5% deduction being made for goods on hand at two inventories up to one of 90% for those on hand at seven inventories.

18. The following statement shows the invoice cost of goods in stock at the present inventory arranged according to the number of times each class has been inventoried. It shows also the rate of deduction from invoice value for each class.

Find the present value of each class (*a* to *h*), the total invoice cost (*i*), the total inventory (*j*), and the average rate of deduction (*k*).

Times inventoried	Invoice cost	Rate of deduction	Present value
1	$3856	none	(*a*)
2	2430	5%	(*b*)
3	1884	10	(*c*)
4	1060	20	(*d*)
5	582	33⅓	(*e*)
6	365	50	(*f*)
7	219	90	(*g*)
8	87	100	(*h*)
Total	(*i*)	(*k*)	(*j*)

To obtain (*k*) find the per cent the difference between (*i*) and (*j*) is of (*i*).

19. A druggist spent $2000 for fixtures. At the end of the year he deducted 20% of their cost for deterioration. The next year he deducted 20% of their estimated value at the beginning of the year. This he did each year thereafter. What was their estimated value at the end of the eighth year?

SIGHT EXERCISES

1. A milk dealer delivers 200 quarts of milk a day. His daily expense is as follows:

Labor	$3.20
Horse feed and shoeing, repairs, etc.	1.30
Depreciation of horses, wagons, etc.	.40
Interest, ice, bad bills, insurance, etc.	1.10

(*a*) Find the total expense. (*b*) The expense a quart. (*c*) The daily profit when the milk costs 8 cents to the dealer and is sold for 16 cents. (*d*) What per cent of the gross receipts is profit?

2. What is the weight of 200 quarts of milk in cans, the milk weighing 2 pounds a quart and an empty can of 10 gallons' capacity weighing 28 pounds?

A Wholesale Business

WRITTEN EXERCISE

The balance sheet of a wholesale grocer shows receipts and expenditures as follows for a year:

Sales during the year			$800,000
Cost of stock		$648,000	
Salary of buyer	$2,500		
Other expenses	700		
Buying expense		(*a*)	
Salaries, etc., of salesmen	18,400		
Advertising, etc.	1,600		
Selling expense		(*b*)	
Receiving and shipping	9,280		
Packages and wrapping	320		
Outward freight, etc.	3,200		
Expense of handling		(*c*)	
Salary of executive	4,000		
Pay of clerks, etc.	5,600		
Postage, etc.	2,040		
Telephone and telegraph	400		
Expense of collection	560		
Other expenses	800		
Expense of management		(*d*)	
Interest on loans		12,000	
Rent	3,200		
Heat and light	400		
Taxes	1,600		
Insurance	900		
Repairs	400		
Depreciation	800		
Expense of upkeep		(*e*)	
Miscellaneous expenses		900	
Bad debts		2,400	
Total expenditures			
Find the net profit			(*f*)

CHAPTER FIVE

PARTNERSHIP

WRITTEN EXERCISES

1. A and B entered into partnership, A contributing \$4000 and B \$5000. Their agreement provided that each should receive from the profits 6% interest on his contribution and that the remaining profits should be divided equally. If the business showed a profit of \$4000 at the end of a year, what should each receive as his share including interest on his contribution?

2. West & Irwin, fellow clerks, earning \$1500 and \$1800, respectively, resigned their positions and formed a partnership. West invested \$3500 and Irwin, \$2500, agreeing to share profits in proportion to their previous salaries, after first receiving 6% interest on their respective investments. What should each receive as his share of a profit of \$5200 for the year, including interest on his capital?

METHOD

Profit \$5200

Less interest due West, 6% of \$3500 (*a*)

" " " Irwin, 6% of 2500 (*b*)

Remainder to be divided (*c*)

West receives $\frac{1500}{1500 + 1800}$ or $\frac{5}{11}$ of (*c*), in addition to (*a*)

Irwin receives $\frac{1800}{1500 + 1800}$ or $\frac{6}{11}$ of (*c*), in addition to (*b*)

3. What would have been the respective shares of West and Johnson in dividing the profit given in Example 2, if each first received interest on his investment, then the amount of his previous salary, and finally one half of the remainder of the profits?

4. What profit would West and Johnson, respectively, receive if the entire profits in Example 2 were divided in proportion to the share invested by each?

5. A invested $6000 in a business, B, $10,000, and C, his services. Out of the profits of $7000, C was allowed $1200 a year, and each of the others 10% of the sum invested by him. The remainder of the profits was divided equally among the three; find the share of each.

6. What would be the share of each in Example 5 if C received $1200 a year and 15% of the gross profits of $7000, and the remainder was divided between A and B in proportion to their respective investments?

In a regular partnership to which the members contribute different amounts, the profits are seldom apportioned in proportion to the respective sums invested when each partner contributes his services to the business.

7. The business of Brown, Jones & Company was managed by Mr. Thompson, who was paid for his services a salary of $3000 and 10% of the profits. On Jan. 1, 1920, Brown's investment was $30,000; Jones' was $20,000, and Smith's (the Co.), was $10,000. The profits of the year after the deduction of all expenses, including the salary of Mr. Thompson, but not his commission, were $20,000. What was the share of each partner?

8. A, B, and C engage in a business venture. A contributes \$10,000 for six months and \$10,000 additional for the next six months; B contributes \$30,000 for four months and then withdraws \$10,000, leaving \$20,000 for the next eight months. C contributes \$40,000 for the year. At the end of the year the business is sold for \$100,000. Give the sum to which each is entitled on the basis of the return of the sum he had in the business at the end of the year, and a share of the profit in proportion to his average investment.

METHOD

Received for business	\$100,000	
A's capital at end of year	20,000	} Deduct
B's " " " " "	20,000	} Deduct
C's " " " " "	40,000	} Deduct
Profit	\$20,000	

A's contribution, \$10,000 for 6 mo. = \$60,000 for 1 mo.
" " 20,000 " 6 " = 120,000 " 1 "

A's aggregate contribution equals = \$180,000 for 1 mo.
B's contribution, \$30,000 for 4 mo. = \$120,000 for 1 mo.
" " 20,000 " 8 " = 160,000 " 1 "

B's aggregate contribution equals \$280,000 for 1 mo.

C's contribution of \$40,000 for year equals \$480,000 for 1 mo.
The sum of the three contributions for 1 month is
\$180,000 + \$280,000 + \$480,000, or \$940,000

A's fraction of this total is $\frac{180000}{940000}$ or $\frac{18}{94}$ or $\frac{9}{47}$

A's share is \$20,000 + $\frac{9}{47}$ of \$20,000
B's " " 20,000 + $\frac{14}{47}$ of "
C's " " 40,000 + $\frac{24}{47}$ of "

9. During 1921, the investments of the members of the firm of Brown, Jones & Co. were as follows:

Brown		*Jones*	
Jan. 1 invested	$30,000	Jan. 1 invested	$20,000
May 1 added	10,000	Jun. 1 withdrew	5,000
Aug. 1 withdrew	20,000	Sept. 1 withdrew	5,000

Co. (*Smith*)

Jan. 1 invested	$10,000
Jul. 1 added	10,000
Oct. 1 added	10,000

Find the average investment of each member of the firm during 1921.

METHOD

Brown's investment

$30,000 for 4 months	(Jan. to May)	equals	$120,000 for 1 mo.
40,000 " 3 "	(May to Aug.)	"	120,000 " 1 "
20,000 " 5 "	(Aug. to Jan.)	"	100,000 " 1 "
Aggregate equals			$340,000 for 1 mo.

½ of which is Brown's average investment for the year.

10. (*a*) Find the ratio the average investment of each member of the firm of Brown, Jones & Company bears to the total of the average investments. (*b*) Find the ratio each one's aggregate investment bears to the sum of the aggregates. (*c*) Find the capital each member has in the business at the end of the year. If the entire business were sold at the end of the year for $75,000, find each partner's share (*d*) of the profit, (*e*) of the sum received for the business.

SECTION VI

FINANCING BUSINESS

CHAPTER ONE

REMITTING MONEY

A boy who orders a rifle from a dealer in a distant city, or a girl who writes for a tennis outfit, may send the price of the article by means of a money order, obtainable at any money-order post office.

Domestic Money Order

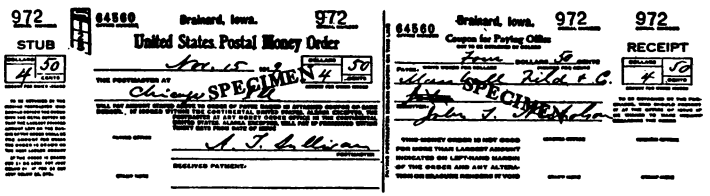
John T. Nicholson of Brainard, Iowa, sends Marshall Field & Company, Chicago, a postal money order for four dollars and a half for a pair of skates.

He fills out an application blank, which he gives to the postmaster, with four dollars and fifty cents and the prescribed fee.

Fees for Domestic Money Orders

For orders from			$0.01	to	$2.50	— 3¢
"	"	"	2.51	"	5.	— 5¢
"	"	"	5.01	"	10.	— 8¢
"	"	"	10.01	"	20.	— 10¢
"	"	"	20.01	"	30.	— 12¢
"	"	"	30.01	"	40.	— 15¢
"	"	"	40.01	"	50.	— 18¢
"	"	"	50.01	"	60.	— 20¢
"	"	"	60.01	"	75.	— 25¢
"	"	"	75.01	"	100.	— 30¢

The postmaster fills out the order blank shown in the accompanying illustration. He retains the stub at the left and gives Master Nicholson the rest of the slip, including the receipt at the right, which the latter detaches and retains, mailing the remainder to Mar-



shall Field & Company. This firm collects the money by depositing the order in its bank.

A recipient of a money order, who has no bank account, obtains the money at the post office upon identification, or he may indorse it and obtain the money from a merchant who knows him.

Telegraphic Transfers of Money

Mrs. Aubrey of Seattle, while stopping at a Pittsburgh hotel, finds herself in need of funds. She telegraphs to her husband at his office in Seattle to send her $300 by telegraph. He gives an express company the order to make the transfer, paying $300, with the fee and the cost of the telegram to the company's Seattle agent. The express company delivers the cash to Mrs. Aubrey at her Pittsburgh hotel in less than two hours.

The following are the rates charged by the express company to make telegraphic transfers:

For transfers up to \$50 — \$0.50
For transfers over \$50 to \$75 — 0.60
For transfers over \$75 to \$100 — 0.85

For each additional \$100 up to \$3000, add 25¢ to rate for \$100. For each additional \$100 over \$3000, add 20¢ to rate for \$3000.

SIGHT EXERCISES

1. Give the sum paid by Mr. Aubrey to the express company, including 90 cents for the cost of the telegram.

2. Give the sum paid to the express company to make each of the following transfers, adding 50 cents for the cost of the telegram:

a \$60	*b* \$70	*c* \$150	*d* \$1000	*e* \$3000
f \$40	*g* \$90	*h* \$750	*i* \$2500	*j* \$5000

A telegraph company charges 85 cents for the transfer of \$100 and an additional charge for a 15-word message.

3. Give the cost of a 15-word message when the rate is 50 cents for the first 10 words and 3 cents for each additional word. (This is generally expressed as 50–3.)

4. Give the cost of a 15-word message at each of the following rates:

a 60–4	*b* 100–7	*c* 75–5	*d* 30–2
e 25–2	*f* 125–8	*g* 35–2	*h* 40–3

5. Give the cost of each of the following messages:

Words	*Rate*	*Words*	*Rate*	*Words*	*Rate*
13	60–4	25	75–5	16	35–2
20	25–5	18	40–3	32	30–2

Bank Drafts

J. M. Morse of Billings, Oklahoma, buys a motor truck from F. B. Lee & Company, Joliet, Illinois, to be delivered on the cars at the latter place on receipt of a draft for $4000 on a Chicago bank.

Mr. Morse purchases from his bank the following draft, paying 15 cents per $1000 *premium.*

A Sight Draft

$4000 $^{00}/_{100}$ Billings, Oklahoma, Nov. 5, 1919.

At sight pay to the order of F. B. Lee & Company

Four thousand 00/100.....Dollars

Value received, and charge to the account of The Citizens Bank.

To Commercial National Bank

Chicago, Ill.

R. B. Frazier
Cashier

Mr. Morse mails this draft to F. B. Lee & Company, who ship the motor truck. His check would not be so satisfactory to F. B. Lee & Company, as they might have to pay their bank the expense of collecting the money from the Billings bank. In the case of the draft, F. B. Lee & Company receive the full price of the truck, Mr. Morse paying the Citizens Bank $4000.60 for the draft.

The daily papers in large cities give the *rates of exchange* on drafts bought in their respective cities. This rate is sometimes given as a per cent; $\frac{1}{8}\%$ premium, $\frac{1}{16}\%$ discount, etc. When it is stated as 15 cents discount, 40 cents premium, it means the rate per $1000.

A draft on a Billings bank bought in Chicago might be obtained at a discount of, say, 10 cents per $1000.

SIGHT EXERCISES

1. Give the cost of a sight draft for $4000 when the rate is 15 cents discount.

$4000 − 4 × 15¢

2. Give the cost of each of the following sight drafts:

	Face	Rate		Face	Rate
a	$1000	10¢ premium	*b*	$2000	⅛% discount
c	3000	15¢ discount	*d*	4000	3/16% premium
e	5000	25¢ premium	*f*	6000	¼% discount

The foregoing sight draft is one form of a check drawn by the Citizens Bank of Billings on the Commercial Bank of Chicago, in which the former bank has funds, directing the latter to pay to the order of F. B. Lee $4000.

The following form is gradually replacing the one given above:

A CASHIER'S CHECK

The Citizens Bank
Billings, Oklahoma, Nov. 5, 1919

Commercial National Bank
Chicago, Illinois

Pay to the order of F. B. Lee & Company
Four Thousand 00/100.....Dollars

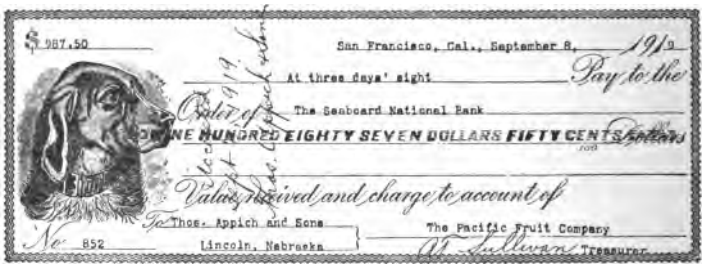
R. B. Frazier
Cashier

The other form is retained for drafts payable *after sight.* For the employment of the sight draft in obtaining payment for grain, etc., upon delivery, see p. 362.

Collections by Draft

On August 15, 1919, the Pacific Fruit Company, of San Francisco, sold to Thos. Appich & Sons, Lincoln, Nebraska, goods to the amount of $987.50, on a credit of 30 days. The latter agreed to settle on Sep. 14 by paying on that day a draft drawn upon them by the sellers.

A Time Draft


$ 987.50 San Francisco, Cal., September 8, 1919
At three days' sight Pay to the
Order of The Seaboard National Bank
ONE HUNDRED EIGHTY SEVEN DOLLARS FIFTY CENTS Dollars
Value received and charge to account of
To Thos. Appich and Sons The Pacific Fruit Company
No. 852 Lincoln, Nebraska A. Sullivan Treasurer

On Sep. 8, the Pacific Fruit Company *draws* upon Thos. Appich & Sons by making out the foregoing draft, which it deposits in its bank, the Seaboard National. This bank mails it for collection to the National Bank of Commerce, at Lincoln. On Sep. 11, a messenger from the latter bank presents the draft at the office of Thos. Appich & Sons for *acceptance*. They *accept* it by writing the word "Accepted" across the face of the draft with the date, and the firm's signature. On Sep. 14, three days after the acceptance, Thos. Appich & Sons pay the money and receive the draft. The National Bank of Commerce sends its check to the Seaboard National Bank, and the latter

credits the account of the Pacific Fruit Company with the sum received less the expenses of collection.

WRITTEN EXERCISES

1. Make a copy of the Pacific Fruit Company's three days' draft on Thos. Appich & Sons.

2. On the back write the "endorsement for collection" of the National Bank of Commerce.

3. Across the face write Thos. Appich & Sons' acceptance.

4. Write Thos. Appich & Sons' check on the First National Bank for the amount of the draft.

5. Across the face of the check write the certification of the bank.

In the foregoing draft the Pacific Fruit Company is called the *drawer*, Thos. Appich & Sons are called the *drawees*, and the Seaboard National Bank is called the *payee*.

If Thos. Appich & Sons decline to accept the draft upon presentation by the National Bank of Commerce, the latter returns it to the Seaboard National Bank, which notifies the drawer. If Thos. Appich & Sons accept the draft, and fail to pay the amount at its maturity, the National Bank of Commerce *protests* the draft before returning it.

Acceptance

A sight draft is payable upon presentation, except in the very few states that allow three days of grace.

In these states a sight draft must be presented for acceptance. When accepted, it is payable in three days thereafter.

A time draft is payable at the specified time after acceptance. To collect from Thos. Appich & Sons the amount of its bill on Sep. 14, and to give them proper notice, the seller arranged that the draft would be presented for acceptance on Sep. 11.

In arranging with Thos. Appich & Sons to settle by accepting and paying a draft, the seller wished to avoid a possible delay in obtaining its money through a neglect of the former to mail their check on time. It felt certain that they would not be likely to decline to accept and pay the draft, as such action would tend to hurt their reputation with the bank that presented the draft.

SIGHT EXERCISES

1. Give (*a*) the discount on the foregoing draft at $\frac{9}{16}\%$ of \$987.50. (*b*) The proceeds.

$\frac{1}{2}\%$ of \$987.50	\$4.9375
$\frac{1}{16}\%$ of "	.6172 $\frac{1}{8}$ of $\frac{1}{2}\%$

2. Give the proceeds of each of the following notes discounted at 6% on the day it is drawn:

Face of Note	Term of Discount	Face of Note	Term of Discount
a \$450	60 days	*b* \$1200	15 days
c 540	90 days	*d* 1320	30 days
e 660	30 days	*f* 1800	45 days

3. Give the proceeds of each of the following drafts less the bank's deduction:

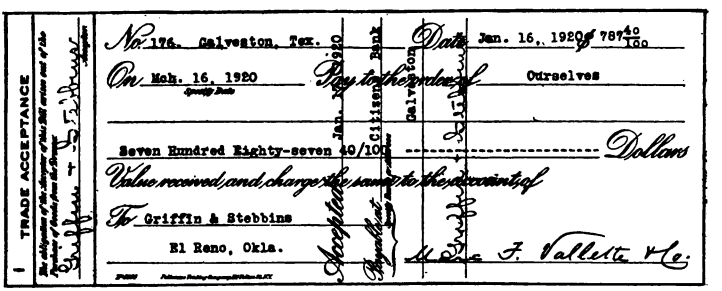
Face of Draft	Rate of Deduction	Face of Draft	Rate of Deduction
a \$2400	$\frac{1}{8}\%$	*b* \$1600	$1\frac{3}{8}\%$
c 1280	$\frac{1}{4}\%$	*d* 3200	$1\frac{5}{16}\%$
e 1680	$\frac{3}{16}\%$	*f* 2040	$1\frac{1}{4}\%$

4. Give the date on which each of the following drafts becomes due:

	Date of Acceptance	Number of days sight		Date of Acceptance	Number of days sight
a	Aug. 15, 1921	60	*b*	Jul. 29, 1919	15
c	Feb. 10, 1920	30	*d*	Mar. 16, 1920	40
e	Dec. 23, 1921	90	*f*	Feb. 23, 1921	60

Trade Acceptances

A manufacturer with moderate capital and a growing business, sometimes finds it difficult to secure from local banks loans sufficiently large for his requirements. His customers pay him promptly as bills become due,



but they are unwilling to take advantage of his offered deduction for cash payment.

By means of the Federal Reserve banks which will rediscount commercial paper previously discounted by their local banks, the latter are enabled to make safe loans to any reasonable extent, especially to a seller of goods offering for discount paper for the price of these goods, made by the purchaser and endorsed by the seller.

The preceding type of draft, a *trade acceptance,* shows the form.

This trade acceptance is an agreement made by Griffin & Stebbins of El Reno, Oklahoma, to pay to Marc F. Valette & Company, of Galveston, Texas, $787.40, on March 16, 1920, for goods bought on January 16, 1920. The draft is made by the sellers on the day the goods are sold and is immediately accepted by the purchasers, who designate their bank as the place of payment.

The sellers make it payable to "Ourselves" so that the firm name will appear on the back as an indorsement.

The form differs from that of an ordinary time draft by the statement signed by Griffin & Stebbins to the effect that the money represents a debt for goods bought by them.

Marc F. Vallette & Company's bank, the Texas Guarantee & Trust Company, of Galveston, discounts this acceptance on Jan. 18, 1920, at 6%, and places the proceeds to the credit of Marc F. Vallette & Company.

On Feb. 16, this bank, needing additional funds for loans, rediscounts the acceptance at 5% at the Federal Reserve Bank of Dallas.

Just before the acceptance becomes due, the latter bank sends it to the Citizen's Bank for collection. The latter, on Mar. 16, charges the account of Griffin & Stebbins with $787.40 (if there are sufficient funds to their credit) and sends its check for this amount to the Federal Reserve Bank.

In discounting this acceptance, each bank treats it as a promissory note.

WRITTEN EXERCISES

1. (*a*) How many days are there between Jan. 18 and Mar. 16, 1920 (leap year)? Find (*b*) the bank discount at 6% for this time on the foregoing trade acceptance. (*c*) The proceeds.

2. (*a*) For how many days did the Federal Reserve Bank discount the acceptance (Feb. 16 to Mar. 16)? (*b*) What did it deduct from the face of the acceptance, for this term at 5%? (*c*) Find the proceeds paid to the Texas Guarantee & Trust Company.

3. (*a*) How much did this bank receive from Marc F. Vallette & Company, as discount on the acceptance? (*b*) How much discount did it pay to the other bank? (*c*) Find the net sum received by it as interest on the acceptance, for the time it held the latter (Jan. 18 to Feb. 16).

Foreign Transfers

A person wishing to send $100 or less to a son in Europe may do so by means of a *postal money order*. A large sum is generally transferred by a *banker's bill* (check) or by a *bill of exchange*. The amount is stated, as a rule, in the money of the country to which the bill is sent.

International Money Orders

Domestic rates are charged on a money order made payable to a U. S. soldier in any military camp, to a sailor or a marine on any U. S. vessel, or to any

person in Bermuda, Canada, Canal Zone, Mexico, and many islands in the West Indies. The Domestic forms (p. 402) are used for these orders.

An application for a money order payable in other countries, Austria, Belgium, Chile, Denmark, France, etc., for example, should be made on an International form. The rates are as follows:

From $0.01 to $10	10¢	From $50.01 to $60	60¢
10.01 to 20	20¢	60.01 to 70	70¢
20.01 to 30	30¢	70.01 to 80	80¢
30.01 to 40	40¢	80.01 to 90	90¢
40.01 to 50	50¢	90.01 to 100	$1.00

English Money

12 pence (*d.*)	1 shilling (*s.*)
20 shillings	1 pound sterling (£)

A farthing is one fourth of a penny. It is generally written as a fraction.

The coin value of £1 in U. S. gold is $4.8665.

WRITTEN EXERCISES

1. (*a*) Taking the value of £1 as $4.87 what sum in U. S. money will pay a bill in Liverpool amounting to £9 5s.? (*b*) How much U. S. money will pay for a money order (including fee) to settle the bill?

2. Find the sum required to pay for a money order and fee to settle the following accounts and pay the money order fee:

a Paris, 273.50 francs, at 19.4 cents
b Brussels, 378.65 francs, at 19.4 cents
c Madrid, 135.90 pesetas, at 19.4 cents

d Rome, 463.75 lire, at 19.4 cents
e Geneva, 312.60 francs, at 19.4 cents
f Athens, 295.30 drachmas, at 19.4 cents

NOTE: The coin value of each of these units is 19.3 cents, the extra mill being charged to cover the rate of exchange.

Bills of Exchange

Chas. E. Teale & Company of Peoria wish to pay in London for an invoice of cloth purchased of John M. Stafford amounting to £384 14s. 6d.

They purchase the following sight *bill of exchange* from their bank.

£384 14s. 6d. Peoria, Ill., Dec. 9, 1920

At sight of this *original* bill of exchange (duplicate unpaid) pay to the order of John M. Stafford
Three Hundred Eighty-four pounds, fourteen shillings, six pence
Value received, and charge to account of

To Strachan & Rector
London
England

First National Bank
Peoria, Ill., U. S. A.
per Chas. W. Lyon
Cashier

The bank gives Teale & Company two bills, an original and a duplicate, marked 1 and 2, respectively. They send the former to Mr. Stafford, and retain the other as a receipt, sending it later if the first is lost by a disaster to the vessel carrying the mail.

The cost of the draft depends on the rate of exchange, which varies from time to time. The daily quotations give the cost of cable transfers, sight bills, and sixty-day bills.

WRITTEN EXERCISES

1. Chas. E. Teale & Company pay for this bill of exchange by means of their check drawn on the First

National Bank. Find the face of the check at the rate of \$4.88½ a £.

ONE METHOD

Value of £250	\$1221.25	¼ of \$4885.—
£125	610.625	½ of £250
£9		9 times \$4.885
10s.		½ of \$4.885
2s. 6d.		¼ of 10s.
2s.		1/10 of \$4.885
Value of £384 14s. 6d.		

Test by multiplying \$4.885 by 384.725 changing 14s 6d. to the decimal of a pound. Do this by subtracting .115 times 384.725 from 5 times 384.725.

2. Find the cost (*a*) of a cable transfer to Liverpool of £178 16s. 9d. at \$4.90 per £ and an additional charge of \$3.75 for the cable message. (*b*) Of a 60-day bill for £236 9s. 4d. at \$4.86¾ a £ sterling.

A Documentary Bill of Exchange

Arthur Brown, a cutlery manufacturer of Sheffield, England, sells Franklin Bros. of Chicago an invoice of goods amounting to £265 7s. 8d. including insurance on the goods for 60 days. He consigns the goods to the buyers, and obtains a through bill of lading to Chicago, the freight charges to be paid by Franklin Bros.

He draws a sight bill on Franklin Bros. for the equivalent of the foregoing amount in U. S. money,

to which bill he attaches the policy of insurance and the bill of lading. These combined documents constitute a *documentary bill.*

Mr. Brown sells this documentary bill to Hatton & Hatton, Sheffield dealers in foreign exchange. The following is the bill itself:

> Sheffield, England, May 3, 1920
>
> At sight of this original bill of exchange, second unpaid, pay to the order of Hatton & Hatton
>
> .....Dollars
>
> Value received, and charge to the account of
>
> To Franklin Bros. Arthur Brown
> Chicago, Ill., U. S. A.

He assigns to Hatton & Hatton the bill of lading and the policy of insurance. They indorse the bill of exchange and the other documents to their Chicago correspondent, and send to the latter, for collection, the bill with the other papers.

4. (*a*) How much does Arthur Brown insert in the foregoing bill for the sum to be collected in U. S. money, at the rate of \$4.85½ a £? (*b*) How much does he receive from Hatton & Hatton for the bill of exchange, at the rate of \$4.84¾ a £?

The quotations for French exchange give the number of francs for \$1. The following are specimen rates charged in St. Louis for Paris transfers:

Cable Transfers	Sight Bills	60-day Bills
5.18¼ @ 5.17⅞	5.18⅞ @ 5.18¼	5.20¾ @ 5.20⅛

5. What is the cost a franc (*a*) when 5.18¼ francs is obtained for a dollar? (*b*) When 5.20¾ francs is obtained for a dollar?

6. A Paris merchant draws a 60-day bill on Oct. 5, 1919, on a Denver creditor. It catches a mail steamer sailing 4 days later, which makes the voyage in 6 days. Three days thereafter it reaches the drawee who then accepts it. The latter pays it on maturity. If the proceeds of the bill require the same time for their return to Paris as was consumed in reaching Denver, on what day was the sum received by the drawer of the bill?

The quotations for German exchange give the value in U. S. money of 4 marks. The following are recent quotations:

Cable Transfers	Sight Bills	60-day Bills
93⅞ @ 94⅛	93⅜ @ 93½	93⅛ @ 93¾

7. What is the cost of a cable transfer to Hamburg of 1870.65 marks at 94¼ cents for 4 marks, and a 15-word message at 35 cents a word?

8. A Berlin merchant sent to a Seattle purchaser an invoice amounting to 2784.60 marks, less 7½, 5, and 2½%. The seller added the freight on 215 kilos to Bremen at 5.20 marks a kilo, and insurance at 1% on the net cost of the goods and the freight charge. How much must the purchaser pay for a sight bill at 93½ cents for 4 marks to cover the cost of the goods delivered on the steamer at Bremen, and the insurance?

CHAPTER TWO

BANKS AND BANKING

Banks of Deposit and Discount

The following is a statement of the condition of the Fairfax National Bank at the close of business June 30, 1920:

RESOURCES		LIABILITIES	
Bonds and Mortgages	$14,597.—	Capital stock	$125,000.—
Public Securities	380,218.56	Surplus	150,000.—
Other securities	250,708.91	Undivided profits	(*b*)
Loans	1,411,007.25	Deposits	2,257,933.56
Real Estate	50,534.90	Reserved for taxes	4,626.—
Accrued interest	14,501.14	Accrued interest	8,946.—
Due on acceptances	39,599.70	Cashier's checks	3,938.57
Cash on hand and in bank	438,906.68	Acceptances	39,599.70
Total	(*a*)	Total	(*a*)

WRITTEN EXERCISES

1. Find the total resources (*a*). Insert this as the total of the liabilities and find (*b*), the undivided profits.

2. Rewrite the foregoing statement of resources and liabilities, expressing the items in thousands of dollars.

First write the totals as even thousands. Next, add the thousands' column of the resources to ascertain how much must be carried to this column to make the new total. Then, rewrite the items as thousands, increasing by 1 thousand the necessary number of items, selecting those having the largest excess, and rejecting the remaining figures of the others.

Thus, write "Bonds and Mortgages, $15,000; Public Securities, $380,000; Other Securities, $251,000; etc.

A savings bank depositor desiring to withdraw any portion of his funds may be required by the bank to wait 60 days for his money. The check of a depositor

in a bank of deposit (state bank, national bank, or trust company) must be paid on proper presentation.

Experience has shown that under ordinary conditions less than 5 % of a bank's deposits are demanded on any one day. When it is evident that larger calls for money are likely to be made, a bank can withdraw some of its deposits in other banks, sell some securities, rediscount in the Federal Reserve Bank some acceptances, collect some of its demand loans, etc.

The following is a condensed form of a collateral time note:

$24,000 $^{00}/_{100}$ Butte, Montana, May 14, 1920

On Sep. 14, 1920, for value received we promise to pay to Merchants & Miners Bank

or order, at its banking house———————————

Twenty-four Thousand 00/100———————Dollars

with interest at six (6) per cent

having deposited with said Bank, as collateral security for the payment of this liability, the following property:

Two hundred fifty shares of stock of the Pennsylvania Railroad Company

with this condition that the Merchants & Miners Bank has the right to call for such additional security as it may deem proper, and, on failure to respond forthwith to such call, this obligation shall immediately thereupon become due and payable, and the said Bank is hereby given full authority to sell and deliver the whole or any part of said securities, and upon such sale the said Merchants & Miners Bank after deducting all legal costs and expenses may apply the residue to pay this liability, returning the overplus to the undersigned. And the undersigned agrees to pay the holder hereof any deficiency upon demand.

Wilcox & Wilcox

Loans on Collateral

A bank is always willing to lend money to the extent of at least 80 % of the value of acceptable property deposited with it as security. To be acceptable, the property must be such as can be sold by the bank at once, in case the conditions of the loan are

not observed, one of them being the promise of the borrower to deposit additional security at the call of the bank. Bonds or stocks are generally employed in some sections; in others, warehouse receipts showing the ownership of grain, cotton, etc.

SIGHT EXERCISES

1. (*a*) Give the value of 250 shares of stock at $132 per share. (*b*) What is 80% of this value?

2. When a bank is willing to lend 80% of the value of the collateral, how much of the latter will secure a loan of $24,000?

3. For how many days is the foregoing collateral note drawn?

4. Give the discount on the note at 6%.

5. Give the interest at 6% (*a*) on $3000 for 30 days. (*b*) On $90,000 for 1 day. (*c*) On $1 for 90,000 days.

6. Give the interest at 6% on $1 for

a 18,000 days *b* 21,000 days *c* 36,000 days *d* 144 days

7. John Martin has 200 shares of stock worth $150 a share. How much can he borrow on the security of this stock if a bank will loan him 80% of the value?

8. (*a*) To borrow $30,000, what should be the market value of the security? (*b*) How many bales of cotton at $125 a bale would equal this sum? (*c*) When corn is selling at $1.50 a bushel, a warehouse receipt for how many bushels would be required as security for the same loan?

INTEREST PAYMENTS

At large trade centers brokers every few days borrow money payable on demand with interest. At

the end of each month the bank renders a statement of the interest due to date by the depositor, and notifies him that his account is debited with the total interest items due.

INTEREST STATEMENT

Minneapolis, Minn., Dec. 31, 1920

THE FLOUR CITY NATIONAL BANK

To Jones & Cooke, Dr.

Your account has been debited with the following interest charges to date:

1920		Loans	Days	1 Day	Rate	Interest
Dec.	1	$3000	30	$90,000	6%	——
	5	5000	26	130,000	"	——
	8	7500	23	etc.	"	——
	12	6000	19	etc.	"	——
	22	12000	9	etc.	"	——
	27	8000	5	etc.	"	——
Totals		(*a*)	..	(*b*)	..	(*c*)

WRITTEN EXERCISES

1. Find the interest due for December, 1920.

METHOD

The interest on $3000 for 30 days is the same as the interest on $90,000 for 1 day; on $5000 for 26 days, it is the same as that on $130,000 for 1 day.

In the column headed "1 day" write the product of the face of the loan by the number of days for which interest is due. Obtain (*b*) the sum of this column, which gives the number of dollars on which 1 day's interest is due.

$$(c) = \frac{(b) \times 1 \text{ (da.)} \times .06}{360}$$

2. (*a*) Find the total amount, principal, and interest, due by Jones & Cooke to the Flour City National Bank on Dec. 31, 1920. (*b*) Find the total of the interest if the rate had been 5%.

Change in Rate of Interest

As the demand for loans increases, the interest rate advances, and a bank may "call" a demand loan unless the borrower agrees to pay a higher rate. The following example shows several increases during a month.

3. Find the interest charged on Oct. 31, on a demand loan of $20,000, the rate being 5% from Oct. 1 to Oct. 5, inclusive; 5¼% from Oct. 6 to Oct. 10, inclusive; 5½% from Oct. 11 to Oct. 20, inclusive, and 5¾% from Oct. 21 to Oct. 31, inclusive.

Bank Tables

The large amount of work an interest clerk is called upon to do, requires the employment of tables to facilitate his calculations.

The following is a portion of a table used to determine the number of days between any two dates in a year of 365 days. An extra day must be added when Feb. 29 (leap year) falls between the dates.

To find the time between any day in January and the corresponding day in another month, use the January line. The time between Jan. 13, 1921, and Aug. 13, 1921, is 212 days, found in the January line and the August column.

DATE TABLE

	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	
Mar.	306	337	365	31	61	92	122	153	184	214		
Apr.	275	306	334	365	30	61	91	122	153			
May	245	276	304	335	365	31	61	92				
Jun.	214	245	273	304	334	365	30					
Jul.	184	215	243	274	304	335						
Aug.	153	184	212	243	273							
Sep.	122	153	181	212								
Oct.	92	123	151									
Nov.	61	92										
Dec.	31											

The time between Jan. 1 and Aug. 13 is 224 days, 12 days more; between Jan. 13 and Aug. 1 it is 200 days, 12 days less; between Jan. 5, 1924, and Aug. 5, 1924 (leap year), it is 213 days.

ORAL EXERCISES

1. Find the time between the following dates:

a Jan. 5, 1921 and Sep. 5, 1921
b Feb. 13, 1922 and Oct. 13, 1922
c Mar. 7, 1922 and Jan. 8, 1923
d Apr. 20, 1923 and Sep. 15, 1923
e May 9, 1923 and Mar. 9, 1924
f Jun. 25, 1924 and Jan. 31, 1925
g Aug. 8, 1924 and May 1, 1925
h Sep. 28, 1925 and Apr. 20, 1926
i Oct. 6, 1925 and Mar. 2, 1926
j Nov. 30, 1926 and Feb. 28, 1927
k Jul. 2, 1926 and Jun. 7, 1927
l Dec. 16, 1927 and Jan. 31, 1928

2. Give the number required to complete (*a*) the February line; (*b*) the February column in the Date Table.

3. (*a*) How many days does each number in the February column exceed the number in the corresponding line in the January column? (*b*) How many days is each number in the February line less than the corresponding number in the January line?

4. Give the numbers required to complete (*a*) the successive lines; (*b*) the successive columns.

Interest Tables

To make it possible for a clerk to determine the interest on as large as possible a number of accounts, banks supply books showing the interest for 1, 2, 3, 4, etc., to 360 days; on sums of \$10,000, \$11,000, \$12,000, etc., to \$990,000; at each of the customary rates.

The following is an extract from the pages showing the interest for 196 days, on a small number of principals, at a few rates:

196 days — Year of 360 days

Principal	4%	5%	6%	7%
\$1000	\$21.77,8	\$27.22,2	\$32.66,7	\$38.11,1
2000	43.55,6	54.44,4	65.33,3	76.22,2
3000	54.44,4	81.66,7	98.	114.33,3
4000	87.11,1	108.88,9	130.66,7	152.44,4
5000	108.88,9	136.11,1	163.33,3	190.55,6
6000	130.66,7	163.33,3	196.	228.66,7
7000	152.44,4	190.55,6	228.66,7	266.77,8
8000	172.44,2	217.77,8	261.33,3	304.88,9
9000	196.	245.	294.	343.

SIGHT EXERCISES

1. Give to the nearest cent the interest on each of the following for 196 days:

a	\$10,000 at 4%	*b*	\$5000 at 6%	*c*	\$900 at 4%	*d*	\$50 at 7%
e	20,000 at 5%	*f*	6000 at 7%	*g*	800 at 5%	*h*	40 at 6%
i	30,000 at 6%	*j*	7000 at 5%	*k*	700 at 6%	*l*	30 at 5%
m	40,000 at 7%	*n*	8000 at 4%	*o*	600 at 7%	*p*	20 at 4%

2. From the table, give the interest for 196 days, to the nearest cent on

a $1000 at 2% *b* $2000 at 2½% *c* $3000 at 3% *d* $4000 at 3½%

NOTE: The bank clerk obtains these four results directly from his book which gives interest at the foregoing rates: 2½%, 3½%, etc.

WRITTEN EXERCISES

1. Find the interest for 196 days, to nearest cent, on $1234 at 4%.

METHOD

Take from the book the following items, at 4%:

Interest at 4% on $1000	—	$21.778
" " " " 200		4.3556 (1/10 of $2000)
" " " " 30		.5444 (1/100 of $3000)
" " " " 4		.0871 (1/1000 of $4000)
" " " " $1234	?	Ans.

NOTE: In practice, write only the four interest items. Check by finding the interest for $617 and multiplying the result by 2.

The table used by the clerk requires but two items, that for $1200, and that for $34.

2. Find the interest for 196 days on

a $2345 at 4% *b* $3456 at 5% *c* $4567 at 6% *d* $4567 at 7%
e 9876 at 4% *f* 8765 at 5% *g* 7654 at 6% *h* 6543 at 7%

Check each result.

ACCURATE INTEREST

In England, all interest is calculated on the basis of 365 days to the year. The United States Government uses the same basis, as do banks in making interest payments.

The following extract shows the exact interest for 264 days on a few sums, at specified rates:

264 days — Year of 365 Days

Principal	4%	5%	6%	7%
$1000		$36.16,4	$43.39,7	$50.63,0
2000		72.32,9	86.79,5	101.26,0
3000		108.49,3	130.19,2	151.89,0
4000		144.65,8	173.58,9	202.52,1
5000		180.82,3	216.98,6	253.15,1
6000		216.98,6	260.38,4	303.78,1
7000		253.15,1	303.78,1	354.41,1
8000		289.31,5	347.17,8	405.04,1
9000		325.47,9	390.57,5	455.67,1

WRITTEN EXERCISES

1. Fill out the 4% column.

METHOD

$$\text{Int. on } \$1000 = \frac{\$1000 \times 264 \times .04}{\frac{365}{73}} = \frac{\$2112}{73} = ?$$

Carry out the quotient to five decimal places. Multiply this successively by 2, 3, etc., to 9. Write the results to the nearest mill.

Check the interest on $6000 and $9000, respectively, by comparing each with the interest on $3000; etc.

2. Find the interest for 264 days on:

a $1234 at 5% *b* $2345 at 6% *c* $3456 at 7% *d* $4567 at 5%
e 5678 at 6% *f* 6789 at 7% *g* 9876 at 5% *h* 8765 at 6%

Certificate of Deposit

A bank that pays 2% interest on a customer's daily balances will pay 3%, for instance, on money left with it for 3 months or more on a special deposit.

John T. Collins having in the Mechanics Bank a balance of $3,800, for which he has no use for three months or more, withdraws $2500 from his account and obtains from the bank the following:

Certificate of Deposit

2500^{00}/_{100}$ Woodrow, Mont., *Jan. 25, 1920*

MECHANICS BANK

This certifies that *John T. Collins*
has deposited with this bank
Twenty-five hundred 00/100.....Dollars
payable on or after Apr. 25, 1920 to the order of
John T. Collins.

3 with interest at 3 per cent upon the return of this certificate property indorsed.

Joseph Stewart Jr. Asst. Cashier — *Francis Weekes* Vice Pres.

3. Find the exact (accurate) interest on the foregoing certificate from Jan. 25, 1920, to Apr. 25, 1920.

METHOD

Find the interest on $\frac{\$2500 \times .03 \times 91}{365}$ 91 days, taking 365 days to the year. Cancel two ciphers in $2500, and the decimal point in .03. Cancel 25 and 365.

4. Mr. Collins, finding he needs money on March 25, returns the certificate on this date. The bank allows him but 2% interest for the time it has had the money. How much does it pay Mr. Collins, principal and exact interest at 2%?

Interest on Daily Balances

Some banks and trust companies pay interest on daily balances when these are in excess of a certain sum. An account that has less than an average balance of $200 is probably carried by a bank at a loss.

The following table shows the interest for 1 day, taking 365 days to the year:

Interest for 1 day — Year of 365 days

Principal	2%	2½%	3%	3⅓%
$100,000	$5.47,9	$6.84,9	$8.21,9	$9.58,9
200,000	10.95,9	13.69,9	16.43,8	19.17,8
300,000	16.43,8	20.54,8	24.65,8	28.76,7
400,000	21.91,8	27.39,7	32.87,7	38.35,6
500,000	27.39,7	34.24,7	41.09,6	47.94,5
600,000	32.97,7	41.09,6	49.31,5	57.53,4
700,000	38.35,6	47.94,5	57.53,4	67.12,3
800,000	43.83,6	54.79,5	65.75,3	76.71,2
900,000	49.31,5	61.64,4	73.97,3	86.30,1

5. A depositor's balance for 20 days is $900; for the next 12 days it is $1200; for the next 15 days it is $1600. Find the interest at 2% for the 47 days.

METHOD

$900 for 20 days = $18,000 for 1 da.
1200 " 12 " = 14,400 " 1 "
1600 " 15 " = 24,000 " 1 "
= $56,400 for 1 da.

To find the interest use the table

6. Find the exact interest on the following:

a $12,000 for 75 da. at 2%
b $3000 for 235 da. at 2½%
c $14,000 for 61 da. at 3%
d $4000 for 186 da. at 3½%
e $16,000 for 39 da. at 2%
f 5000 for 127 da. at 3½%
g $18,000 for 47 da. at 3%
h 6000 for 206 da. at 2½%

Savings Accounts

Postal Savings Certificates

In a place remote from banks, a person ten years of age or over can obtain interest on his savings by means of postal savings certificates, obtained through any post office.

These certificates are issued in denominations of $1, $2, $5, $10, $20, $50, $100, $200, and $500, each bearing the name of the depositor, the number of his account, the date of issue, the name of the depository office, and the date on which interest begins (the first day of the month next following the day on which the deposit is made).

Interest is paid at the end of a full year; if not collected, it accrues annually. No interest is paid on accrued interest.

If a certificate is lost or destroyed, the depositor may obtain a new one.

Postal Savings Bonds

A depositor may exchange certificates into U. S. postal savings bonds on Jan. 1 or Jul. 1 by making application one month previously.

These bonds are issued in denominations of $20, $100, and $500. They bear interest at 2½%, payable semiannually.

Savings Banks

H. M. Devoe's account in the savings department of the Mississippi Valley Trust Company shows deposits and withdrawals as follows:

Date		Deposits	Interest	Withdrawals	Balances
1921					
Jan.	1				327 49
Apr.	9	250			577 49
May	8			75	502 49
Jul.	1		6 54		509 03
Sep.	10			50	459 03
Nov.	4	100			559 03
1922					
Jan.	1		(*a*)		(*b*)
Mar.	13	75			(*c*)
May	24	150			(*d*)
Jul.	1		(*e*)		(*f*)

This company allows interest semiannually on January 1 and July 1, on even dollars of deposits that have been in the bank for 6 months at the interest date.

On July 1, 1921, 2% was allowed on $327 (rejecting the cents), the interest being added to the balance. On January 1, 1922, 2% of $509 was allowed (*a*); on July 1, 1922, 2% of $559 was allowed (*e*).

WRITTEN EXERCISES

1. Write from the book the answers (*a*) to (*f*).

2. Copy and complete the foregoing account by allowing 1% quarterly.

Insert two other interest dates, April 1 and October 1. At each date insert 1% of the smallest sum in the bank during the quarter.

Other banks, while allowing interest on April 1 and October 1, do not enter interest at these dates, but add it to the interest due on January 1 and July 1, respectively.

3. Copy and complete the foregoing account by calculating the interest due on April 1 and October 1 as in Example 2. Do not credit these interest items to the account until the following July 1 and January 1, respectively.

Calculate the interest due on April 1, but do not enter it until July 1, combining it then with the interest from April 1 to July 1. This makes a slight difference in some of the "balances" and, therefore, in the interest.

4. Copy and complete the foregoing account by allowing interest on January 1 and July 1, at the rate of 4% annually on all sums three months in the bank at those dates.

Check up the interest allowance in your bank book or in those of your parents. Read carefully the interest rules printed on one of the cover pages.

CHAPTER THREE

STOCKS AND BONDS

Forming a Corporation

Wishing to provide employment for residents of Accotink, some of the progressive citizens determined to establish a cannery. They interested the farmers in the vicinity, who agreed to furnish a portion of the funds, and to supply fruits and vegetables at a fair price.

It was found that a beginning could be made with $50,000, and a *charter* was obtained from the Legislature, authorizing the establishment of the Accotink Canning Company, with a capital of $50,000.

Capital Stock
$50,000

500 Shares of
$100 each

THE ACCOTINK CANNING COMPANY

Incorporated 1918

Stock Certificate No. 44

This certifices that Louis Laplace

is entitled to 25 shares of $100 each of the stock of

THE ACCOTINK CANNING COMPANY

fully paid and non-assessable, transferable on the books of this corporation only by the holder hereof in person or by attorney upon the surrender of this certificate.

SEAL

In witness whereof this corporation has caused this certificate to be signed by its duly authorized officers and to be sealed with the seal of the Corporation, this 24th day of February, 1918

Lud Hopkins
Secretary and Treasurer

Chas. Appich
President

This capital *stock* of $50,000 was divided into *shares* of $100 each, the purchaser of one or more shares receiving a stock certificate in the preceding form.

Directors and Officers

The stockholders elected twelve (12) directors to serve for a year from March 1, 1918. To this Board of Directors, given the general management of the corporation and the selection of officers to take charge of the details.

PREPARATORY EXERCISES

1. At the end of a year, the profits of the Accotink Canning Company were found to be $5510.85. About what per cent of $50,000 does this represent?

2. The directors decided to distribute $6 per share among the stockholders. (*a*) How much was thus distributed? (*b*) How much of the profits remained for working capital, etc.?

3. How much of the profits did Mr. Laplace receive, who owned 45 shares?

4. Mr. Laplace sold 15 of his shares to Mr. Beattys at $140 a share. How much did the latter pay for them?

5. At the end of the next year, the directors distributed $7 a share. What per cent of $140 did Mr. Beattys receive?

Par Value of Stock

By the *par value* of a stock is generally meant its cost to the original contributors to the capital. This is generally fixed at $100 a share.

The par value of a share of the Pennsylvania Railroad is $50 a share.

When the stock sells below its par value ($100 in the case of most stocks), it is said to be sold at a *discount;* when it sells above its par value, it is said to be sold at a *premium.*

Dividends

At stated times the directors meet to *declare a dividend.* This means that they determine the sum per share to be paid as a dividend to the stockholders.

In the case of the Accotink Canning Company for example, the books showed earnings for the year of $5510.85. Of this amount $1000 had been spent for new machinery, and $750 more was needed for the purpose. To have funds in hand for emergencies, same it was agreedto distribute $3000 among the stockholders. This made the dividend $6 a share.

To each *stockholder of record* was mailed a check for the amount of his dividend at the rate of $6 for each share owned by him according to the Company's books.

Prices of Stocks

The daily papers give the prices at which stocks are bought and sold at the Stock Exchange. The following were the rates for a few stocks:

Adams Express Co.	51¼	American Beet Sugar	70½
Baltimore & Ohio	57¾	Delaware & Hudson	104⅛
General Electric	148⅜	Illinois Central	96⅝
Louisville & Nashville	116⅞	Union Pacific	128¾
United States Steel	110⅛	Western Union	83⅜

These prices mean that on a certain day, stock of

Adams Express Co. sold at $51.25 a share, of U. S. Steel, $110.12½ a share, etc.

The Stock Broker

A person desiring to buy or to sell stock generally finds it advisable to do so through a stock broker, who charges for his services ⅛% of the par value of $100 a share. When the broker buys Baltimore & Ohio Railroad stock for $57.75 a share, he charges his client $57.87½, adding 12½ cents a share as his commission. When he sells stock of the General Electric Co. for $148.37½ a share, he remits to his client $148.25 a share, deducting his commission of 12½ cents a share.

Note: In all examples in stocks take as the par value $100 unless another value is given.

SIGHT EXERCISES

1. A broker bought for a client stocks for which he paid the following prices:

a 70½	*b* 83¾	*c* 97¼	*d* 101⅞	*e* 93
f 56⅛	*g* 64½	*h* 73⅜	*i* 174⅝	*j* 86¼

Give the cost of each per share to the client, after he pays the broker's commission of ⅛%.

2. Stock was sold through a broker at the following rates:

a 68	*b* 75½	*c* 86⅛	*d* 97¼	*e* 118⅜
f 57⅝	*g* 82¾	*h* 95½	*i* 88⅞	*j* 129

Give the price per share received by the seller in each case, after the deduction of the broker's commission of ⅛%.

3. A broker filled orders for stocks, buying the following quantities at the prices specified:

a 25 shares at 83⅞	*b* 4 shares at 71⅛
c 88 shares at 99⅝	*d* 6 shares at 60⅜
e 50 shares at 80¾	*f* 8 shares at 87½

Give the cost of each lot when ⅛% commission is added to each purchase.

4. Give the price received by the seller of each of the following lots, after the deduction of the broker's commission of ⅛%:

a 25 shares at 127⅞	*b* 4 shares at 81⅜
c 96 shares at 100	*d* 6 shares at 70⅝
e 50 shares at 144¼	*f* 8 shares at 62½

5. How much commission does a broker receive who sells 25 shares at 127⅞, and 96 shares at 100?

Ignore the prices, since a broker's commission is the same whether he sells a $100 share for $27⅛ or for $127⅛.

WRITTEN EXERCISES

1. A broker bought for a customer 176 shares of General Electric at 148½. How much did the stock cost the latter including the broker's commission?

METHOD

The stock cost the customer $148.62½ a share, including commission. Find the product of 176 times $148.62½ by aliquot parts. Test by multiplying 176 by 148⅝.

100 sh.	$14,862.50
50 "	7,431.25
25 "	3,715.625
1 "	148.625
Ans.	$16,158.—

2. Find the amount a purchaser should pay for each of the following blocks of stock, adding a commission of ⅛% to the given rates:

a 125 shares at 163¼	*b* 75 shares at 57⅞
c 287 shares at 148⅝	*d* 63 shares at 64½
e 144 shares at 126¾	*f* 98 shares at 76⅞

3. A broker sold for Mr. Jenkins 275 shares of Illinois Central at 96⅞. How much should Mr. Jenkins receive after the deduction of the commission?

METHOD

Mr. Jenkins receives	250	shares	@	$96.75		$24187.50
$96⅞ − $⅛ a share,	25	"	"	"		2418.75
or $96.75. Test	275	"	"	"	Ans.	$26606.25

4. Find the sum due each seller of the following blocks of stock after deduction of the broker's commission, the respective quantities and selling prices being

a 216 shares at 112¾	*b* 86 shares at 76½
c 154 shares at 106⅞	*d* 79 shares at 59⅜
e 375 shares at 184½	*f* 38 shares at 64¼

5. M. E. Kelley sent his broker $10,000 with instructions to buy stock of the Pacific Lumber Co. How many shares at 86¾ could be bought, and how much money should the broker return his principal after deducting his commission at ⅛%?

6. Find the maximum number of shares at each of the following rates that can be bought for $10,000, and the balance remaining after paying for the shares and the commission of ⅛%.

a 94⅜ *b* 86¾ *c* 74⅝ *d* 116⅛ *e* 127

7. Mr. Guiry ordered his broker to sell a sufficient number of shares of Midvale Trolley Co. to realize $10,000 after the deduction of the usual commission. How many shares at 114⅞ must be sold?

METHOD

$10,000 ÷ $114¾ = 40,000 ÷ 459 = 87 +

Ans. 88 shares.

8. Find the number of shares of each of the following that must be sold to realize $10,000.

a 96½ *b* 87¾ *c* 64⅞ *d* 109¼ *e* 137⅞

Preferred Stock

In 1921, needing more money to extend the business of the Accotink Canning Company, its stockholders authorized the issue of 500 shares of *preferred* stock, on which an annual dividend of $6 a share was to be paid before any dividend payment was made to holders of the original (*common*) stock. Each stockholder was permitted to buy for $100 each the same number of shares as he held of the common stock. Owners of the latter unable or unwilling to buy preferred stock could sell their rights.

SIGHT EXERCISES

1. (*a*) How much should an outsider pay for 6% preferred stock to enable him to obtain 5% annually on his investment?

6% ÷ ? = 5%

(*b*) How much a share could he afford to pay for the right to purchase the preferred stock?

2. If the company's profits the next year were \$9000, how much would be left after paying the holders of 500 shares of preferred stock \$6 a share and the holders of 500 shares of common stock \$7 a share?

3. The following year the amount available for the payment of dividends was only \$5750. (*a*) How much would remain for the holders of common stock after the payment of \$6 a share to the holders of the preferred stock? (*b*) How many dollars a share could be paid the former?

Bonds

Finding that it could use to advantage a large sum of money, the Accotink Canning Company offered for sale bonds maturing in 20 years, to the amount of \$50,000, bearing interest at 5% a year, payable semi-annually.

As security the company mortgaged its property, worth \$80,000, to the Old Dominion Trust Company for the benefit of the bondholders.

The bonds were issued in denominations of \$100, \$500, and \$1000.

SIGHT EXERCISES

1. How much will be required annually to pay 6% on bonds amounting to \$50,000, 6% dividends on preferred stock of \$50,000, and 7% dividends on common stock of \$50,000?

2. In order to pay the principal of \$50,000 in 20 years, how much must a company set aside semi-annually out of its gross profits?

3. How much must it set aside semiannually to allow for 2% annual depreciation on equipment valued at $43,750?

Public Securities

Bonds are issued to raise money to build schools, improve roads, install water systems, etc., etc.

Investors can be certain that the interest on these bonds will be paid as it becomes due, and the principal at the time the bond matures.

As the benefits from the foregoing improvements will continue for some time it is only fair that the payment of the cost thereof should be spread over a series of years.

Interest Payments

An examination of a Liberty Bond will show that it specifies the principal, the rate of interest, the time of each interest payment, and the date when the principal is payable.

Bonds are issued in two forms, registered and coupon. The registered bond shows the name of the owner. His address is kept by the Treasury Department, and the check for the interest is mailed to him. If he wishes to sell the bond he transfers it by assignment, which must be recorded by the Washington authorities, so that they may send interest checks to the new owner.

One advantage of the registered bond is that the owner suffers no financial loss if it is destroyed or stolen. A disadvantage to a person desiring to obtain cash at once, is the necessity of waiting a few days for the intending purchaser to obtain title.

Coupon Bonds

A coupon bond is generally payable to the holder, making it possible to sell it by handing it over to the purchaser.

A 20-year bond with interest payable semiannually contains as a part of it 40 interest coupons, one of which is detached each half year. They are numbered from 1 to 40; each shows the amount of the half-yearly interest and the date when it is due. When this day arrives the holder of the bond detaches the current coupon and cashes it through his bank.

Denominations

Bonds, registered and coupon, are issued in various denominations: \$50, \$100, \$500, \$1000, \$5000, \$10,000, etc.

Bond Quotations

When the price of a bond is given as 115, this means 115% of the face of the bond. If bought or sold through a broker, there is a charge of ⅛%.

A bond quotation gives the rate payable for a bond bought on an interest day. If bought thereafter, the buyer pays the "accrued" interest; that is, the interest earned by a bond from the day the last interest was paid until the day of purchase.

Accrued Interest

When a person on Aug. 15 sells a bond paying interest on Jan. 1 and Jul. 1, he is entitled to the interest it has earned during these 45 days between Jul. 1 and

Aug. 15. He turns the bond over to the buyer with the coupon covering six months' interest from Jul. 1. If the bond purchased is a 4% one for $1000 and the price is 115, the purchaser pays $1151.25 + 45 days' interest on $1000 at 4%.

In the following examples take the interest for the accrued time, at the rate paid by the bond, on the basis of 360 days to the year.

The pupil should, however, know that in large transactions the buyer may insist upon actual interest, 365 days to the year. The most equitable way is to take the number of days in the interest period and to calculate the accrued interest on the basis of the number of days in this period.

WRITTEN EXERCISES

1. At 112⅛ plus brokerage, find the cost of bonds to the amount of $15,000, bearing interest at the rate of 5%, payable semiannually on March 1 and September 1, when the purchase is made (*a*) June 20, (*b*) September 6.

METHOD

(*a*) Accrued interest on $15,000, Mar. 1 to Jun. 20, 111 da. at 5% is $231.25

(*b*) For 5 da. at 5%, it is $10.42.

The cost of the bonds on Mar. 1 or on Sep. 1 after the removal of the proper coupon, would be 1.12¼ times $15,000 or $16,837.50

Total cost (*a*) $16,837.50 + $231.25 = Ans.

Total cost (*b*) 16,837.50 + 10.42 = Ans.

2. Find the amount paid for each of the following purchases of bonds. Add brokerage to the given price.

	Bought	Price	Face Val.	Int. rate	Int. payable
a	Jun. 28	112½	$15,000	6%	Jan. 1
b	Sep. 20	97⅜	12,000	3%	Jul. 1
c	Nov. 19	104¼	20,000	5%	Oct. 1
d	Dec. 13	96⅝	18,000	3%	Dec. 1
e	May 12	101⅞	24,000	4%	Mar. 1

Income Rate on Investments

When a person buys a 3% bond for 90, including brokerage, and holds it until its maturity, 5 years later, his total income from a $100 bond would be $15 for 5 years' interest plus $10, the difference between $90, the cost of the bond, and the $100 he received for it when it was paid off.

This total income of $25 in 5 years represents an average of $5 a year, which was obtained from an investment of $90, making the annual rate 5⅝%.

These figures ignore the interest obtained by the reinvestment of each interest item as it is collected. Large investors take this into account, also the fact that the interest is payable quarterly, semiannually, or annually. They ascertain the income rates from bond tables, which involve calculations that can be made only by experts.

SIGHT EXERCISES

1. Ignoring the matter of interest on interest, how much less than $100 must a buyer pay for a 3% bond to receive $4 profit a year when the bond matures in (*a*) 1 year; (*b*) 2 years; (*c*) 3 years?

2. How much more than $100 can a buyer pay for a 5% bond maturing in (*a*) 1 year; (*b*) 2 years; (*c*) 3 years?

CHAPTER FOUR

FINANCING THE GOVERNMENT

The Taxpayer

Everybody contributes to the expenses of running the government. He may not receive a bill, but he pays taxes when he buys anything the price of which includes a tax paid by someone else. If he is not the owner of a house, a portion of his rent is used by his landlord to pay the tax. Everybody, therefore, should be interested in the proper use of government receipts.

The Budget

The residents of a rural school district meet annually to determine the sum to be raised for educational purposes.

The legislative department of a county, a city, or a state fixes the sums to be raised for its special purposes. Each body receives estimates from the officers in charge of the various activities, and finally determines the sum to be raised by taxation.

State Revenues

WRITTEN EXERCISES

1. Using the following data, write from the book the total of a state's revenues.

Direct Taxes	$45,510.43
Indirect Taxes	
Excise	114,787.50
Corporations	5,894,051.60
Inheritance	1,280,660.49
Stock transfers	985,902.38
Secured debt	635,902.53
Mortgages	745,132.12
Motor vehicles	262,747.—
Other revenue receipts	643,982.05
Total	$

2. The direct taxes are collected from the counties. What per cent of the revenue is obtained in this way?

3. Write the total of the following yearly

EXPENDITURES OF A STATE

Executive	$18,798.80	Defensive	$108,490.76
Administrative	250,244.41	Penal	111,178.29
Legislative	153,004.94	Curative	687,904.95
Judicial	200,755.71	Charitable	322,434.56
Regulative	382,962.83	Protective	160,371.90
Educational	250,249.90	Constructive	201,494.44
Agricultural	224,516.88	General	78,596.88

THE CITY BUDGET

The following are the approximate appropriations made by the city of Belle Haven for the year 1921:

Department of Finance	$43,420
Department of Water and Sewers	39,380
Department of Public Works	42,450
Department of Buildings	10,800
Department of Charities and Correction	32,650
Department of Police	35,400
Fire Department	31,250

Department of Parks	24,000
Department of Health	12,250
Judicial Purposes	10,000
Department of Street Cleaning	11,350
Interest on City Debt	44,500
Sinking Fund	24,000
Expenses of Administration	48,000
Department of Education	90,450
Sundry Expenses	8,450

4. Write the total appropriations for the year.

5. Find the per cent of the total appropriation allowed for (*a*) Educational purposes. (*b*) Charities and Correction. (*c*) Fire department. (*d*) Police purposes. (*e*) Parks. (*f*) Health. (*g*) Interest and Sinking Fund.

Valuations

For purposes of taxation, the value of the property in the city is fixed by officials called *assessors*. They visit each parcel once a year; they are furnished with maps of every block, showing the character of the improvements; and they endeavor in every way to keep acquainted with the changes in the value of the *real property* during the year. They then fix the *assessed value*, which is sometimes as low as one half its actual value.

Personal property is also assessed, and its valuation is added to that of the real estate, the sum of both representing the total valuation for purposes of taxation.

SIGHT EXERCISES

1. (*a*) When property worth 400 millions of dollars is valued by the assessors at 300 millions, what per cent

of the actual value is the assessed value? (*b*) What would be the assessed value of Mr. Ritchie's house, at this rate, if its actual value is $4000?

2. (*a*) If the sum to be raised by taxation is 3 millions, give the tax rate when the valuation is 300 millions. What would be (*b*) Mr. Ritchie's valuation? (*c*) His tax bill?

3. If the valuation were made 200 millions, what would be (*a*) the tax rate? (*b*) Mr. Ritchie's valuation? (*c*) His taxes?

4. Why would there be no difference in his taxes when there was a change in the valuation of his property?

Equalization

Whether property is assessed at its full value or at any per cent of it, the tax payment on any parcel is the same. All that an owner can desire is that all the parcels should be assessed at the same per cent of their value. If he feels that his valuation is proportionately greater than that of his neighbors, he can appeal to the Board of Assessors.

WRITTEN EXERCISES

1. The budget requirements for city purposes are $2,973,468; for county purposes, $387,596; and for state purposes $294,810. The city will receive from revenues $843,495. How much remains to be derived from taxation?

2. If the assessed valuation of the real and personal property to be taxed is $275,483,500, (*a*) what must

be the tax on each $100 to raise the amount needed as shown in the preceding example? (Give the result correct to five decimal places.) Find the tax to the nearest cent on property assessed (*b*) at $2000, (*c*) at $30,000, (*d*) at $400,000, (*e*) at $5,000,000.

3. Find H. DeW. Slater's tax at $1.80 per $100 on personal property, as follows: furniture, $500; clock, $10; watch, $25; vehicle, $100; horse, $175.

4. For the guidance of county assessors, the State Board of Equalization established the following classifications and valuation of an acre for land acreages in Nevada for the following year:

Cultivated

1st	class	$90	2d	class	$65
3d	"	50	4th	"	35

Meadow

1st	class	(1 ton or more to the acre)	$30
2d	"	(less than 1 ton to the acre)	18

Pasture

1st	class	$30	2d	class	$20
3d	"	11	4th	"	7

5. The classification of cultivated land is determined by the production from an acre as follows:

	1st class	2d class	3d class	4th class
Alfalfa	5 tons	3 to 5 tons	2 to 3 tons	under 2 tons
Hay	1½ tons	under 1½ tons	——	——
Grain	1 ton	1400 to 2000 lb.	800 to 1400 lb.	under 800 lb.

(*a*) Find the assessed value of 160 acres of land owned by Stephen Luken, 40 acres of which produced 180 tons of alfalfa; 40 acres, 70 tons of hay; 40 acres, 1200 bushels of wheat; and 40 acres of first-class pasture. (*b*) Find the average valuation per acre.

6. Neville Hart had 156 sheep which were assessed at \$9 each; 68 hogs at \$12; 42 pigs at \$4; 35 cattle at \$38; 6 horses at \$275; and 3 mules at \$195. What was the total valuation of the foregoing?

Find the amount of the following tax bill:

Mr. *Leonard Gross*..... } Mt. Vernon District
To the Treasurer of Fairfax County, Dr. }

State Taxes..... 35¢ on \$100 .....
County Taxes..Levy, 30¢; Pensions, 5¢; County Schools, 20¢; District Schools, 15¢; Road Tax, 25¢. Total 95¢ on \$100. — 1921

SUBJECTS OF TAXATION	State Taxes	County Taxes	Total
STATE CAPITATION TAX			1 50
Personal Property } B Val. \$3620			
" " } C " 600			
Total Valuation \$ (*a*)	(*f*)	(*g*)	(*h*)
Dog tax, 2 at 50¢ each			(*i*)
160 A. val. \$40 \$6400			
40 " " 30 (*b*)			
20 " " 25 (*c*)			
20 " " 18 (*d*)			
Total Valuation (*e*)	(*j*)	(*k*)	(*l*)
		Total tax Add 5%	(*m*) (*n*)
			(*o*)

Received payment in full

Curtis G. Edgerton, Treasurer

NOTE: Enter the capitation tax and the dog tax only in the "Total" column. Insert at (*a*) the total value of the personal property, at (*f*) the tax at 35¢, at (*g*) the tax at 95¢, and at (*h*) the sum of (*f*) and (*g*). Do the same with the tax on the real estate. To (*m*) add 5% of itself for delay in payment.

UNITED STATES REVENUES

WRITTEN EXERCISES

1. During the year preceding the war the receipts of the United States Government from all sources were $1,153,044,639.10. The total disbursements during the same period were $1,072,894,093.23. Find the balance.

2. Among the items of revenue were:

Customs		$213,185,845.63
Internal Revenue		(*a*)
Ordinary	$303,486,474.94	
Emergency	84,278,302.13	
Income tax		
Corporation	56,993,657.98	
Individual	67,943,594.63	
Sales of public lands		1,887,661.80
Consular fees		1,466,572.72
Profits and coinage		4,354,613.12
Tax on bank circulation		3,838.034.25
Sale of two battleships		12,535,275.96
Patent fees		2,329,510.36
Forest reserve fund		2,883,783.73
Receipts, Dist. of Columbia		9,132,976.52
Items not enumerated		(*b*)
		(*c*)

Insert at (*a*) the total of the four items of internal revenue receipts. Write at (*c*) the total receipts as given in the preceding example. Insert at (*b*) the difference between (*c*) and the sum of the other items, adding the latter and subtracting their total from (*c*) in one operation.

Duties

In ordinary times about one fourth of the total receipts of the Government are obtained from *duties*. These are the taxes paid on imported goods.

The rates of duty are fixed by the *tariff*. This is an act of Congress specifying the duty to be paid on each class of imports.

The Tariff

On some articles the tariff fixes an *ad valorem* duty. This is a certain per cent on the foreign cost. Bicycles and motorcycles pay 25%, for instance; breech-loading shot guns and rifles, 35%; silk ribbons, 40%; pen knives, 35%.

On other articles there is a *specific* duty of so much a square yard, pound, ton, etc. The rate on window glass, for instance, ranges from ⅛¢ to 2¢ a pound, according to its size, the lowest being for that not exceeding 150 square inches in surface. On sugar, the rate is 71/100¢ a pound with higher rates for the better grades. Grapes are charged 25 cents a cubic foot; lemons and oranges 15 cents a package not exceeding 1¼ cu. ft., 25 cents a package not exceeding 2½ cu. ft., etc., and ½¢ a pound in packages containing over 5 cu. ft.

Double Duties

A few articles pay both a specific and an ad valorem duty. Perfumery which contains alcohol is charged 40¢ a pound and 60%; that without alcohol pays only 60%. Lead pencils are charged 36¢ a gross and 25%; sweet chocolate, 2¢ a pound and 25%.

The Free List

Many articles are admitted free of duty; among them are agricultural implements, blooded cattle, bagging, binding twine, books, plants, trees, tea, coffee, wool, etc.

A Dutch Invoice

B. E. McAveney & Co. import two cases of dry goods. They receive the following invoice from the sellers:

Rotterdam, Mar. 20, 1920

Invoice of two (2) cases of dry goods marked ◇M/T 1609–10

Sold to Messrs. A. W. Ross & Co., Omaha
By Bergen & Van Brunt

and shipped Mar. 23, 1920, from Amsterdam per S.S. *Victory*.

◇M/T	1609	840 *m* Dress Goods	*fl* 1.80			
		360 " Laces	1.85			
		180 " Embroideries	2.10			
	1610	1200 " Sateens	1.97½			
		Less 5%				
					fl.	

The first column shows the mark on both cases; the second gives the number on each. No. 1609 contains three kinds of goods, and No. 1610 one kind. The length of each kind is given in meters (*m*) and the price in florins (*fl*)

WRITTEN EXERCISES

1. Copy the foregoing invoice, inserting the extension for each item. From the footing deduct 5%, and insert the net amount due in florins.

2. Find the equivalent value in U. S. money at the gold value of the florin, 40.2 cents.

3. At 39.37 inches to the meter find the number of yards (*a*) of dress goods. (*b*) Of laces. (*c*) Of embroideries. (*d*) Of sateens.

4. (*a*) Find the cost of each of the four items in the foregoing invoice after the deduction of the discount of 5%. (*b*) Express the net cost in U. S. money to the nearest dollar.

Paying Duties

The goods imported by B. E. McAveney & Co. were landed in New York and sent *in bond* in a sealed car to Omaha. Upon their arrival, the importers filed at the custom house their bill of lading and the foregoing invoice with the following entry:

Omaha, Nebr., Apr. 15, 1920

Entry of Merchandise imported by B. E. McAveney & Co.
Invoice dated Rotterdam, Mar. 20, 1920
Arrived at New York, Mar. 31, 1920

Marks	Nos.	Contents	35%	60%	15%	40%	Total
M T	1609/10	Dress Goods Laces Embroideries Sateens	(*a*)	(*b*)	(*c*)	(*d*)	(*e*)
			(*f*)	(*g*)	(*h*)	(*i*)	(*j*)

(*f*) 35% (*k*)
(*g*) 60% (*l*)
(*h*) 15% (*m*)
(*i*) 40% (*n*)
(*o*)

5. Copy the foregoing entry. Insert (*a* to *d*), the net cost of the various items, and at (*e*) the total.

Insert (*f* to *j*) the value in U. S. money, omitting cents. Calculate the duty on each item (*k* to *n*) at the given rates. Find the total amount (*o*).

Two copies of this entry, when complete, are handed to the entry clerk, who verifies the calculations, affixes his initials at (*o*), and passes one copy along to the cashier. He also designates the package to be sent to the appraiser's stores with the invoice. Here the package is opened, the goods measured, their character determined, and the rate of duty noted on the invoice. The latter is returned to the custom house. If the figures on the entry are correct, a *liquidating clerk* certifies thereto by affixing his initials in red. If the appraiser changes the values or the rates, the liquidating clerk makes out a new duty statement in red ink, and the difference to be collected from the importer or to be returned to him.

6. Find the duty on each of the following:

Classification of goods	Cost at place of purchase	U. S. coin value of foreign money	Rate of duty
a Opera glasses	1463.90 francs	19.3¢	35%
b Watches	£183 16s. 10 d.	$4.8665	30%
c Pickled fish	1237.85 kroner	26.8¢	25%
d Vases	2460.50 lire	19.3¢	45%

7. What is the duty on 48,648 pounds of sugar testing 87°, the rate being 71/100¢ a pound for sugar testing 75°, and 26/1000¢ additional for each degree above 75?

CHAPTER FIVE

PROTECTING THE INDIVIDUAL

A fire loss of $1000. or $100,000, which might greatly embarrass an individual, is easily shared by a multitude. For some such sum as $2.50 in one case or $250 in the other, an insurance company will provide for the payment of the loss if it happens during the year.

Fire Insurance

The contract is evidenced by a *policy.* This sets forth that the specified insurance company (*the underwriter*) in consideration of a certain sum (*the premium*) agrees to insure John Doe (*the insured*) for a specified *term,* from —— to ——, against all *Direct Loss and Damage by Fire* and by removal from premises endangered by fire to an amount not exceeding —— *Dollars* to the following property:

Insurance Rates

The rates for a given locality are generally fixed by a Board of Underwriters, who take into consideration all the conditions. The following shows those for $100 of insurance for 1 year on certain types of houses in a given section, also on their contents:

Occupied as	Residence		Apartment		Store and Dwelling	
Construction	Brick	Frame	Brick	Frame	Brick	Frame
Building	10¢	16¢	15¢	20¢	20¢	40¢
Household goods	16	20	20	24	24	40

Factors Determining Rates

A rate is provided for every type of building and for all varieties of goods. The size of a building, the nature of the roof, the water supply of the vicinity, the character of the business carried on, the proximity to other buildings — all are considered.

An extra per cent is sometimes added to the regular rates when the water supply is decreased; a per cent is deducted when the insured installs fire-fighting equipment; hose, fire-extinguishers, sprinklers, etc.

WRITTEN EXERCISES

1. Find the premium for insuring a house for $7000 and household goods for $5000 for 3 years at 2½ times the annual rate of 24 cents for the former and 32 cents for the latter.

2. A manufacturer paid 75 cents insurance on a building and 90 cents on stock, the former being insured for $20,000 and the latter for $80,000. *a* What was the insurance per year? He installed a sprinkler equipment at a cost of $6000. If his insurance was reduced 75% thereby, (*b*) how much did he save a year in excess of 6% interest on the cost of the equipment and 6% additional allowed for its depreciation?

3. A dealer in dry goods occupied the 7th, 8th, and 9th floors of a loft building. He insured goods to the amount of $48,000 on the 7th floor, at 75¢; to the amount of $45,000 on the 8th floor, at 80¢; and to the amount of $65,000 on the 9th floor, at 85¢. Find (*a*) the total cost of the insurance. (*b*) The average rate paid on the total amount insured.

4. A merchant insures his goods while in a warehouse for 25 days at 19 % of the yearly rate of 72 cents. What is the premium for insurance to the amount of $25,000?

$$250 \times .19 \times \$.72$$

Represent the number of $100 in $25,000 as 250; 19% as .19; and 72¢ as the decimal of a dollar. Write the answer.

Rates for less than a year are fixed for a locality by the Board of Underwriters. The following table gives the rates for one section. Insurance generally begins at 12 M and terminates at the same hour.

SHORT-TERM RATES

Time	%	Time	%	Time	%	Time	%	Time	%
1 da.	2	8 da.	9	15 da.	13	1 mo.	20	6 mo.	70
2 "	4	9 "	10	16 "	14	45 da.	27	7 "	75
3 "	5	10 "	10	17 "	15	2 mo.	30	8 "	80
4 "	6	11 "	11	18 "	16	75 da.	37	9 "	85
5 "	7	12 "	11	19 "	16	3 mo.	40	10 "	90
6 "	8	13 "	12	20 "	17	4 mo.	50	11 "	95
7 "	9	14 "	13	25 "	19	5 mo.	60	12 "	100

5. A person who has taken out insurance for a year from March 3, surrenders his policy of $10,000 on the morning of April 17. If the rate was 54 cents per year, what should the insurance company return?

The rate for 45 days (March 3 to April 17) being 27 % of that for a year, the company would refund 73 % of 100 times 54 cents. If the policy were surrendered on the afternoon of April 17, the company would retain 30 %, the next higher rate, returning 70%. Periods other than those specified are not considered by insurance companies using the foregoing short-term rates.

7. An insurance company canceled policies as follows:

	Issued	Face	Term	Rate	Canceled
a	Jan. 4, 1921	$16,000	75 da.	55¢	Feb. 16, 1921
b	Feb. 5, 1922	20,000	3 mo.	70¢	Apr. 10, 1922
c	Mar. 9, 1921	25,000	25 da.	26¢	Mar. 20, 1921
d	Apr. 6, 1922	12,000	1 yr.	32¢	Sep. 30, 1922
e	May 8, 1921	15,000	10 mo.	90¢	Jul. 15, 1921

Find (I) the premium originally paid on each; (II) the premium retained by the company; (III) the sum returned to the insured upon the surrender of his policy.

Since the great majority of fires cause only a partial loss, most insurers take out a policy for a sum less than the value of the property insured. A man, for instance, may insure for $5000 property worth $10,000. If the policy contains no provision to the contrary, he will be reimbursed in full for any loss not exceeding $5000.

Co-insurance

The laws of many states require that a provision similar to the following be inserted in every policy issued in these states:

"*This company shall not be liable for a greater proportion of any loss or damage to the property described herein than the sum hereby insured bears to eighty per centum (80%) of the actual cash value of said property at the time such loss shall happen. . . .*"

This means that the holder of policy of $5000 on property worth $10,000 will receive only $\frac{5}{8}$ of any loss he incurs not exceeding the face of the policy. To secure payment in full up to the amount of his insurance he must insure for $8000.

SECTION VII

BUSINESS MEASUREMENTS

CHAPTER ONE

COMMON TABLES

A housekeeper buys milk by the quart; a dealer, by the 100 pounds. While a miller pays for wheat at a given price a bushel, the quantity is determined by the weight of the grain.

Small dealers now sell by weight such vegetables as potatoes, onions, tomatoes, etc.

In some states the use of dry measures in selling goods is forbidden by law.

The following tables include the weights and measures in common use.

Measures of Length

12 inches (in. or ″)	= 1 foot (ft. or ′)
3 feet	= 1 yard (yd.)
5½ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)

Measures of Surface

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Measures of Volume

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd)
128 cubic feet	= 1 cord

Avoirdupois Weight

16 ounces (oz.)	= 1 pound (lb.)
2000 pounds	= 1 ton (T.)
2240 pounds	= 1 long ton

Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Liquid Measure

2 pints (pt.)	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)

SIGHT EXERCISES

1. At 60 pounds of potatoes to the bushel, what should be the weight of a peck?

2. There are 231 cubic inches in a gallon. How many cubic inches are there in a quart, liquid measure?

3. A cubic foot of water weighs 1000 ounces. (*a*) How many pounds does it weigh? (*b*) Give the weight of a gallon of water, assuming that there are 7½ gallons to the cubic foot.

4. A section of land is a square mile. How many acres are there in a quarter section?

5. How do two panes of glass compare in area when the dimensions of one are $6'' \times 8''$ and those of the other are each 1½ times as great?

6. How long does it take a soldier to travel 2½ miles at the rate of 3 miles an hour?

7. When soldiers march 88 yards a minute, (*a*) how many minutes do they take to march a mile (1760 yd.)? (*b*) How many miles an hour do they march?

8. A carrier pigeon flew from Rheims to Paris, 81 miles, in 4½ hours. How many miles an hour did it fly?

WRITTEN EXERCISES

1. At 2150.42 cubic inches to the bushel (*a*) how many cubic feet are there in a bushel? (Give answer to two decimal places.) (*b*) How much does this differ from the approximate rate of 1.25 cubic feet?

2. Find the difference in cubic inches between 1¼ cubic feet and 2150.42 cubic inches.

3. At $4.8665 to the pound, find the value of £247 17s. 6d.

4. Express 4.835 miles in miles, rods, and yards.

5. Find the average height of forty boys four of whom measure 5 ft. 5 in. each; eight, 5 ft. 6 in. each; twelve, 5 ft. 7 in. each; and sixteen, 5 ft. 8 in. each.

6. A regiment made a forced march, starting at 8 A.M. It rested from 8:50 to 9, from 9:45 to 10, from 10:40 to 11, from 11:30 to 11:40, from 12:10 to 1, from 1:50 to 2, from 2:45 to 3, from 3:40 to 4, from 4:30 to 4:40, and reached its journey's end at 5:10, covering a distance of 19½ miles. (*a*) How many hours elapsed between the start and finish? (*b*) How much time was spent in rest? (*c*) Find the average rate of travel an hour while on the march.

7. Find the total weight of the following items of an officer's baggage in a summer campaign:

1 barrack bag	1 lb.		1 pr. woolen breeches	1 lb.	9 oz.
1 mosquito bar		14 oz.	2 " cotton drawers	1 "	11 "
1 canvas basin		7 "	1 flannel shirt		15 "
1 bedding roll	11 "	12 "	1 pr. marching shoes	2 "	10 "
1 blanket	5 "	2 "	5 " stockings		10 "
1 canvas bucket	2 "		3 cotton undershirts	1 "	8 "
1 bedsack	1 "	14 "	1 clothing roll	3 "	14 "
1 mosquito headnet		14 "	3 handkerchiefs		2 "
1 lantern	2 "	4 "	1 sweater	2 "	
1 pack carrier		8 "	1 poncho	3 "	13 "
1 comb		2 "	1 housewife		4 "
1 pkg. paper		15 "	1 mirror		6 "
1 cake soap		6 "	1 shaving outfit	1 "	4 "
3 face towels	1 "		1 toothbrush and dentrifice		4 "

8. When a soldier's pace is 30 inches, (*a*) how many paces does he take in going a mile? (*b*) When he goes 3 miles in an hour, how many paces does he take in a minute?

9. How many miles does a horse walk in an hour (*a*) when it walks a mile in 16 minutes? (*b*) When it takes 120 steps of 33 inches each in a minute?

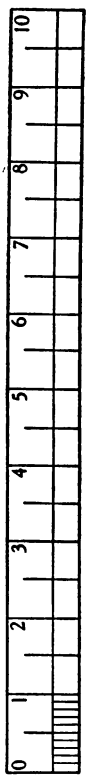
Metric Measures and Weights

The basis of the metric system is the *meter*, which is one forty-millionth of the earth's circumference passing through the poles.

Long Measure

10 millimeters (^{mm})	= 1 centimeter (^{cm})
10 centimeters	= 1 decimeter (^{dm})
10 decimeters	= 1 meter (^{m})
10 meters	= 1 decameter (^{dam})
10 decameters	= 1 hectometer (^{hm})
10 hectometers	= 1 kilometer (^{km})

The subdivisions of each metric unit are denoted by the Latin prefixes, *deci*, *centi*, *milli*, which indicate tenths, hundredths, thousandths, respectively. The multiples are denoted by the Greek prefixes, *deca*, *hecto*, *kilo* which indicate ten, hundred, thousand, respectively.



The denominations in common use are the *millimeter*, to express the thickness of wire, for instance; the *centimeter*, to express the width of ribbon; the *meter*, to express ordinary lengths; and the kilometer to express long distances.

Writing Metric Numbers

To express 84 centimeters, write either 84^{cm} or $0.^{m}84$, just as you would write either 84¢ or \$0.84 to express 84 cents. To indicate 3 meters, 8 decimeters, write $3^{m}.80$, expressing the decimeters as centimeters; just as you would use \$3.80 to express 3 dollars 8 dimes.

SIGHT EXERCISES

1. The meter is 39.37 inches. Taking 40 inches as its length, how long is (*a*) the decimeter? (*b*) The centimeter?

The accompanying strip is 1 decimeter long, divided into 10 centimeters, the first of which is subdivided into ten millimeters, the others showing subdivision of 5 millimeters each.

2. Give the approximate number of centimeters to the inch.

3. About what size collar in centimeters should be

bought in Paris by an American soldier who wears 16-inch collars?

4. About what is the bore of a 77 millimeter gun?

5. Taking the kilometer as ⅝ mile, how many miles are equal to (*a*) 40 kilometers, (*b*) 200 kilometers?

6. When the bore of a cannon measures 155 millimeters, what is it approximately in inches?

WRITTEN EXERCISES

1. An importer bought 1879 meters of silk; how many yards did he buy?

2. How many centimeters is it in width if it is 27 inches wide?

3. Find the difference between a kilometer and ⅝ mile (*a*) in inches. (*b*) In feet.

4. A merchant bought 7200 meters of silk at 12 fr. 50 a meter and sold it at $2.50 a yard. Find his profit.

5. How far does a cavalry squad travel from 7:45 A.M. to 11:15 A.M. if it travels 2 hectometers in 2 min. 30 sec?

Dry and Liquid Measure

The unit for measuring liquids, grain, etc. in small quantities is the *liter* (l). Its multiples and its subdivisions are indicated by the prefixes used with the meter: deci, centi, deca, etc The liter may be considered as a hollow cube 1 decimeter long, 1 decimeter wide, and 1 decimeter high.

Grain in large quantities is sold by the *hectoliter*.

SIGHT EXERCISES

1. There are 231 cubic inches in a gallon. (*a*) How many cubic inches are there in a quart? (*b*) How many

cubic inches are there in a cube 4 inches long, 4 inches wide, 4 inches high? Taking the latter as the equivalent of a liter, how many more cubic inches does this contain than the liquid measure quart?

2. A liter is equivalent to .908 dry quart. (*a*) How many quarts are there in a hectoliter? (*b*) About how many bushels?

3. About how many gallons are there in a hectoliter at .9463 liquid quarts to the liter?

Metric Weights

The unit of weight is the *gram*, used in one or the other of its denominations for weighing everything from diamonds to iron ore.

The prefixes are the same as in the other tables.

A kilogram is the weight of a liter of water.

The most commonly used denomination is the *kilogram* (generally called a kilo). This is equivalent to 2.2046 pounds. The metric ton of 1000 kilos (the tonneau) is used in selling articles we sell by the ton. The druggist weighs some drugs in *milligrams*.

SIGHT EXERCISED

Give the difference between the long ton of 2240 pounds and the metric ton of 1000 kilos of 2.2046 pounds each.

WRITTEN EXERCISE

Find the weight (*a*) of a gallon of water (231 cu. in.) at 62.5 pounds to the cubic foot. (*b*) Of a quart of

water. (*c*) Of a liter of water, taking 1.0567 quarts as the equivalent of a liter.

Square Measure

PREPARATORY EXERCISES

1. What is the area (*a*) of a piece of land 12 meters long, 12 meters wide? (*b*) Of a piece of leather $1^{m.}$ 2 long, $1^{m.}$ 2 wide? (*c*) Of a piece 12^{dm} long, 12^{dm} wide?

2. (*a*) How many square decimeters are there in a square meter? (*b*) How many square centimeters are there in a square decimeter?

100 square centimeters (^{cmq})	= 1 square decimeter (^{dmq})
100 square decimeters	= 1 square meter (^{mq})
100 square meters	= 1 square decameter (^{damq})

Observe that in the table of square measure there are 100 divisions of one denomination to 1 of the next higher.

Farm Areas

In giving the area of a field, the term *are* (^{a}) is used, which is equivalent to 100 square meters. The *are* has one multiple, the *hectare* (^{ha}), and one subdivision, the *centiare* (^{ca}).

WRITTEN EXERCISES

1. What is the duty on 1000 tiles measuring 75^{cm} square at 1½¢ per square foot. (1 sq. meter = 1.196 sq. yd.)

2. How many square yards are there in a roll of cloth 48^{m} long 105^{cm} wide?

3. Find the number of pounds in a barrel of oil containing 45 gallons when 1^{l} weighs $.8^{kg}$.

4. Find the number of square yards in a square meter.

5. How many square meters are there in an acre (4840 sq. yd.)?

6. Find the number of acres (*a*) in an are. (*b*) In a hectare.

Cubic Measure

Table

1000 cubic centimeters (^{cmc})	= 1 cubic decimeter	(^{dmc})
1000 cubic decimeters	= 1 cubic meter	(^{mc})
1000 cubic meters	= 1 cubic decameter	(^{damc})
etc.		etc.

Observe in the table of cubic measure that there are 1000 divisions of one denomination to 1 of the next higher.

In some countries the abbreviation for cubic meter is (^{cbm}). In others, exponents are used for the square meter ($^{m^2}$) and the cubic meter ($^{m^3}$).

Measuring Firewood

Firewood is sold by the *stere* (st), which is a cubic meter. This has one multiple, the *decastere* (^{dast}); and one subdivision, the *decistere* (^{dst}).

The practice of selling wood by weight is increasing. The buyer of wood by the stere or by the cord cannot be sure of the amount of space occupied by the "voids." When he buys by weight, he requires that the wood be dry.

WRITTEN EXERCISES

1. Find the contents in cubic meters of a tank $3.^{m}$ 75 long, 2^{m} wide, $1^{m}.50$ deep.

2. How many liters of water would this tank contain?

3. What would be the weight of the water in kilograms?

4. How many decaliters of grain would a bin of the foregoing dimensions contain?

5. Find the weight (*a*) of a stere of white pine assuming that the wood occupies only 75 % of the space, and that the weight of the wood is 38 % of the weight of the same volume of water. (*b*) Of a stere of white oak with the same per cent of "voids," when its weight is 80 % of that of the same volume of water.

6. What is the weight of a cubic foot of each of the foregoing at the rate of 1000 ounces of water to the cubic foot?

CHAPTER TWO

AREAS AND VOLUMES

Lines and Angles

A *straight line* (Fig. 1) keeps the same direction throughout its length. A *broken line* (Fig. 2) is made

FIG. 1

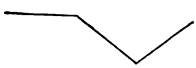


FIG. 2

FIG. 3

up of two or more straight lines. *A curved line* (Fig. 3) changes its direction continually.

When two straight lines meet at a point, they are said to form an angle (Fig. 4). In Fig. 5 are shown two angles formed by two lines. These are called supplementary angles. When these supplementary

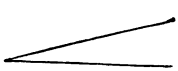


FIG. 4

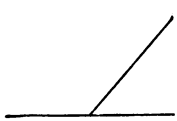


FIG. 5

FIG. 6

angles are equal, each is said to be a *right angle* (Fig. 6). The angle in Fig. 5 that is smaller than a right angle is called an *acute angle;* the larger one is called an *obtuse angle*. The term *oblique angles* is used to denote those that are not right angles; the one in Fig. 4 and the two in Fig. 5 are oblique angles.

The size of an angle is expressed in degrees (and subdivisions). The lines forming the four equal angles in Fig. 7 at the center of the circle divide the circumference into four equal parts. As a circle contains 360°, each portion contains 90°, and eachangle is said to contain 90°.

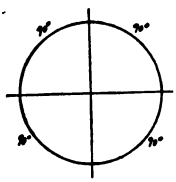


FIG. 7

Circular Measure

60 seconds (″) = 1 minute (′); 60 minutes = 1 degree (°); 360 degrees = 1 circle.

ORAL EXERCISES

1. How many degrees are there in the angle made by the hands of a clock (*a*) at 3 o'clock? (*b*) At 9 o'clock?

2. (*a*) How many degrees does the minute hand move in going from XII to III? (*b*) How many degrees beyond III does the hour hand move in 15 minutes? (*c*) How many degrees are there in the angle made by the hands of a clock at 3:15? (*d*) How many degrees are made by the hands of a watch at the same hour? (*e*) By the hands of the town-clock?

Determining Dimensions

A person that wishes to ascertain an area must know what lines to measure and how to measure. Some practice can be had about the class-room, the school building, and the grounds. A yard stick or a two-foot rule will do for the first, a steel tapeline for

the others. A 4-rod chain composed of 100 links is used to some extent.

Some longer distances may be approximated by pacing. Each pupil should determine the length of his ordinary step: (*a*) by measuring a single one; and (*b*), by measuring the distance covered by 20 or more, and finding the average, doing this several times and comparing results.

Area of Rectangle

A rectangle is a *quadrilateral* (figure of four sides) having four right angles. By drawing a rectangle 4 inches by 3 inches and dividing it into 1 inch squares, you will see that there are 3 rows of squares, each containing 4 squares, a total of 12 squares. Since each is 1 square inch, the rectangle contains 12 square inches. This is called its *area.*

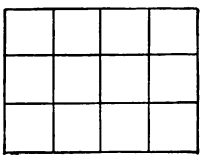


FIG. 8

To find the area of a rectangle in square units, multiply its length in the linear unit by its width in the same unit.

This may be stated as follows:

Area of rectangle = Length × Width

SIGHT EXERCISES

1. Give the areas of rectangles having dimensions as follows:

a 25 ft. by 96 ft. *b* 88′ × 99′ *c* 64 yd. × 12½ yd.
d 44 rd. by 25 rd. *e* 98″ × 32″ *f* 66 ft. × 16⅔ ft.

2. What is the area, in square feet, of a rectangle (*a*) 12 feet long, 9 inches wide? (*b*) 3 yards long, 18 inches wide?

Change (*a*) to 12 ft. $\times$ ¾ ft. (*b*) 36 ft. $\times$ 1½ ft.

3. Give the areas in square feet.

(*a*) 25 ft. by 32 yd. (*b*) 33 yd. $\times$ 88 ft. (*c*) 22 yd. $\times$ 33⅓ ft.

WRITTEN EXERCISES

1. (*a*) How many square feet will 1416 bricks cover when each is laid on its side, which measures 4″ $\times$ 2½″? (*b*) How many square yards will 1296 tiles cover, when each is 7 inches square?

METHOD

$$(a)\ \frac{1416 \times 1 \times 5}{3 \times 24} = ?\ \text{(sq. ft.)}$$

$$(b)\ \frac{1296 \times 7 \times 7}{36 \times 36} = ?\ \text{(sq. yd.)}$$

In (*a*) change 4″ to ⅓ ft. and 2½″ to ⅝ ft.
In (*b*) change 7″ to ⅞ yd.

2. Find the area, in square yards, of a piece of carpet 89 yards long 27 inches wide.

3. How many square feet will be covered by 68 boards 18 feet long 8 inches wide?

4. Find the number of acres in a field (*a*) 144 rods long, 32 rods wide. (*b*) 320 yards long, 186 yards wide.

METHOD

(*a*) $\frac{144 \times 32}{160} = ?$ (A) (*b*) $\frac{320 \times 186}{4840} = ?$ (A)

Since there is no linear unit corresponding to the acre, indicate the area of (*a*) in square rods (144×32) and the division of this product by 160, the number of square rods in an acre. In (*b*) divide the product in square yards by 4840, the number of square yards in an acre. Cancel in each example.

5. A field is 924 yards long by 792 yards wide. (*a*) How many acres does it contain? (*b*) How many rods of barbed wire, 4 wires high, will be needed to inclose it? (*c*) How many posts 6 feet apart will be required? (*d*) If boards are used, how many would it take when the boards are 12 ft. long and the fence is 3 boards high?

Parallelograms

A quadrilateral having its opposite sides and opposite angles, respectively equal, each to each, is called a *parallelogram* (Fig. 9).

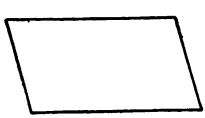


FIG. 9

The area of any parallelogram is equal to that of a rectangle having the same length and width. This may be seen in Fig. 10, which shows a right triangle $A(x)D$ cut from the left side of the parallelogram and transferred to $B(y)C$ at the right, forming the rectangle $AByx$. The area

of the latter is the length of the parallelogram (AB) multiplied by its width (Ax). The width of a parallelogram is generally called its *altitude*, and is the perpendicular distance between the sides constituting its length. Either of the latter sides is called the *base*.

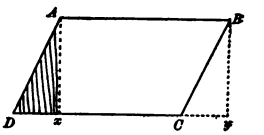


FIG. 10

Area of parallelogram = Length × Width

This is generally stated as the product of the base by the altitude.

NOTE: The line that measures the width must be perpendicular to the length. For this reason it is sometimes called the *perpendicular*.

NAMES OF QUADRILATERALS

While a rectangle is a parallelogram, a parallelogram is not always a rectangle. To distinguish between them, a rectangle is sometimes called an *oblong*, while the term *rhomboid* is used to denote a parallelogram containing oblique angles. An equilateral rectangle is called a *square;* an equilateral rhomboid is called a *rhombus*. A quadrilateral that has two parallel sides is called a *trapezoid;* one that has no sides parallel, a *trapezium*.

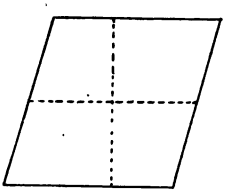


FIG. 11

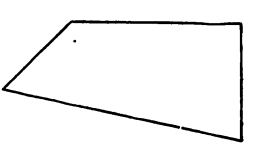


FIG. 12

In Fig. 11, the broken line halfway between the parallel sides represents the "average length" of the trapezoid, and the broken line perpendicular to

it represents the width. The average length is one-half the sum of the lengths of the two parallel sides.

Area of trapezoid = ½ (Sum of Parallel sides × Perpendicular)

NOTE: The words "perpendicular," "altitude," and "width" have the same meaning in these examples.

DIMENSIONS OF A PARALLELOGRAM

If a person desires to obtain the area of *ABCD*, (Fig. 13) which has opposite sides equal, and is, therefore, a parallelogram, he may measure any convenient side as a base, say *DC*. In this case he will measure *Bx* as the perpendicular. To do this, he must locate *x*, the point at which a perpendicular from *B* will intersect the base *DC*. This he can approximate by the use of a mounted T-square having pins *F* and *G*, near the extremities of one arm, and *N* and *S* near the extremities of the other.

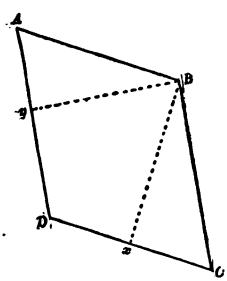


FIG. 13

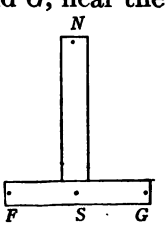


FIG. 14

Keeping on the line *DC* at each end of which a stake is placed, he locates the point *x*, by sighting *B* through *S* and *N*. If he can then see *D* through *G* and *F*, and *C* through *F* and *G*, the hole made by the staff supporting his *T*-square will mark the point *x*.

The Trapezoid

In Fig. 15, vw, one half the sum of AB and DC (the parallel sides), is multiplied by Ay (the length) or Bx to find the area of the trapezoid.

Since y must be located to measure Ay, the perpendicular between the parallel lines AB and DC, it will be unnecessary to locate v and w in field work. To check the result, x may be located and Bx measured. If these two lines are parallel, the length of vw may be determined by taking the half sum of the parallel sides, AB and DC.

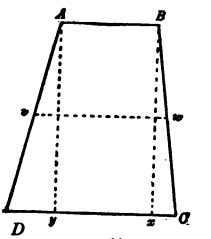


FIG. 15

In this case the length Ay is multiplied by the average width, vw.

Triangles

The accompanying figures show three triangles, ABC, DEF, and GIH, with broken lines drawn parallel

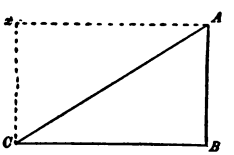


FIG. 16

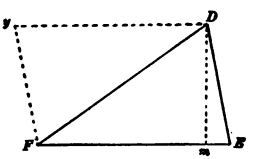


FIG. 17

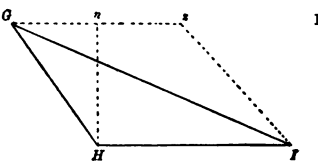


FIG. 18

to two sides of each triangle to form a parallelogram that has twice the area of its corresponding triangle.

Taking *CB*, *FE*, and *HI* as the respective bases of the triangles, their altitudes will be *AB*, *Dm*, and *Hn*, respectively.

Since the area of each triangle is one-half that of the corresponding parallelogram, the area of a triangle may be thus expressed:

Area of triangle = ½ (Base × Altitude)

The triangle *ABC*, having a right angle at *B*, is called a *right triangle;* the other two are called *oblique-angled* triangles. *DEF*, having three angles acute, is called an *acute-angled* triangle; *GIH*, containing an obtuse angle, is called an *obtuse-angled* triangle.

The line *AC*, *DF*, or *FD*, connecting the opposite angles of a parallelogram (Figs. 16–18), is called its *diagonal.*

Powers and Roots

PREPARATORY EXERCISES

1. Give the area of a square whose base measures

a 5 in. *b* 6 ft. *c* 7 rd. *d* 9 yd. *e* 12 mi. *f* 20 m.

2. Give products:

a 13×13 *b* 21×21 *c* 30×30 *d* 25×25 *e* 99×99

To indicate that a number is to be multiplied by itself, write above it to the right a small 2, called an exponent. The result is called the *square* of the number.

3. Give squares as follows:

a 13^2 *b* 21^2 *c* 30^2 *d* 32^2 *e* 40^2 *f* 41^2 *g* 80^2

To indicate that a number is to be used three times as a factor, use 3 as an exponent. The result is called the *cube* of the number.

4. Give cubes as follows:

a 2^3 *b* 3^3 *c* 4^3 *d* 5^3 *e* 6^3 *f* 7^3 *g* 8^3 *h* 9^3 *i* 10^3

When a number is taken 4 times, 5 times, etc., as a factor, the result is called the 4th power, 5th power, etc.

Square Root

5. Give the base of a square whose area contains

a 144 sq. in. *b* 100 sq. ft. *c* 81 sq. yd.
d 64 sq. mi. *e* 49 sq. rd.

The answer to each of the foregoing requires the finding of one of the two equal factors that make the given number. This factor is called the *square root* of the number. The sign of square root is $\sqrt{\ }$

6. Give square roots as follows:

a $\sqrt{25}$ *b* $\sqrt{100}$ *c* $\sqrt{144}$ *d* $\sqrt{400}$ *e* $\sqrt{169}$ *f* $\sqrt{900}$

$\sqrt[3]{\ }$ indicates the cube root, $\sqrt[4]{\ }$ the 4th root, etc.

WRITTEN EXERCISES

1. What is the side of a square plot that contains 2304 square feet?

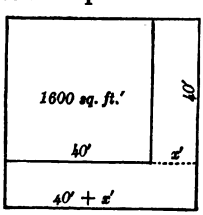


FIG. 19

Draw a square. Since it is evident that the square root of 2304 is between 40 and 50, lay off a portion measuring 40 ft. square, in one corner. This contains 1600 sq. ft. The remainder of the plot may be considered to comprise two rectangles each x ft. wide, one of them being 40 ft. long and the other 40 ft. $+ x$ ft. long. The two combined form a single rectangle x ft. wide and 80 ft. $+ x$ ft. long, containing 704

$$\begin{array}{r} 2304 \\ \underline{1600}\ (40^2) \\ 80 + x)704 \\ 88 \times \underline{8\ 704} \end{array}$$

sq. ft. Since 704 contains 80 more than 8 times, try 8 as the value of x taking 88 ft. for the length of the combined rectangles and 8 ft. for the width. The product of 88 and 8 being 704, 8 is the value of x and 40 + 8, or 48, is the length in feet of a side of the square.

METHOD

Divide 2304 into periods of two figures each, beginning at the right. Write 16, the largest square in 23, under the latter, and 4, its square root, over 23. Deduct 16 from 23. To 7, the remainder, annex 04. To the right of 704 write 8, twice the tens' figure of the root. Taking this as a trial divisor, divide it into 70 for the ones' figure. Try 8, writing it after the trial divisor and also after the 4 tens of the root. Multiply 88 by 8, writing the product under 704. Since the product agrees with the latter, 8 is correct, and the root is, therefore, 48. Write ft. in a parenthesis. Test by multiplying 48 by 48.

Ans.	4 8 (ft)
	23'04
	16
88	704
	704

2. Find roots:

$a\ \sqrt{8281}$ $b\ \sqrt{5184}$ $c\ \sqrt{2809}$ $d\ \sqrt{1225}$ $e\ \sqrt{729}$

3. (*a*) Multiply 7.9 by 7.9. How many decimal places are there in the product? (*b*) Find the square of .29. How many decimal places does it contain?

4. Extract the square root of each of the following:

$a\ \sqrt{5.29}$ $b\ \sqrt{.0841}$ $c\ \sqrt{13.69}$ $d\ \sqrt{34.81}$

5. Extract the square root of 136,161.

METHOD

Proceed as in the previous example. Separate 136161 into periods of two figures each. Write 3 in the root and 9 under 13. Subtract. Bring down 61. Take twice 3 for the first figure of the first trial divisor. Divide 46 by 6 for the second figure of the root. Note, however, that 7 times 67 would be greater than 461, and try 6, writing it in the root and annexing it to the other 6. Multiply 66 by 6, subtract. Bring down. For the second trial divisor, take twice 36, the portion already obtained, which gives 72 as the first two figures. Divide 65 by 7 for the third figure of the root. Write 9 in the root and annex 9 to 72. Multiply 729 by 9.

```
      3  6  9 Ans.
     13'61'61
      9
 66   461
      396
729    6561
       6561
```

6. Find the root of each of the following:

a $\sqrt{103041}$ *b* $\sqrt{178929}$ *c* $\sqrt{88804}$ *d* $\sqrt{443556}$

The Right Triangle

PREPARATORY EXERCISES

NOTE: In making graphs, drawing to scale, etc., the use of cross-ruled paper is very helpful and saves much time.

1. Draw three right triangles as follows:

a On a scale of ½″ to 1′, having base and perpendicular of 3 ft. and 4 ft., respectively.

b On a scale of ¼″ to 1′, having base and perpendicular of 5 ft. and 12 ft., respectively.

c On a scale of ⅛″ to 1′, having base and perpendicular of 8 ft. and 15 ft., respectively.

2. Measure the hypotenuse of each triangle.

The results will show that the hypotenuse in (*a*) will be 5 ft.; in (*b*), 13 ft.; and in (*c*), 17 ft.

3. Give the length of each in the scale drawing.

Observe the following:

$3^2 = 9$	$5^2 = 25$	$8^2 = 64$
$4^2 = 16$	$12^2 = 144$	$15^2 = 125$
$5^2 = 25$	$13^2 = 169$	$17^2 = 189$

that is, the square of the hypotenuse is equal to the sum of the squares of the other sides.

$$\text{Hypotenuse}^2 = \text{Base}^2 + \text{Perpendicular}^2$$

Application of Square Root

1. Find the length of the missing side in each of the following triangles:

a Perpendicular, 45; base, 24; hypotenuse, ?

b Perpendicular, 70; base, ?; hypotenuse, 74

c Perpendicular, ?; base, 30; hypotenuse, 78

d Perpendicular, 40; base, 42; hypotenuse, ?

2. How many rods of fence will be needed to enclose a field in the form of a right triangle having a base of 48 rods and a perpendicular of 64 rods?

3. How far from the foot of a building is the foot of a ladder 50 feet long that reaches a window 48 feet above the ground?

4. Find the diagonal of a rectangular field (Fig.16) 165 yards long, 144 yards wide.

Areas of Oblique-angled Triangles

Owing to the difficulty, at times, of measuring the altitude of a triangle, it becomes necessary, in finding its area, to use the lengths of its sides.

5. Find the area of a triangle whose sides are, respectively, 21 rods, 24 rods, and 27 rods.

METHOD

21
24
27
2)72
36

$\sqrt{36 \times (36-21) \times (36-24) \times 36-27} =$

$\sqrt{36 \times 15 \times 12 \times 9} =$

$\sqrt{58320} = 241.4953$

Ans. 241.5 (sq. rd.)

Take the square root of the continued product of the half sum of the three sides (36) by the difference between this half sum and each of the respective sides.

6. The following is a right triangle. Find its area by the foregoing method. Test the result by finding the half product of its base and altitude. The sides are 63 ft., 65 ft., and 16 ft.

7. The following is an *isosceles* triangle; that is, one having two equal sides. The sides are 29 yd., 29 yd., 40 yd. Find its area.

8. In an isosceles triangle, the unequal side is called the base. A perpendicular let fall from the opposite angle bisects the base. Find the perpendicular (altitude) of the triangle in the last example, and obtain its area by the use of the base and the altitude as the dimensions.

9. A triangle having three equal sides is called an *equilateral* triangle. (*a*) Find the area of an equilateral triangle with sides of 100 yards. (*b*) Find the altitude.

10. Find the square root of the following:

$a\ \sqrt{.5}$ $b\ \sqrt{1.6}$ $c\ \sqrt{.225}$

NOTE: In finding the square root of a decimal the latter must have an even number of decimal places. Change the foregoing to (*a*) $\sqrt{.50}$; (*b*) $\sqrt{1.60}$; (*c*) $\sqrt{.2250}$. In pointing off a mixed decimal whose root is to be found, begin at the decimal point, and point off in two directions.

AREAS OF POLYGONS

A polygon of three sides is called a *triangle;* of four sides, a *quadrilateral;* of five sides, a *pentagon;* of six sides a *hexagon;* of eight sides, an *octagon;* etc.

In the case of a regular polygon, the sides are all equal, as well as the angles.

Give the name of a *regular* triangle. Of a *regular* quadrilateral.

A regular polygon may be divided by lines into as many equal triangles as the polygon has sides each triangle having its apex at the center. Computers' tables give the number to be multiplied by the square of the length of the side to give the area. This number is .4330 for the equilateral triangle; 1, for the square, 2.5980, for the regular hexagon; 4.8284 for the regular octagon.

To find the area of an irregular polygon, divide it into triangles, two less than the number of sides.

WRITTEN EXERCISES

1. Find the area (*a*) of an equilateral triangle having sides of 17 feet. (*b*) Of a regular octagon having 6-inch sides. (*c*) Of a regular hexagon having sides of 11 inches.

2. *ABCD* is a trapezium. To find its area, the line *AC* has been measured and found to be 42 rods long. The perpendiculars *Bx* and *Dy* measure, respectively, 24 rods and 32 rods. Find the area of the trapezium, which is in square rods, ½ of (42 × 32) + ½ of (42 × 24). Shorten the work by multiplying ½ of (24 + 32) by 42.

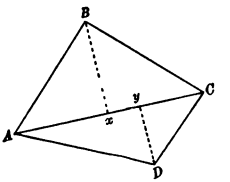


FIG. 20

3. Find the sum of the areas of the two triangles in the foregoing trapezium by determining the area of each triangle from the following:

In *ABC*: *AB* 34 rods, *BC* 20 rods, *AC* 42 rods

" *ADC*: *AD* 40 rods, *DC* 26 rods, *AC* 42 rods

Compare the two results.

4. A room is 18 feet wide, 24 feet long, and 9 feet high. (*a*) How many square yards are there in the ceiling? (*b*) How many square feet are there in the floor? (*c*) Find the number of square yards in one side wall (18′ × 9′). (*d*) In the opposite wall after the deduction of the space occupied by a door (6′ 9″ × 4′). (*e*) Find the number of square yards in an end wall, deducting for two windows, each 6′ × 3′. (*f*) In the opposite wall, deducting for a door of the size given above. (*g*) Find the number of running feet of baseboard in the room, deducting the space occupied by the doors. (*h*) Find the number of square feet, when the baseboard is 9″ high.

5. Determine, by making the necessary measurements, (*a*) the number of square yards of plastering required for your classroom. (*b*) The number of cubic

feet of air space. (*c*) The number of square feet of floor space. (*d*) The area of the exposed window glass.

6. A box of window glass contains 50 square feet as nearly as possible. Find the number of panes in a box for each of the following sizes:

a 6″× 8″ *b* 8″×10″ *c* 8″×12″ *d* 9″×12″ *e* 9″×16″
f 10″×15″ *g* 10″×16″ *h* 12″×12″ *i* 12″×15″ *j* 12″×18″

Board Measure

Lumber is sold by the *board foot.* A board foot is 1 foot wide, 1 foot long, and 1 inch thick. When the thickness is less than an inch, it is taken as 1 inch. A board 12 ft. long, 1 foot wide, and 1 inch thick contains 12 board feet; if of the same length and thickness and 8 inches wide, it contains 8 board feet; 16 feet long, 6 inches wide, and 1 inch thick, 8 board feet; etc.

Board feet = Feet long × feet wide × inches thick

SIGHT EXERCISES

1. Give the number of board feet in planks, scantlings, etc., having dimensions as follows:

	Length	Width	Thickness		Length	Width	Thickness
a	12′	½′	1″	*b*	16′	¾′	2″
c	18′	⅔′	2″	*d*	12′	⅓′	1″

Dealers' Tables

In a dealer's tables, the number of board feet in a board, scantling, plank, joist is given, for various lengths in feet and widths and thicknesses in inches.

Length in ft.	Width and thickness in inches								
	2×2	2×6	2×9	2½×6	2½×8	3×3	3×6	4×7	4×10
8	2⅔	8	12	10	13⅓	6	12	18⅔	26⅔
10									
12									
14									
16									
18									

2. From the foregoing dimensions give the number of board feet (*a*) by lines. (*b*) By columns.

WRITTEN EXERCISES

1. Find the number of board feet in each of the following:

a 2 sills, $4'' \times 4'' \times 10'$ *b* 2 plates, $2'' \times 4'' \times 10'$
c 2 " $4'' \times 4'' \times 14'$ *d* 2 " $2'' \times 4'' \times 14'$
e 16 pieces, $2'' \times 4'' \times 12'$ for studs, rafters, roosts.

2. Draw to a convenient scale the end view of a shed 13 feet high in front, 8 feet high in the back, and 12 feet deep. Find (*a*) the area of the end. (*b*) The length of the edge of the roof, if it projects 6 inches beyond the front and the back of the shed. (*c*) Find the area of the roof if the length of the shed is 18 feet and the roof projects 6 inches beyond each side also.

SHINGLES

Shingles vary in width. A bundle of 250 shingles contains a total width of 250 times 4 inches, or 1000 inches. When shingles are laid to form a roof, each row so overlaps the under one as to leave only a

portion of the length of the latter exposed to the weather, generally 4 inches. Since a shingle is considered as 4 inches wide, the space covered by each is $4'' \times 4''$.

3. (*a*) How many shingles with $4'' \times 4''$ exposed, will cover a square foot? (*b*) How many bundles of 250 shingles will be required for a roof $14' \times 19'$? (*c*) Find the cost of the shingles at \$30 a 1000 (4 bundles of 250 each). A whole bundle must be bought for any excess.

4. How many gallons of paint will be required to paint the sides and the back of the shed in Ex. 2 at the rate of a gallon to 45 square yards for the first coat, to 50 square yards for the second coat, and to 55 square yards for the third coat? (Give results to the nearest ½ gallon).

5. How many hours of work will a painter require for three coats, if he takes an 8 hour day for each 100 square yards in the first coat and for each 80 square yards in the second and third coats? (Give result to nearest ¼ day.)

The Circle

The circle is a plane surface bounded by a curved line called the *circumference*. Every point on the latter is equi-distant from a point (*C*) called the *center*. The line *mn* passing through the center of a circle and beginning and terminating in its circumference, is called a *diameter*. Each of the semi-diameters *Co*, *Cm*, and *Cn*, is called a *radius*. A portion

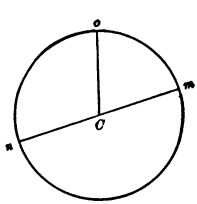


FIG. 21

of the circumference, *om* or *on*, is called an *arc*. A portion, *oCm*, or *oCn*, of the area of a circle, bounded by an arc and two radii is called a *sector*.

The circumference of a circle whose diameter is 1 inch has been found to be 3.1416 inches. This ratio of the circumference to the diameter is expressed by the Greek letter π (pronounced pi).

$$\text{Circumference} = \pi \times \text{Diameter}$$

You can find this ratio approximately by measuring the circumference of a cylindrical tumbler with a tape line and comparing this length with that of the diameter of the tumbler.

Diameter and Circumference

Note: In the following exercises take $3\frac{1}{7}$ as the value of π.

SIGHT EXERCISES

1. Give the diameter of the trunk of a tree when its circumference is 22 inches.

22 in. ÷ $3\frac{1}{7}$

2. A bicycle wheel has a diameter of 28 inches. How far will the bicycle travel during one revolution of the wheel?

3. Give the circumference of a circle whose diameter is $1\frac{3}{4}$ inches.

4. How long will be the circumference of a circle drawn by a compass when the points are $\frac{7}{8}$ inch apart?

5. What is the circumference of the bottom of a tent when its diameter is 7 feet?

6. Give the circumference described by the minute hand of a clock if the hand is 8 feet long.

WRITTEN EXERCISES

1. How many revolutions are made by a bicycle wheel in going a mile (5280 ft.) when the radius of the wheel is 28 inches?

2. What is the diameter of a circle whose circumference is 1 mile?

3. A circular running track is 16½ feet wide, and its interior circumference is ½ mile. Find the length of the circumference of the outer side of the track.

4. Find the difference between the length of the circumference of a circular pond 375 yards in diameter when π is taken as $3\frac{1}{7}$ and when it is taken as 3.1416.

Area of Circle

When a circle (Fig. 22) is divided into a large number of equal parts, and these are arranged as is shown

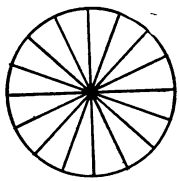


FIG. 22

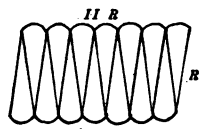


FIG. 23

in Fig. 23, they form a parallelogram whose altitude is R, the radius of the circle, and whose base measures πR, its semi-circumference. The area is, therefore, $\pi R \times R$, or πR^2.

Area of Circle = π × Square of Radius

WRITTEN EXERCISES

1. Taking $3\frac{1}{7}$ as the value of π, find the areas of circles, as follows:

a Diameter, 14 ft. *b* Radius, 21 in. *c* Diameter, 35 yd.

2. (*a*) Find the area in square yards inclosed by a circular running track having a circumference of ½ mile. (*b*) Find the area inclosed by the outer circumference of the track, when the width of the track is 16½ feet. (*c*) Find the area of the track, which is the difference between (*a*) and (*b*).

Rectangular Solids

A solid having six faces, the opposite ones of which are equal and parallel, is called a *parallelopipedon*. When the faces are rectangles, it is called a *right parallelopipedon*. When the faces are equal, it is called a *cube*.

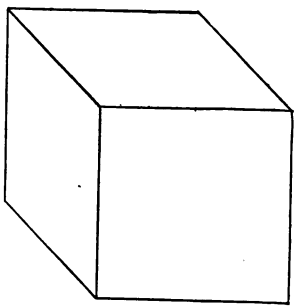


FIG. 24.

The term *solid* is applied to bodies that are hollow. The *volume* of a solid may mean the quantity it will hold.

Volumes

The number of cubic units in the volume of a rectangular solid is equal to the combined product of its three linear units of the corresponding kind.

Volume of Rectangular Solid = Length × Breadth × Height

SIGHT EXERCISES

1. Give the number of cubic units in the volume of each of the following rectangular solids, their dimensions in linear units being

a $12\frac{1}{2} \times 17 \times 8$	*b* $12 \times 19 \times 33\frac{1}{3}$	*c* $16 \times 41 \times 25$
d $66\frac{2}{3} \times 11 \times 6$	*e* $12 \times 37 \times 16\frac{2}{3}$	*f* $12 \times 31 \times 75$

2. How many cubic inches will a canteloupe crate hold, when its dimensions are $12'' \times 12'' \times 22''$? An orange crate measures $12'' \times 12'' \times 24''$; how many cubic feet does it contain?

WRITTEN EXERCISES

1. Find the number of gallons that can be contained in a tank 17 ft. 6 in. long, 12 ft. 3 in. wide, and 4 ft. 7 in. deep.

$$\frac{210 \times 147 \times 55}{231}$$

Express each dimension in inches. Divide by 231 (cu. in.) the contents of a gallon.

2. Give the capacity, in bushels, of a bin measuring $24' \times 16' \times 14'$. Take 2150.4 cubic inches to the bushel.

$$\frac{24 \times 12 \times 16 \times 12 \times 14 \times 12}{2150.4}$$

Multiply each dimension by 12 to express it in inches.

3. How many cubic feet are there in a bale of cotton whose dimensions are $54'' \times 27'' \times 45''$. Give answer to nearest cubic foot.

4. How long a piece of bagging, 54 inches wide, will be required (*a*) to wrap the four sides of the bale? (*b*) To cover the two ends? (*c*) How much bagging

will be saved by compressing the bale to one measuring 54″ x 27″ x 22½″?

5. At 55 pounds to the cubic foot, how many pounds of anthracite coal are there in a bin 6 ft. 8 in. wide, 10 ft. 6 in. long, when the depth of the coal is 4 ft. 6 in.?

The Prism

A solid having two equal and parallel faces and the remaining faces parallelograms is called a *prism*. One of these parallel faces is called the *base* of the prism. When the other faces are rectangles, the prism is said to be a *right* prism. A prism is designated as *triangular*, *quadrilateral*, *hexagonal*, according to the number of sides in the base.

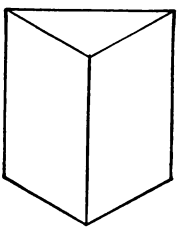


FIG. 25

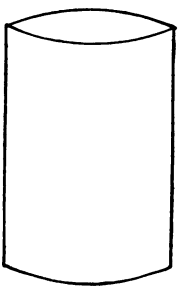


FIG. 26

The Cylinder

The *cylinder* has two parallel circular bases and a curved *lateral surface*. The lateral surface of a prism or a cylinder, that is, the surface exclusive of that of the bases, is also called its *convex surface*.

Lateral Surface of Prism or Cylinder

PREPARATORY EXERCISES

A factory is making a number of hollow prisms and cylinders 20 inches high. For the lateral surface of each a strip of sheet iron is taken, 20 inches wide.

1. Give the length of the strip required for the lateral surface of each of the following regular prisms when the length of each side of the base is 7 inches, making no allowance for overlapping:

a Triangular *b* Square *c* Hexagonal

2. What is the area of the strip in each case?

3. How long must be the strip for a cylinder 14 inches in diameter?

Lateral Surface of Prism (Cylinder) = Perimeter (Circumference) of Base × Height

WRITTEN EXERCISES

1. How many square yards of painting are required to give three coats to the outside of a cylindrical silo 28 feet in diameter and 36 feet high?

2. Find the convex surface of a marble octagonal shaft 6 ft. 8 in. high, when each side of the base measures 4 inches.

VOLUME OF PRISM; OF CYLINDER

Volume of Prism (Cylinder) = Area of Base × Height

WRITTEN EXERCISES

1. Find the volume in cubic feet of a silo 28 feet in diameter and 35 feet high.

2. At 231 cubic inches to the gallon, find the capacity of a standpipe 42 feet high and 14 feet in diameter.

Pyramid and Cone

Surface

The lateral faces of a right pyramid are isosceles triangles, the base of each being a side of the base of the pyramid, and the vertices of the triangles meeting at a common point called the apex. Pyramids are triangular, square, etc. The area of ABC is one-half the product of AB by BC.

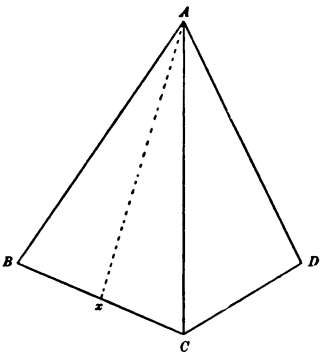


FIG. 27

To make a hollow paper cone take a sector, $HIyJ$ (Fig. 28), bring together the radii HI and HJ, which makes the arc IyJ the circumference of the base of the cone (Fig. 29). The area of the sector is one-half

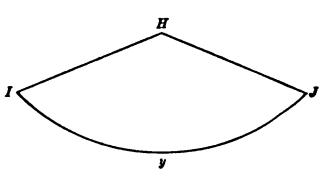


FIG. 28

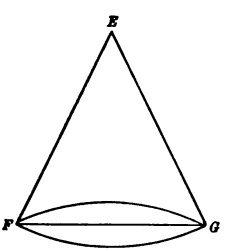


FIG. 29

the product of the arc IyJ by the radius HI. In Fig. 27 the line Ax represents the slant height of the pyramid; in the cone, Fig. 29, any straight line

drawn from the vertex to the circumference of the base is its slant height.

SIGHT EXERCISES

1. Give the area of a lateral face of a square pyramid when a side of its base measures 24 inches and its slant height 99 inches.

NOTE: Remember that the *slant height* of the pyramid is the *altitude* of a triangular face.

2. When the base is a square 50 feet on a side, and the slant height is 47¾ feet, give the surface of the four lateral faces.

3. What is the lateral surface of a cone, the circumference of the base being 49 feet and its slant height 50 feet?

Lateral Surface of Regular Pyramid (Cone) = ½ Perimeter (Circumference) of Base × Slant Height

WRITTEN EXERCISES

1. Find the lateral surface of a triangular pyramid, having a slant height of 37 yards and each side of the base 23 yards.

2. Find the entire surface of a square pyramid (including the base) when its slant height is 16½ feet, and each side of the base is 7 feet 3 inches.

3. Find the lateral surface of a cone having a base 14 feet in diameter and a slant height of 22 feet.

4. Draw a rectangle $3'' \times 1\frac{1}{4}''$. On two adjacent sides construct isosceles triangles having sides of $3\frac{1}{8}''$. Measure the altitude of each triangle.

Take these sides as the scale drawing of two of the lateral faces of a rectangular pyramid having a base 24 inches by 14 inches, with edges measuring 25 inches each. Calculate the slant height of each of these faces.

(*a*) Find the lateral surface of the pyramid. (*b*) Its entire surface.

Volume of Pyramid. Of Cone

By making a hollow pyramid of any height (altitude), and a prism having the same base and altitude, respectively, as the pyramid, it will be found that the prism will contain the contents of the pyramid three times.

Volume of Pyramid (Cone) = ⅓ (Area of Base × Altitude)

WRITTEN EXERCISES

1. Find the volume (*a*) of a square pyramid, each side of the base measuring 23 inches and the altitude 45 inches. (*b*) Of a rectangular pyramid of the same height when the sides of the base are 12 inches and 14 inches, respectively.

2. Find the volume of a cone 15 feet high, with a base 7 feet in diameter.

3. How many bushels of wheat are there in a freight car 40 feet long and 8½ feet high when the depth of the grain is 5¼ feet?

4. How many square yards of canvas are required for a conical tent 14 feet in diameter at the base and having a slant height of 12 feet?

5. (*a*) How many square feet of boards will be required to inclose a rectangular plot 174 yards long and 126 yards wide with a rectangular fence 6 feet high? (*b*) How many boards 12 feet long 8 inches wide will be needed? (*c*) How many board feet, if the boards are 1 inch thick?

6. (*a*) How many square feet are there in the foregoing plot? (*b*) A walk 4 feet wide is made inside the fence; what is the area of the plot inside the walk? (*c*) How many square feet does the walk contain? (*d*) How many square feet are there in a 4-foot walk along the fence on the outside?

7. (*a*) How many square yards are there in the space covered by a wall 3 feet wide inclosing a cellar 24 feet wide and 48 feet long? (*b*) How many cubic yards of material are there in the wall, if the latter is 9 feet high? (*c*) What is the outside perimeter of the wall? (*d*) The inner perimeter? (*e*) The average of the two?

8. How many cubic feet of material are there in a sewer pipe 4 feet long whose inner diameter is 13 inches and the outer diameter 15 inches?

9. In framing a diploma that measures $18'' \times 12''$, a girl uses a cardboard "mat" that covers one inch of each side of the diploma and shows 3 inches. One inch of each side of the mat is covered by the frame, which is 2 inches wide. Find (*a*) the outer dimensions of the framed picture; (*b*) the dimensions of the mat; (*c*) the area of the opening; and (*d*) the number of running feet of frame needed, making allowance for the waste at the corners.

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